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Objective Mathematics

{Mains & Advance}

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He is actively involved in the field of online teaching to the engineering aspirants and is associated with *iProf Learning Solutions India (P) Ltd* for last 3 years. As a premium member of www.wiziq.com (an online teaching and learning portal), he has delivered many online lectures on different topics of mathematics at IIT-JEE and AIEEE level.{some of the free online public classes at wizIQ can be accessed at http://www.wiziq.com/LKS }.

Since last 2 years many engineering aspirants have got tremendous help with the blog "*mailtolks.blogspot.com*" and with launch of the site "mathematicsgyan.weebly.com", engineering aspirants get the golden opportunity to access the best study/practice material in mathematics at school level and IIT-JEE/AIEEE/BITSAT level. The best part of the site is availability of e-book of "OBJECTIVE MATHEMATICS for JEE- 2013" authored by Er. L.K.Sharma, complete book with detailed solutions is available for free download as the PDF files of different chapters of JEE-mathematics.

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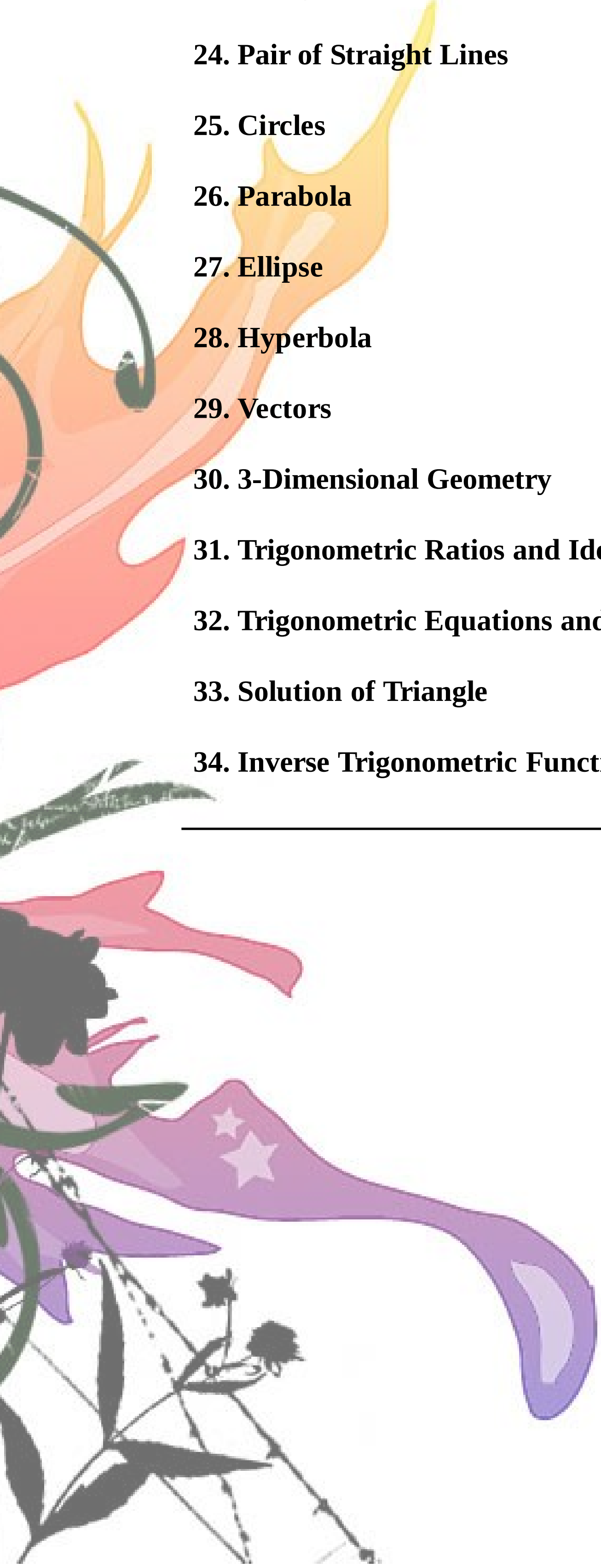
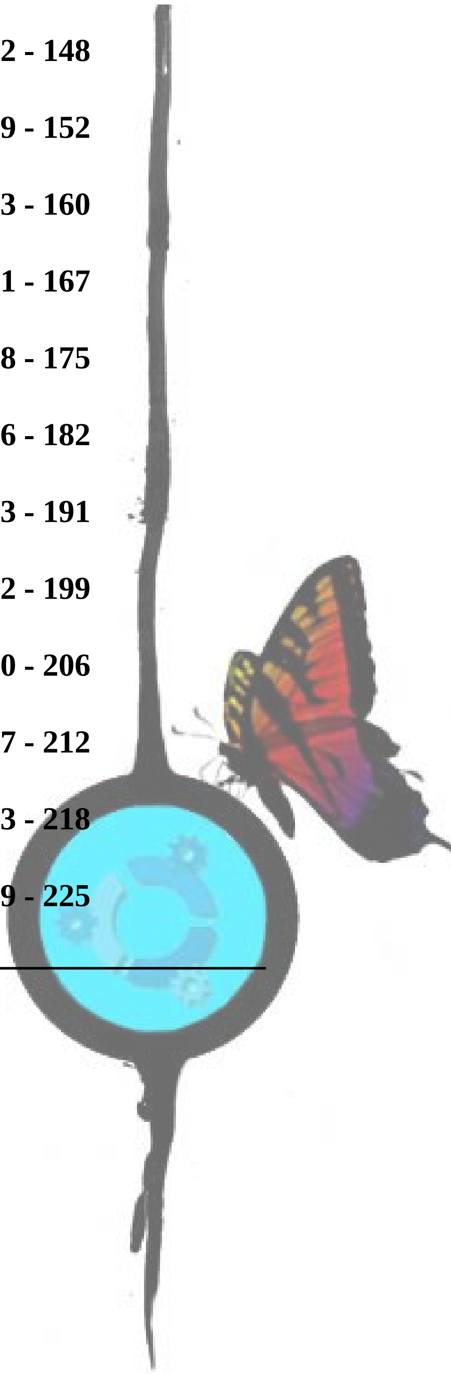




Chapter 1

Quadratic Equations

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-25)

1. If the equation $|x - n| = (x+2)^2$ is having exactly three distinct real solutions , then exhaustive set of values of $'n'$ is given by :

(a) $\left[-\frac{5}{2}, -\frac{3}{2}\right]$ (b) $\left\{-\frac{5}{2}, -2, -\frac{3}{2}\right\}$

(c) $\left\{-\frac{5}{2}, -\frac{3}{2}\right\}$ (d) $\left\{-\frac{9}{4}, -2, -\frac{7}{4}\right\}$

2. Let a , b , c be distinct real numbers , then roots of $(x-a)(x-b) = a^2 + b^2 + c^2 - ab - bc - ac$, are :

(a) real and equal (b) imaginary

(c) real and unequal (d) real

3. If $2x^3 - 12x^2 + 3\lambda x - 16 = 0$ is having three positive real roots , then $'\lambda'$ must be :

(a) 4 (b) 8

(c) 0 (d) 2

4. If a , b , c are distinct real numbers , then number of real roots of equation

$$\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} = 1$$

is/are :

(a) 1 (b) 4

(c) finitely many (d) infinitely many

5. If $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ have a common root and $\frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1}$ are in A.P. , then a_1 , b_1 , c_1 are in :

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

6. If all the roots of equations

$$(a-1)(1+x+x^2)^2 = (a+1)(x^4+x^2+1)$$

are imaginary , then range of $'a'$ is :

(a) $(-\infty , -2]$ (b) $(2 , \infty)$

(c) $(-2 , 2)$ (d) $(-2 , \infty)$

7. Total number of integral solutions of inequation

$$\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(7-2x)^6} \leq 0 \text{ is/are :}$$

(a) four (b) three

(c) two (d) only one

8. If exactly one root of $5x^2 + (a+1)x + a = 0$ lies in the interval $x \in (1 , 3)$, then

(a) $a > 2$

(b) $-12 < a < -3$

(c) $a > 0$

(d) none of these

9. If both roots of $4x^2 - 20px + (25p^2 + 15p - 66) = 0$ are less than 2 , then $'p'$ lies in :

(a) $\left(\frac{4}{5} , 2\right)$ (b) $(2 , \infty)$

(c) $\left(-1 , \frac{4}{5}\right)$ (d) $(-\infty , -1)$

10. If $x^2 - 2ax + a^2 + a - 3 \geq 0 \ \forall \ x \in R$, then $'a'$ lies in

(a) $[3 , \infty)$ (b) $(-\infty , 3]$

(c) $[-3 , \infty)$ (d) $(-\infty , -3]$

11. If $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$ have a common root , then value of $'a'$ is

(a) 2 (b) –2

(c) 0 (d) 1

12. If $x^2 + px + 1$ is a factor of $ax^3 + bx + c$, then

(a) $a^2 + c^2 + ab = 0$

(b) $a^2 - c^2 + ab = 0$

(c) $a^2 - c^2 - ab = 0$

(d) $a^2 + c^2 - ab = 0$

13. If expression $a^2(b^2-c^2)x^2 + b^2(c^2-a^2)x + c^2(a^2-b^2)$ is a perfect square of one degree polynomial of x , then a^2 , b^2 , c^2 are in :

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

14. The value of α for which the quadratic equation

$$x^2 - (\sin\alpha - 2)\,x - (1 + \sin\alpha) = 0$$

has roots whose sum of squares is least , is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

15. If $\cos\theta$, $\sin\phi$, $\sin\theta$ are in G.P. , then roots of $x^2 + 2(\cot\phi)x + 1 = 0$ are :

(a) equal (b) real

(c) imaginary (d) greater than 1

16. If $-3 < \frac{x^2 + ax - 2}{x^2 + x + 1} < 2$ holds $\forall\ x \in R$, then $'a'$ belongs to :

(a) $[-2 , 1)$ (b) $(-2 , 1)$

(c) $R - [-2 , 2]$ (d) $(-2 , 2)$

17. The number of real solutions of the equation $\sqrt{2x + \sqrt{2x + 4}} = 4$ is/are :

(a) 0 (b) 1

(c) 2 (d) 4

18. Let α , β be the roots of quadratic equation $ax^2 + bx + c = 0$, then roots of the equation $ax^2 - bx\,(x - 1) + c(x - 1)^2 = 0$ are :

(a) $\frac{\alpha}{1-\alpha} , \frac{\beta}{1-\beta}$ (b) $\frac{\alpha}{\alpha+1} , \frac{\beta}{\beta+1}$

(c) $\frac{1-\alpha}{\alpha} , \frac{1-\beta}{\beta}$ (d) $\frac{1+\alpha}{\alpha} , \frac{1+\beta}{\beta}$

19. If the equation $x^5 - 10a^3x^2 + b^4x + c^5 = 0$ has 3 equal roots , then :

(a) $b^4 = 5a^3$ (b) $2c^5 + a^2b^3 - 5 = 0$

(c) $c^5 + 6a^5 = 0$ (d) $2b^2 - 5a^3c = 0$

20. If a , b and c are not all equal and α , β are the roots of $ax^2 + bx + c = 0$, then value of $(1 + \alpha + \alpha^2)(1 + \beta + \beta^2)$ is :

(a) zero (b) positive

(c) negative (d) non-negative

21. The equation $(x)^{\frac{3}{4}(\log_2 x)^2 + (\log_2 x) - \frac{5}{4}} = \sqrt{2}$ has :

(a) exactly two real roots (b) no real root

(c) one irrational root (d) three rational roots

22. If real polynomial $f(x)$ leaves remainder 15 and $(2x + 1)$ when divided by $(x - 3)$ and $(x - 1)^2$ respectively , then remainder when $f(x)$ is divided by $(x - 3)(x - 1)^2$ is :

(a) $2x - 1$ (b) $3x^2 + 2x - 4$

(c) $2x^2 - 2x + 3$ (d) $3x + 6$

23. Let $a \in R^+$ and equation $3x^2 + ax + 3 = 0$ is having one of the root as square of the another root , then $'a'$ is equal to :

(a) 2/3 (b) –3

(c) 3 (d) 1/3

24. If the quadratic equation

$$a^2(x + 1)^2 + b^2(2x^2 - x + 1) - 5x^2 - 3 = 0$$

is satisfied for all $x \in R$, then number of ordered pairs (a , b) which are possible is/are :

(a) 0 (b) 1

(c) finitely many (d) infinitely many

25. The smallest value of $'k'$ for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real and distinct and have values at least 4 , is :

(a) 1 (b) 2

(c) –1 (d) 3

26. Let $f(x) = (x - 3k)(x - k - 3)$ be negative for all $x \in [1 , 3]$, where $k \in R$, then complete set of values of $'k'$ belong to :

(a) $\left(-\frac{1}{2} , \frac{1}{2}\right)$ (b) $\left(0 , \frac{1}{3}\right)$

(c) $\left(\frac{1}{3} , 3\right)$ (d) $(-3 , 0)$

27. Let $A = \{y : 4 \le y < 150 , y \in N\}$ and $\alpha \in A$, then total number of values of $'\alpha'$ for which the equation $x^2 - 3x - \alpha = 0$ is having integral roots , is equal to :

(a) 8 (b) 12

(c) 9 (d) 10

28. Let $\alpha , \beta , \gamma \in R^+$ and $(ln\ 3)^{\alpha}$, $(ln\ 3)^{\beta}$, $(ln\ 3)^{\gamma}$ form a geometric sequence. If the quadratic equation $\alpha x^2 + \beta x + \gamma = 0$ has real roots , then absolute value of $\left\{\sqrt{\frac{\alpha}{\gamma}} - \sqrt{\frac{\gamma}{\alpha}}\right\}$ is not less than :

(a) 4 (b) $2\sqrt{3}$

(c) $3\sqrt{2}$ (d) $2\sqrt{2}$

29. Let $a\,,b\,,c \in R$ and $f(x) = ax^2 + bx + c$, where the equation $f(x) = 0$ has no real root. If $y + k = 0$ is tangent to the curve $y = f(x)$, where $k \in R^+$, then :

(a) $a - b + c > 0$ (b) $c = 0$

(c) $4a - 2b + c \geq 0$ (d) $a - 2b + 4c < 0$

30. Let $a\,,b\,,c$ be the sides of a scalene triangle and $\lambda \in R.$ If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ac) = 0$ are real , then :

(a) maximum positive integral value of λ is 2

(b) minimum positive integral value of λ is 2

(c) values of λ lies in $\left[-\frac{2}{3}\,,\frac{2}{3}\right]$

(d) $\lambda \in (-\infty\,,4/3)$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 31-35)

31. Let $|a| < |b|$ and $a\,,b$ are the real roots of equation $x^2 - |\alpha|x - |\beta| = 0.$ If $1 + |\alpha| < b$, then the equation

$log_{|a|}\left(\frac{x}{b}\right)^2 = 1$ has

(a) one root in $(-\infty\,,a)$ (b) one root in $(b\,,\infty)$

(c) one root in $(a\,,b)$ (d) no root in $(a\,,b)$

32. Let $p\,,q \in Q$ and $\cos^2\frac{\pi}{8}$ be a root of the equation $x^2 + px + q^2 = 0$, then :

(a) $\left[|\sin\theta| + |\cos\theta|\right] + p = 0$ for all $\theta \in R$, where [.] represents the greatest integer function.

(b) Value of $log_2|q| = -\frac{3}{2}$

(c) $8q^2 - 4p = 0$

(d) $\left[|\sin\theta| + |\cos\theta|\right] + 2p = 0$ for all $\theta \in R$, where [.] represents the greatest integer function.

33. Let $S = \left\{\alpha : \alpha^2 - 5\alpha - 6 \leq 0\,,\alpha \in R\right\}$ and $a\,,b \in S.$

If the equation $x^2 + 7 = 4x - 3\sin(ax+b)$ is satisfied for at least one real value of x , then

(a) minimum possible value of $2a + b$ is $-\pi/2$

(b) maximum possible value of $2a + b$ is $7\pi/2$

(c) minimum possible value of $2a + b$ is $\pi/2$

(d) maximum possible value of $2a + b$ is $11\pi/2$

34. If all the four roots of the bi-quadratic equation $x^4 - 12x^3 + \alpha x^2 + \beta x + 81 = 0$ are positive in nature , then :

(a) value of α is 45

(b) value of β is 108

(c) value of $2\alpha + \beta = 0$

(d) value of $\frac{\beta}{\alpha} log_{0.5}5 = log_2 25$

35. Let $\alpha\,,\beta$ be the real roots of the quadratic equation $x^2 - ax - b = 0$, where $a\,,b \in R.$

If $A = \left\{x : x^2 - 4 < 0\,;\,x \in R\right\}$ and $\alpha\,,\beta \in A$, then which of the following statements are incorrect :

(a) $|a| > 2 - \frac{b}{2}$

(b) $|a| < 2 - \frac{b}{2}$

(c) $|a| < 4$

(d) $|a|^2 + 4b > 0$

Assertion Reasoning questions : (Questions No. 36-40)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

36. Let $a\,,b\,,c \in R\,,a \neq 0\,,f(x) = ax^2 + bx + c$, where $\Delta = b^2 - 4ac.$ If $f(x) = 0$ has $\alpha\,,\beta$ as two real and distinct roots and $f(x+k) + \lambda f(x) = 0\,,\lambda\,,k \in R$, has exactly one real root between α and β , then

Statement 1 : $0 < |ak| < \sqrt{\Delta}$

because

Statement 2 : the values of *'k'* don't depend upon the values of $'\lambda'$.

37. **Statement 1 :** If $a, b, c \in R$, then at least one of the following equations ..... (1), (2), (3) has a real solution

$x^2 + (a - b)x + (b - c) = 0$ .... (1)

$x^2 + (b - c)x + (c - a) = 0$ .... (2)

$x^2 + (c - a)x + (a - b) = 0$ .... (3)

because

Statement 2 : The necessary and sufficient condition for at least one of the three quadratic equations, with discriminant $\Delta_1, \Delta_2, \Delta_3$, to have real roots is $\Delta_1 \Delta_2 \Delta_3 \geq 0$.

38. **Statement 1 :** If the equation

$$x^2 + \lambda x - \frac{\pi}{2} + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$$

is having real solution, then value of 'λ' must be $2 \log_{\frac{1}{2}} 8$

because

Statement 2 : $\sin^{-1}(x) + \cos^{-1}(x) - \frac{\pi}{2} = 0$ for all $x \in [-1, 1]$.

39. **Statement 1 :** If equation $x^2 - (\lambda + 1)x + \lambda - 1 = 0$ is having integral roots, then there exists only one integral value of 'λ'

because

Statement 2 : $x = 2$ is the only integral solution of the equation $x^2 - (\lambda + 1)x + \lambda - 1 = 0$, if $\lambda \in I$.

40. Let $f(x) = ax^2 + bx + c$, $a, b, c \in R$ and $a \neq 0$.

Statement 1 : If $f(x) = 0$ has distinct real roots, then the equation $(f'(x))^2 - f(x).f''(x) = 0$ can never have real roots

because

Statement 2 : If $f(x) = 0$ has non-real roots, then they occur in conjugate pairs.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $a , b \in R-\{0\}$ and α , β , γ be the roots of the equation $x^3+ax^2+bx-b=0$. If $\frac{2}{\beta}=\frac{1}{\alpha}+\frac{1}{\gamma}$, then answer the following questions.

1. The value of $2b + 9a + 30$ is equal to :

(a) 2 (b) – 5

(c) 3 (d) –2

2. The minimum value of $\frac{(\alpha\beta)^2+(\beta\gamma)^2+(\alpha\gamma)^2}{(\alpha\beta\gamma)^2}$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{9}$

(c) $\frac{1}{8}$ (d) $\frac{1}{3}$

3. The minimum value of $\frac{a+b}{b}$ is equal to :

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$

(c) $\frac{1}{3}$ (d) $\frac{3}{8}$

Comprehension passage (2)
(Questions No. 4-6)

Let α , β be the roots of equation $x^2+ax+b=0$, and γ , δ be the roots of equation $x^2+a_1x+b_1=0$ .If $S=\left\{x : x^2+a_1x+b_1=0 , x\in R\right\}$ and $f : R-S \to R$ is a function which is defined as $f(x)=\frac{x^2+ax+b}{x^2+a_1x+b_1}$, then answer the following question.

4. If $\alpha , \beta , \gamma , \delta \in R$ and $\beta > \delta > \alpha > \gamma$, then

(a) $f(x)$ is increasing in (γ , δ)

(b) $f(x)$ is increasing in (α , β)

(c) $f(x)$ is decreasing in (δ , β)

(d) $f(x)$ is increasing in $(-\infty , \alpha)$

5. If $\alpha , \beta , \gamma , \delta \in R$ and $\gamma < \delta < \alpha < \beta$, then :

(a) $f'(x) > 0 \;\forall\; x \in R-\{\gamma , \delta\}$.

(b) $f(x)$ has local maxima in (γ , δ) and local minima in (α , β).

(c) $f(x)$ has local minima in (γ , δ) and local maxima in (α , β).

(d) $f'(x) < 0 \;\forall\; x \in R-\{\gamma , \delta\}$

6. If $\alpha , \beta , \gamma , \delta$ are the non-real values and $f(x)$ is defined $\forall\; x \in R$, then :

(a) $f'(x) = 0$ has real and distinct roots.

(b) $f'(x) = 0$ has real and equal roots.

(c) $f'(x) = 0$ has imaginary roots.

(d) nothing can be concluded in general for $f'(x)$.

Comprehension passage (3)
(Questions No. 7-9)

Consider the function

$f(x) = (1 + m)x^2 - 2(3m + 1)x + (8m + 1)$,

where $m \in R-\{-1\}$

7. If $f(x) > 0$ holds true $\forall\; x \in R$, then set of values of $'m'$ is :

(a) (0 , 3) (b) (2 , 3)

(c) (–1 , 3) (d) (–1 , 0)

8. The set of values of $'m'$ for which $f(x) = 0$ has at least one negative root is :

(a) $(-\infty , -1)$ (b) $\left(-\frac{1}{8} , \infty\right)$

(c) $\left(-1 , -\frac{1}{8}\right)$ (d) $\left(-\frac{1}{8} , 3\right)$

9. The number of real values of $'m'$ such that $f(x) = 0$ has roots which are in the ratio $2 : 3$ is /are :

(a) 0 (b) 2

(c) 4 (d) 1

Questions with Integral Answer :
(Questions No. 10-15)

10. Let α , β be the roots of the quadratic equation $m^2(x^2-x)+2mx+3=0$, where $m \neq 0$ & m_1 , m_2 are two values of m for which $\left\{\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right\}$ is equal to $\frac{4}{3}$.

If $P=\frac{m_1^2}{m_2}+\frac{m_2^2}{m_1}$, then value of $\left\{-\frac{3P}{17}\right\}$ is equal to ....

11. Let a , b , c , d be distinct real numbers , where the roots of $x^2 - 10\,cx - 11d = 0$ are a and b. If the roots of $x^2 - 10ax - 11b = 0$ are c and d , then value of $\dfrac{1}{605}(a+b+c+d)$ is ....

12. If a , b are complex numbers and one of the roots of the equation $x^2 + ax + b = 0$ is purely real where as the other is purely imaginary , then value of $\left\{\dfrac{a^2-(\bar{a})^2}{2b}\right\}$ is equal to ....

13. If the equation $x^4 - (a+1)\,x^3 + x^2 + (a+1)\,x - 2 = 0$ is having at least two distinct positive real roots , then the minimum integral value of parameter $'a'$ is equal to ....

14. If the equations $ax^3 + 2bx^2 + 3cx + 4d = 0$ and $ax^2 + bx + c = 0$ have a non-zero common root , then the minimum value of $(\,c^2 - 2bd\,)(\,b^2 - 2ac\,)$ is equal to ....

15. If $n \in I$ and the roots of quadratic equation $x^2 + 2nx - 19n - 92 = 0$ are rational in nature , then minimum possible value of $|\,n\,|$ is equal to ....

Matrix Matching Questions :
(Questions No. 16-18)

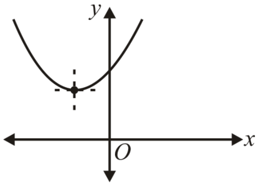
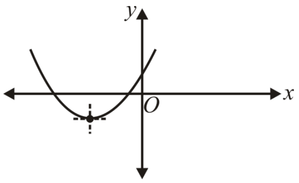
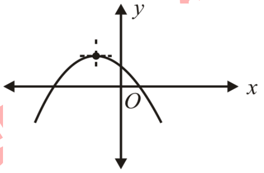
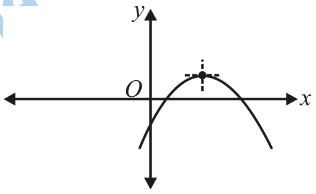
16. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) If roots of $x^2 - bx + c = 0$ are two consecutive integers , then $(b^2 - 4c)$ is	(p) -2
(b) If $x \in [2\,,\,4]$, then least value of the expression $(x^2 - 6x + 7)$ is :	(q) 0
(c) Number of solutions of equation $\big\|\,\|x^2-1\|-3\big\| = 4$ is /are	(r) 2
(d) Minimum value of $f(x) = \|2x-4\| + \|6-4x\|$ is :	(s) 1

17. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) If $(\lambda^2+\lambda-2)x^2+(\lambda+2)x<1 \ \forall \ x \in R$, then λ belongs to the interval	(p) $(0\ ,\ 4)$
(b) If sum and product of the quadratic equation $x^2-(\lambda^2-5\lambda+5)x+(2\lambda^2-3\lambda-4)=0$ are both less than one , then set of possible values of λ is	(q) $\left(-2\,,\,\dfrac{2}{5}\right)$ (r) $\left(1\,,\,\dfrac{5}{2}\right)$
(c) If $5^x + (2\sqrt{3})^{2x} - 169$ is always positive then set of x is	
(d) If roots of equation $2x^2-(a^2+8a+1)x+a^2-4a=0$ are opposite in sign , then set of values of a is	(s) $(2\,,\,\infty)$

18. Let $f(x) = ax^2 + bx + c$, $a \neq 0$, a , b , $c \in R$. If column (I) represents the conditions on a , b , c and column (II) corresponds to the graph of $f(x)$, where $D = (b^2 - 4ac)$, then match columns (I) and (II).

Column (I)	**Column (II)**
(a) a , b , $c \in R^+$ and $D > 0$	(p) 
(b) a , $c \in R^-$ and $b \in R^+$, $D > O$	(q) 
(c) a , b , $c \in R$ and $D > O$	(r) 
(d) a , $b \in R^+$, $c \in R^+$ and $D < 0$	(s) 

ANSWERS

Exercise No. (1)

1. (d)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (b)
6. (c)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (a)
11. (b)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (b)
16. (b)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (b)
21. (c)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (b)
26. (b)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (d)
31. (a , b , d)	**32.** (a , b)	**33.** (a , d)	**34.** (c , d)	**35.** (b , c , d)
36. (b)	**37.** (c)	**38.** (d)	**39.** (c)	**40.** (b)

ANSWERS

Exercise No. (2)

1. (c)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (b)
6. (a)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (4)
11. (2)	**12.** (2)	**13.** (2)	**14.** (0)	**15.** (8)
16. (a) $\to$ s (b) $\to$ p (c) $\to$ r (d) $\to$ s	**17.** (a) $\to$ q (b) $\to$ r (c) $\to$ s (d) $\to$ p	**18.** (a) $\to$ q (b) $\to$ s (c) $\to$ q , r , s (d) $\to$ p		

Chapter 2

Sequences and Series

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-25)

1. If sum of $'n'$ terms of a sequence is given by

$$S_n = \sum_{r=1}^{n} T_r = n(n+1)(n+2) \text{ , then } \sum_{r=1}^{12} \frac{1}{T_r} \text{ is equal to :}$$

(a) $\frac{4}{13}$ (b) $\frac{2}{13}$

(c) $\frac{5}{67}$ (d) $\frac{4}{39}$

2. Let a, b, c be distinct non-zero real numbers such that a^2, b^2, c^2 are in harmonic progression and a, b, c are in arithmetic progression, then :

(a) $2b^2 + ac = 0$ (b) $4b^2 + ac = 0$

(c) $2b^2 - ac = 0$ (d) $4b^2 - ac = 0$

3. Let a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. , if $a < b < c$ and $a + b + c = 3/2$, then value of $'a'$ is :

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$

(c) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (d) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

4. If $a, b, c \in R^+$, then maximum value of

$$\left\{ \frac{bc}{b+c} + \frac{ac}{a+c} + \frac{ab}{a+b} \right\} \text{ is}$$

(a) $\frac{1}{2}(a+b+c)$ (b) $\frac{1}{3}\sqrt{abc}$

(c) $\frac{1}{3}(a+b+c)$ (d) $\frac{1}{2}\sqrt{abc}$

5. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is :

(a) $\frac{n(4n^2-1)c^2}{6}$ (b) $\frac{n(4n^2+1)c^2}{3}$

(c) $\frac{n(4n^2-1)c^2}{3}$ (d) $\frac{n(4n^2+1)c^2}{6}$

6. Let $\alpha \in R^+ - \{1\}$ and $(ln\alpha)^p, (ln\alpha)^q, (ln\alpha)^r, (ln\alpha)^s$ be in G.P. , then pqr, pqs, prs, qrs are in :

(a) A.P. (b) G.P.

(c) H.P. (d) A.G.P.

7. Let $T_1 = \frac{1}{2}$, $T_{r+1} = T_r + T_r^{\,2} \;\; \forall \;\; r \in N$ and

$$S_n = \frac{1}{1+T_1} + \frac{1}{1+T_2} + \frac{1}{1+T_3} + \dots + \frac{1}{T_n+1} \text{ , then}$$

(a) $S_{100} \geq 4$ (b) $S_{100} > 2$

(c) $1 < S_{100} < 2$ (d) $0 < S_{100} < 1$

8. Let $S_n = \sum_{r=1}^{n} r^4$, then $\sum_{r=1}^{n} (2r-1)^4$ is given by :

(a) $S_{2n} - 8S_n$ (b) $S_{4n} - 24S_{2n}$

(c) $S_{2n} - 16S_n$ (d) $S_{4n} - 16S_n$

9. Let $\{x_n\}$ represents G.P. with common ratio $'r'$ such that $\sum_{k=1}^{n} x_{2k-1} = \sum_{k=1}^{n} x_{2k+2} \neq 0$, then number of possible values for $'r'$ is/are :

(a) 1 (b) 2

(c) 3 (d) 4

10. Let x, y be non-zero real numbers and the expression $x^{12} + y^{12} - 48x^4y^4$ is not less than $'k'$, then value of $'k'$ is equal to :

(a) -2^{12} (b) 2^{12}

(c) 2^8 (d) -2^8

11. Let 10 A.M.'s and 10 H.M.'s be inserted in between 2 and 3. If $'A'$ be any A.M. and $'H'$ be the corresponding H.M. , then $H(5-A)$ is equal to :

(a) 6 (b) 10

(c) 11 (d) 8

12. Let $a, b, c \in R^+$ and the inequality $bx^2 + (\sqrt{(a+c)^2 + 4b^2})x + (a+c) \geq 0$ holds true for all real value of $'x'$, then $e^{a+1}, e^{b+1}, e^{c+1}$ are in :

(a) A.P. (b) G.P.

(c) H.P. (d) none of these.

13. Let $'A_n'$ denotes the sum of n terms of an A.P. and $A_{2n} = 3A_n$, then $\dfrac{A_{3n}}{A_n}$ is equal to :

(a) 4 (b) 6

(c) 8 (d) 10

14. If $a \neq 0$, roots of equation $ax^3 + bx^2 + cx + d = 0$ are in G . P. , then :

(a) $ac^3 = db^3$ (b) $a^3c = d^3b$

(c) $a^3b = c^3d$ (d) $ab^3 = cd^3$

15. Let a , b , c be non-zero real numbers and $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ac)$, then a , b , c are in :

(a) A.P. (b) G.P.

(c) H.P. (d) A.G.P.

16. In a set of four numbers , if first three terms are in G.P. and the last three terms are in A.P. with common difference 6 , then sum of the four numbers , when the first and the last terms are equal , is given by :

(a) 20 (b) 14

(c) 16 (d) 18

17. Let the real numbers α, β, γ be in A.P. and satisfy the equation $x^2(x-1) + px + q = 0$, then :

(a) $p \in \left[\dfrac{1}{3}, 3\right]$ (b) $q \in \left[-\dfrac{1}{27}, \infty\right)$

(c) $p \in \left[\dfrac{1}{3}, \infty\right)$ (d) $q \in \left(-\infty, \dfrac{1}{27}\right]$

18. In ΔABC, if all the sides are in A.P. , then the corresponding ex-radii are in :

(a) A.P. (b) G.P.

(c) H.P. (d) none of these.

19. Let $S = \displaystyle\sum_{r=1}^{n} \frac{8r}{4r^4 + 1}$, then $\lim\limits_{n \to \infty} (S)$ is equal to :

(a) 4 (b) 2

(c) 1 (d) 0

20. In a sequence of $(4n + 1)$ terms , the first $(2n + 1)$ terms are in A.P. whose common difference is 2 , and the last $(2n + 1)$ terms are in G.P. whose common ratio is 1/2. If the middle terms of the A.P. and G.P. are equal , then the middle term of sequence is :

(a) $\dfrac{n.2^n}{2^n + 1}$. (b) $\dfrac{(n+1)2^n}{2^n + 1}$.

(c) $\dfrac{n.2^{n+1}}{2^n - 1}$. (d) $\dfrac{(n+1)2^{n+1}}{2^n - 2}$.

21. Let $a_1, a_2, a_3, \ldots\ldots, a_{50}$ be 50 distinct numbers in A.P. , and $\displaystyle\sum_{r=1}^{50} (-1)^{r+1}(a_r)^2 = \left(\sqrt{\frac{5}{7}}\right)^{n/2}(a_1^2 - a_{50}^2)$, where $n \in N$, then value of n is equal to :

(a) 4 (b) 2

(c) 8 (d) 10

22. Let three numbers be removed from the geometric sequence $\{a_n\}$ and the geometric mean of the remaining terms is $\sqrt[5]{2^{37}}$. If $a_n = \left(1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \ldots\ldots\ldots + \infty\right)^n$, then value of $'n'$ can be :

(a) 10 (b) 8

(c) 20 (d) 13

23. Let $x, y \in R^+$ and $x^2y^3 = 6$, then the least value of $3x + 4y$ is equal to :

(a) 12 (b) 10

(c) 8 (d) 20

24. Let $S_n = 1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \ldots..$ n terms and $S = \lim\limits_{n \to \infty}(S_n)$, if $S - S_n < \dfrac{1}{1000}$, then least value of $'n'$ is :

(a) 11 (b) 10 (c) 12 (d) 6

25. Let the sides of a triangle be in arithmetic progression. If the greatest angle of triangle is double the smallest angle , then the cosine value of the smallest angle is equal to :

(a) $\dfrac{3}{8}$ (b) $\dfrac{3}{4}$

(c) $\dfrac{4}{5}$ (d) $\dfrac{1}{4}$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. If $a\ ,\ b \in R^+$, where $a\ ,\ A_1\ ,\ A_2\ ,\ b$ are in arithmetic progression , $a\ ,\ G_1\ ,\ G_2\ ,\ b$ are in geometric progression and $a\ ,\ H_1\ ,\ H_2\ ,\ b$ are in harmonic progression , then which of the following relations are correct ?

(a) $G_1G_2(G_1+G_2)=\dfrac{A_1+A_2}{H_1+H_2}$

(b) $\dfrac{H_1H_2}{G_1G_2}=\dfrac{H_1+H_2}{A_1+A_2}$

(c) $\dfrac{G_1G_2}{H_1H_2}=\dfrac{(2a+b)(2b+a)}{9ab}$

(d) $\dfrac{A_1+A_2}{H_1+H_2}=\dfrac{(2a-b)(2b-a)}{9ab}$

27. Let four consecutive integers form an increasing arithmetic progression and one of these numbers is equal to the sum of the squares of the other three numbers , then :

(a) the smallest number is 0.

(b) the largest number is 2.

(c) sum of all the four numbers is 2.

(d) product of all the four numbers is 0.

28. For two distinct positive numbers , let A_1 , G_1 , H_1 denote the *AM* , *GM* and *HM* respectively. For $n \geq 2\ ,\ n \in N$, if A_{n-1} and H_{n-1} has arithmetic , geometric and harmonic means as $A_n\ ,\ G_n\ ,\ H_n$ respectively , then :

(a) $A_1 > A_2 > A_3 > A_4 > \ldots\ldots\ldots$

(b) $G_1 < G_2 < G_3 > G_4 < \ldots\ldots\ldots$

(c) $H_1 > H_2 > H_3 > H_4 > \ldots\ldots\ldots$

(d) $G_1 = G_2 = G_3 = G_4 = \ldots\ldots\ldots$

29. Let $\{a_n\}$ represents the arithmetic sequence for which $a_1 = |x|\ ,\ a_2 = |\ x-1\ |$ and $a_3 = |x+1|$, then :

(a) $a_n - a_{n-1} = \dfrac{1}{2}$ (b) $a_1 = 2$

(c) $\displaystyle\sum_{n=1}^{10} a_n = 25$ (d) $a_n - a_{n-1} = \dfrac{1}{4}$

30. Let $a_n = \dfrac{3}{4} - \left(\dfrac{3}{4}\right)^2 + \left(\dfrac{3}{4}\right)^3 \ldots\ldots\ldots\ldots (-1)^{n-1}\left(\dfrac{3}{4}\right)^n$ and $b_n + a_n = 1$. If $b_n > a_n$ for all $n > n_0$, where $n \in N$, then possible values of natural number $'n_0'$ can be :

(a) 4 (b) 6

(c) 8 (d) 2

Assertion Reasoning questions : (Questions No. 31-35)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

31. **Statement 1 :** Let three positive numbers in geometric progression represent the sides of a triangle , then the common ratio of the G.P. can be $\dfrac{1}{2}\sin\left(\dfrac{\pi}{5}\right)$

because

Statement 2 : the common ratio of the G.P. in consideration lies in between $\dfrac{1}{2}\sin\left(\dfrac{\pi}{10}\right)$ and $\dfrac{1}{2}\sin\left(\dfrac{3\pi}{10}\right)$.

32. **Statement 1 :** In a triangle *ABC* , if $\cot A$, $\cot B$, $\cot C$ forms an *A.P.* , then $\dfrac{1}{b+a}\ ,\ \dfrac{1}{c+b}\ ,\ \dfrac{1}{a+c}$ also form an *A.P.*

because

Statement 2 : $\dfrac{1}{a^2}\ ,\ \dfrac{1}{b^2}\ ,\ \dfrac{1}{c^2}$ form a *H.P.*

33. **Statement 1 :** If [.] and {.} denote the greatest integer function and the fractional part , then x , $[x]$, $\{x\}$ can never form a geometric progression for any positive rational value of x

because

Statement 2 : $x\ ,\ [x]\ ,\ \{x\}$ can form a G.P. for $x \in R^+$, only if $x = \dfrac{1}{2}\sin\left(\dfrac{7\pi}{10}\right)$.

34. **Statement 1 :** If $a, b, c \in R^+$, then the minimum value of $\{a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)\}$ is equal to $6abc$

because

Statement 2 : for $a_1, a_2, a_3, a_4, \ldots\ldots\ldots\ldots a_n \in R^+$, $(AM)(HM)=(GM)^2 \quad \forall \quad n \in N-\{1\}$

35. **Statement 1 :** Let $S_n = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots\ldots\ldots+\frac{1}{n}$, $n \in N$, then $S_n > ln(n+1)$

because

Statement 2 : $ln\,(n+1) > ln\,(n) \quad \forall \quad n \in N$

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r-1)$.

Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \ldots$

1. The sum $V_1 + V_2 + \ldots + V_n$ is :

(a) $\frac{1}{12}n(n+1)(3n^2-n+1)$

(b) $\frac{1}{12}n(n+1)(3n^2+n+2)$

(c) $\frac{1}{2}n(2n^2-n+1)$

(d) $\frac{1}{3}(2n^3-2n+3)$

2. T_r is always :

(a) an odd number (b) an even number

(c) a prime number (d) a composite number

3. Which one of the following is a correct statement ?

(a) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 5

(b) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 6

(c) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 11

(d) $Q_1 = Q_2 = Q_3 = \ldots$

Comprehension passage (2)
(Questions No. 4-6)

Let P and Q be two sets each of which consisting of three numbers in *A.P.* and *G.P.* respectively. Sum of the elements of set P is 12 and product of the elements of set Q is 8 , where the common difference and the common ratio of *A.P.* and *G.P.* are represented by *'d'* and *'r'* respectively. If sum of the squares of the terms of *A.P.* is 8 times the sum of the terms of *G.P.* , where $d = r$, and $d, r \in I^+$, then answer the following questions.

4. Total number of terms in the set of $P \cap Q$ is/are :

(a) 0 (b) 2

(c) 1 (d) 3

5. Let $Q = \{a, b, c\}$, where $a < b < c$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ are :

(a) real (b) real and unequal

(c) real and equal (d) non-real

6. Sum of all the elements of set $P \cup Q$ is equal to :

(a) 56 (b) 13

(c) 19 (d) 25

Questions with Integral Answer :
(Questions No. 7-10)

7. Let x and y be two real numbers such that the k^{th} mean between x and $2y$ is equal to the k^{th} mean between $2x$ and y when n arithmetic means are placed between them in both the situations. The value of expression $\left\{\frac{n+1}{k} - \frac{y}{x}\right\}$ is equal to ....

8. Let $S_n = \sum_{r=1}^{n} \frac{1}{r}$ and

$$S'_n = \frac{n+1}{2} - \left\{\frac{1}{n(n-1)} + \frac{2}{(n-1)(n-2)} + \ldots \frac{(n-2)}{6}\right\},$$

then value of $\left\{\frac{S'_n}{S_n}\right\}$ is equal to ....

9. Let an *A.P.* and a *G.P.* each has α as the first term and β as the second term , where $\alpha > \beta > 0$. If sum of infinite terms of *G.P.* is 4 and the sum of first n terms of *A.P.* can be written as $n\alpha - \frac{n(n-1)\alpha^2}{k}$, then value of $'k'$ is equal to ....

10. Let sum of the squares of three distinct real number in geometric progression be S^2 and their sum is $\frac{\sqrt{p}}{2}(S)$. If $p \in R^+$, then total number of possible integral values of $'p'$ is/are ....

11. Let $a, b, c, d, e \in R^+$ and $s = a + b + c + d + e$, if minimum value of $\left\{\frac{(s-a)(s-b)(s-c)(s-d)(s-e)}{abcde}\right\}$ is 4^n , then value of n is ....

Matrix Matching Questions :
(Questions No. 12-15)

12. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) Let $\sum_{r=1}^{2009}\left(1+\frac{1}{r^2}+\frac{1}{(r+1)^2}\right)^{\frac{1}{2}}=\alpha+\frac{\alpha}{\beta}$, then sum of all the digits of the number $'\beta'$ is	(p) 1
(b) The largest positive term of the harmonic progression whose first two terms are $\frac{2}{5}$ and $\frac{12}{23}$, is equal to	(q) 4
(c) If $I_n=\int_0^{\pi/4}\tan^n x\,dx$, where $n\in N$, and $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ ..... form an *A.P.*, then common difference of this *A.P.* is	(r) 3
	(s) 11
(d) Value of $(0.16)^{log_{\frac{5}{2}}\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+....\infty\right)}$ is equal to	(t) 6

13. Match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) If p is prime number and $x\in N$, where $log_p\left(\sqrt{x}+\sqrt{x+p}\right)=1$, then first three smallest possible values of x are	(p) in arithmetic progression
(b) If a_1, a_2, a_3, a_4, a_5 are five non-zero distinct numbers such that a_1, a_2, a_3 are in *A.P.*, a_2, a_3, a_4 are in *G.P.* and a_3, a_4, a_5 are in *H.P.*, then a_1, a_3, a_5 are	(q) in geometric progression
	(r) in harmonic progression
(c) $\tan 70^\circ$, $\tan 50^\circ+\tan 20^\circ$ and $\tan 20^\circ$ are	
(d) If a, b are positive distinct real number and α, β, γ are three roots of $\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}$ such that	(s) not is arithmetic progression
$\alpha>\beta>\gamma$ and $\alpha-\beta-\gamma=c$, then a, b, c are	(t) not in geometric progression

14. Match the following columns (I) and (II).

	Column (I)		**Column (II)**
(a)	If sum of first n positive integers is $\frac{1}{5}$ times the sum of their squares , then n is	(p)	3
(b)	If $\sum n$, $\frac{\sqrt{10}}{3}\sum n^2$, $\sum n^3$ are in G.P. , then the value of n is	(q)	7
(c)	If $log_3 2$, $log_3(2^x - 5)$ and $log_3\left(2^x - \frac{7}{2}\right)$ are in A.P. , then value of x is	(r)	4
(d)	Let S_1 , S_2 , S_3 , ... be squares such that for each $n \geq 1$, length of side of S_n equals the length of diagonal of S_{n+1} . If length of S_1 is 1.5 cm , then for which values of n is the area of S_n less than 1 sq. cm.	(s)	6
		(t)	2

15. Match the following columns (I) and (II).

	Column (I)		**Column (II)**
(a)	If altitudes of a triangle are in A.P. , then sides of triangle are in	(p)	A.P.
(b)	If $\begin{vmatrix} a & b & a\alpha - b \\ b & c & b\alpha - c \\ 2 & 1 & 0 \end{vmatrix} = 0$ and $\alpha \neq \frac{1}{2}$, then a , b , c are in	(q)	G.P.
(c)	If $\frac{a_2\, a_3}{a_1\, a_4} = \frac{a_2 + a_3}{a_1 + a_4} = 3\left(\frac{a_2 - a_3}{a_1 - a_4}\right)$, then a_1 , a_2 , a_3 , a_4 are in	(r)	H.P.
(d)	If $(y - x)$, $2(y - a)$ and $(y - z)$ are in H.P. , then $(x - a)$, $(y - a)$, $(z - a)$ are in	(s)	A.G.P.

ANSWERS Exercise No. (1)

1. (a) **2.** (a) **3.** (d) **4.** (a) **5.** (c)

6. (c) **7.** (c) **8.** (c) **9.** (c) **10.** (a)

11. (a) **12.** (b) **13.** (b) **14.** (a) **15.** (c)

16. (b) **17.** (b) **18.** (c) **19.** (b) **20.** (c)

21. (c) **22.** (d) **23.** (b) **24.** (a) **25.** (b)

26. (b , c) **27.** (b , c ,d) **28.** (a , d) **29.** (a , c) **30.** (b , c)

31. (a) **32.** (d) **33.** (a) **34.** (c) **35.** (b)

ANSWERS Exercise No. (2)

1. (b) **2.** (d) **3.** (b) **4.** (b) **5.** (d)

6. (b) **7.** (1) **8.** (1) **9.** (8) **10.** (9)

11. (5)

12. (a) → r
(b) → t
(c) → p
(d) → q

13. (a) → s , t
(b) → q , s
(c) → p , t
(d) → r , s , t

14. (a) → q
(b) → r
(c) → p
(d) → p , q , r , s

15. (a) → r
(a) → q
(a) → r
(a) → q

Chapter
3

Complex Numbers

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-25)

1. If $A(z_1)$, $B(z_2)$ and $C(z_3)$ are the vertices of an equilateral triangle in the clockwise direction , then $arg\left(\frac{z_2+z_3-2z_1}{z_3-z_2}\right)$ is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

2. Let complex numbers z_1 and z_2 satisfy the conditions $|z+6i|=2$ and $|z-4i|=\left(\frac{z-\bar{z}}{2i}\right)$ respectively, then minimum value of $|z_1-z_2|$ is :

(a) 8 (b) 6

(c) 4 (d) 2

3. For non-zero complex number $'z'$, if $|z-2-2i|+2\sqrt{2}=|z|$, then $arg\,(i\,\bar{z})$ is equal to :

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{5\pi}{4}$ (d) $\frac{7\pi}{4}$

4. If α and β are complex numbers , then maximum value of $\frac{|\alpha\bar{\beta}+\bar{\alpha}\beta|}{|\alpha\beta|}$ is :

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) 4

5. If α , β , γ are the roots of cubic equation $x^3 - 3x^2 + 3x + 7 = 0$, $'\omega'$ is non-real cube root of unity , then $\left(\frac{\alpha-1}{\beta-1}+\frac{\beta-1}{\gamma-1}+\frac{\gamma-1}{\alpha-1}\right)$ is :

(a) $\frac{8}{\omega}$ (b) ω^2 (c) $2\omega^2$ (d) $3\omega^2$

6. $f(z)$ is non-real function of complex number $'z'$ and when $f(z)$ is divided by $(z-i)$ and $(z+i)$ the remainders are i and $1+i$ respectively, then the remainder when $f(z)$ is divided by (z^2+1) is equal to :

(a) $\frac{1}{2}+i+z$ (b) $\frac{1}{2}iz+\frac{1}{2}+i$

(c) $iz+1+i$ (d) $\frac{i}{2}+iz$

7. If $|\alpha_k|<3 \;\forall\; 1\le k\le n , k\in N$, and complex number $'z'$ satisfy $1+\alpha_1 z+\alpha_2 z^2+.......\alpha_n z^n=2$, then :

(a) $|z|>\frac{1}{4}$ (b) $|z|<\frac{1}{4}$

(c) $|z|=\frac{1}{4}$ (d) $\frac{1}{3}<|z|<\frac{1}{2}$

8. If $\left|\frac{z_1-3z_2}{3-z_1\bar{z}_2}\right|=1$ and $|z_2|\neq 1$, then $|z_1|$ is equal to :

(a) 3 (b) 1 (c) 2 (d) 4

9. A particle P starts from the point $z_0=1+2i$, where $i=\sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1 . From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i}+\hat{j}$ and then it moves through an angle 90° in anticlockwise direction on a circle with centre at origin to reach a point z_2 . The point z_2 is given by :

(a) $6+7i$ (b) $-7+6i$

(c) $7+6i$ (d) $-6+7i$

10. Consider a square $OABC$, where O is origin and $A(z_0)$, $B(z_1)$, $C(z_2)$ are in anticlockwise sense , then equation of circle inscribed in the square is :

(a) $|z-z_0(1+i)|=2|z_0|$

(b) $\left|z-\frac{1}{2}(1-i)z_0\right|=|z_0|$

(c) $2\left|z-\frac{1}{2}(1+i)z_0\right|=|z_0|$

(d) $2\left|z-(1+i)z_0\right|=|z_0|$

11. If $A(z_1)$, $B(z_2)$ and $C(z_3)$ are the vertices of a triangle ABC inscribed in the circle $|z| = 1$ and internal angle bisector of $\angle A$ meet the circumference at $D(z_4)$, then

(a) $z_4^2 = z_2z_3$ (b) $z_4 = \frac{z_2z_3}{z_1}$

(c) $z_4 = \frac{z_1z_2}{z_3}$ (d) $z_4 = \frac{z_1z_3}{z_2}$

12. Centre of the arc represented by $arg\left(\frac{z-3i}{z-2i+4}\right) = \frac{\pi}{4}$ is given by :

(a) $\frac{1}{2}(5+5i)$ (b) $\frac{1}{2}(5i-5)$

(c) $\frac{1}{2}(9i+5)$ (d) $\frac{1}{2}(9i-5)$

13. If a, b, c are integers not all equal and ω is cube root of unity $(\omega \neq 1)$, then minimum value of the expression $|a+b\omega+c\omega^2|$ is :

(a) 0 (b) 1

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

14. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that $arg\left(\frac{z-z_1}{z-z_2}\right) = \frac{\pi}{4}$, then

(a) $|z-7+9i| = 3\sqrt{2}$ (b) $|z-7-9i| = 2\sqrt{3}$

(c) $|z+7+9i| = 3\sqrt{2}$ (d) $|z-7-9i| = 3\sqrt{2}$

15. If $A(z_1)$, $B(z_2)$ and $C(z_3)$ form an isosceles right angled triangle and $\angle A = \frac{\pi}{2}$, then

(a) $(z_1-z_2)^2 = 2(z_2-z_3)(z_3-z_2)$

(b) $(z_1-z_2)^2 = 2(z_1-z_3)(z_3-z_2)$

(c) $(z_3-z_2)^2 = 2(z_1-z_3)(z_2-z_1)$

(d) $(z_3-z_2)^2 = 2(z_2-z_1)(z_3-z_1)$

16. If complex number 'z' satisfy $|z + 13i| = 5$, then complex number having magnitude-wise minimum argument is :

(a) $-\frac{12}{13}(12+5i)$ (b) $\frac{12}{13}(5+12i)$

(c) $-\frac{12}{13}i(12+5i)$ (d) $\frac{12}{13}i(12-5i)$

17. Let $|z_1| = 30$ and $|z_2 + 5 + 12i| = 13$, then minimum value of $|z_2 - z_1|$ is :

(a) 2 (b) 6

(c) 4 (d) none of these

18. Area of region on the complex plane which is bounded by the curve $|z+2i| + |z-2i| = 8$ is :

(a) $3\sqrt{8}\pi$ (b) $4\sqrt{12}\,\pi$

(c) $16\pi\sqrt{3}$ (d) none of these

19. If z and w are two non-zero complex numbers such that $|zw| = 1$ and $arg\left(\frac{z}{w}\right) = \frac{\pi}{2}$, then $\bar{z}w$ is equal to:

(a) 1 (b) –1

(c) i (d) $-i$

20. Let $x = e^{i\alpha}$, $y = e^{i\beta}$ and $z = e^{i\gamma}$ and $x+y+z=0$, then which one of the following is not correct :

(a) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

(b) $xy+yz+zx=0$

(c) $x^2+y^2+z^2=1$

(d) $x^3+y^3+z^3=3xyz$

21. Let $z = x+iy$ be a complex number where x and y are integers, then the area of the rectangle whose vertices are roots of the equation $(\bar{z})z^3 + z(\bar{z})^3 = 350$ is :

(a) 48 (b) 32

(c) 40 (d) 80

22. Let $z = \cos\theta + i\sin\theta$, then the value of summation $\sum_{r=1}^{15} Im\left(z^{2r-1}\right)$ at $\theta = 2^\circ$ is equal to :

(a) $\frac{1}{\sin 2^\circ}$ (b) $\frac{1}{3\sin 2^\circ}$

(c) $\frac{1}{2\sin 2^\circ}$ (d) $\frac{1}{4\sin 2^\circ}$

23. Let $A(z_1)$, $B(z_2)$ and $C(z_3)$ form triangle ABC on the argand plane such that $\frac{z_1-z_2}{z_3-z_2} = \frac{1-i}{\sqrt{2}}$, then ΔABC is :

(a) equilateral (b) right angled

(c) isosceles (d) scalene

24. If moving complex number $'z'$ satisfy the conditions, $1 \le |z-1+i| \le 2$ and $\frac{\pi}{12} \le \arg(z+i-1) \le \frac{5\pi}{12}$, then area of region which is represented by $'z'$ is :

(a) π (b) $\frac{\pi}{2}$

(c) 2π (d) $\frac{\pi}{3}$

25. A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P, then the position of P in the argand plane is :

(a) $3e^{i\pi/4}+4i$ (b) $(3-4i)e^{i\pi/4}$

(c) $(4+3i)e^{i\pi/4}$ (d) $(3+4i)e^{i\pi/4}$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. Let z_r, where $r \in \{1, 2, 3, ..., n\}$, be the $'n'$ distinct roots of the equation $\sum_{r=1}^{n} {}^{n}C_r x^r = 1$. If there exists some z_r for which $arg\left(\frac{z_r-(\sqrt{2}\,i-1)}{-1-(\sqrt{2}\,i-1)}\right)=\frac{\pi}{4}$, then $'n'$ can be :

(a) 4 (b) 8

(c) 12 (d) 16

27. Let 2 + 3i and –2 + 3i be the two vertices of an equilateral triangle on the complex plane, then the third vertex of triangle can be given by :

(a) $(-3+2\sqrt{3})i$ (b) $(-3-2\sqrt{3})i$

(c) $(3+2\sqrt{3})i$ (d) $(3-2\sqrt{3})i$

28. Let α, β, γ be the complex numbers, and $\alpha z^2+\beta z+\gamma=0$, where $z \in C$. If the quadratic equation in $'z'$ is having

(a) both roots real, then $\frac{\alpha}{\bar{\alpha}}=\frac{\bar{\beta}}{\beta}=\frac{\gamma}{\bar{\gamma}}$.

(b) both roots purely imaginary, then $\frac{\bar{\alpha}}{\alpha}=\frac{-\bar{\beta}}{\beta}=\frac{\bar{\gamma}}{\gamma}$.

(c) both roots real, then $\frac{\alpha}{\bar{\alpha}}=\frac{\beta}{\bar{\beta}}=\frac{\gamma}{\bar{\gamma}}$.

(d) both roots purely imaginary, then $\frac{\alpha}{\bar{\alpha}}=\frac{\beta}{\bar{\beta}}=\frac{-\gamma}{\bar{\gamma}}$.

29. Let $A(z_1)$, $B(z_2)$ and $C(z_3)$ be the vertices of ΔABC on the complex plane, where the triangle ABC is inscribed in circle $|z|=1$. If altitude through A meets the circle $|z|=1$ at D and image of D about BC is E, then

(a) complex point $'E'$ is $z_1+z_2+z_3$.

(b) complex point $'D'$ is $-\frac{z_2 z_3}{z_1}$

(c) complex point $'E'$ is $2(z_1+z_2+z_3)$.

(d) complex point $'D'$ is $-\frac{z_1 z_2}{z_3}$

30. Let P, Q, R be three sets of complex numbers as defined below:

$$P=\left\{z : \text{Re}(z(1-i))=\sqrt{2}\right\}$$
$$Q=\{z : |z-i-2|=3\}$$
$$R=\{z : Im(z)\ge 1\}$$

In the context of given sets, which of the following statements are correct ?

(a) number of elements in the set $P \cap Q \cap R$ are infinite.

(b) If $'z'$ be any point in $P \cap Q \cap R$, then $|z-5-i|^2+|z+1-i|^2=36$

(c) number of elements in the set $P \cap Q \cap R$ is one.

(d) number of elements in the set $P \cap Q$ are two.

Assertion Reasoning questions : (Questions No. 31-35)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

31. **Statement 1 :** Let $'z'$ be the moving complex point on argand plane for which

$$|z-3-2i| = \left| |z| \sin\left(\frac{\pi}{4} - arg\ (z)\right) \right| ,$$

then the locus of $'z'$ is part of an ellipse

because

Statement 2 : Ellipse is the locus of a point for which sum of its distances from two distinct fixed points is always constant , where the constant sum is more than the distance between the fixed points.

32. **Statement 1 :** If $i^2 + 1 = 0$, then value of $\cos^{-1}\left\{\sin(ln(i)^i)\right\}$ is equal to π

because

Statement 2 : $\cos^{-1}(\cos x) = 2\pi - x \quad \forall \quad x \in [\pi\ ,\ 2\pi]$

33. Let the equations $arg(z+4-3i) = -\frac{\pi}{3}$ and $arg(z-2+3i) = \frac{5\pi}{6}$ be represented by the curves C_1 and C_2 respectively on the complex plane , then

Statement 1 : The number of points of intersection of C_1 and C_2 is only one

because

Statement 2 : Two non-parallel lines always intersect at only one point in 2-dimensional plane.

34. Let $z_1 = 5 + 8i$ and z_2 satisfy $|z+2+3i| \le 2$, then

Statement 1 : minimum value of $|iz_2 + z_1|$ is equal to 8

because

Statement 2 : maximum value of $|z_2|$ is $2+\sqrt{13}$

35. **Statement 1 :** Let $m\ ,\ n \in N$ and the equations $z^m - 1 = 0$ and $z^n - 1 = 0$ is having only one common root , then m and n must be different prime numbers

because

Statement 2 : the common root for the equations $z^m - 1 = 0$ and $z^n - 1 = 0$ is 1 if m and n are different prime numbers.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $1\,,\ \alpha_1\,,\alpha_2\,,\alpha_3\,,\ .....\ \alpha_{n-1}$ be the n^{th} roots of unity, then $\alpha_k=\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}$, where $k=0\,,1\,,2\,,3\,,4\,..........\,,n-1$, further $x^n-1=0$ can be expressed as $x^n-1=(x-1)\prod_{k=1}^{(n-1)}(x-\alpha_k)$. Now answer the following questions based on above information

1. Value of $\sum_{k=0}^{17}\left(\cos\frac{k\pi}{8}+i\sin\frac{k\pi}{8}\right)$ is equal to :

(a) 0 (b) 1

(c) $2\cos\frac{\pi}{16}\cdot\left(e^{i\frac{\pi}{16}}\right)$ (d) $2\cos\frac{\pi}{8}\cdot\left(e^{i\frac{\pi}{8}}\right)$

2. Value of $\sum_{k=1}^{n-1}\frac{1}{(4-\alpha_k)}$ is equal to :

(a) $\frac{1+4^{n-1}(3n-4)}{3(4^n+1)}$

(b) $\frac{4+4^n(3n-4)}{12(4^n-1)}$

(c) $\frac{1+(3n-2)4^{n-1}}{4^n+1}$

(d) $\frac{1+4^n(3n-4)}{12(4^n-1)}$

3. If $1\,,\alpha_1\,,\alpha_2\,,\ ....\ \alpha_{n-1}$ forms a polygon on the complex plane , then area of the circle inscribed in the polygon is given by :

(a) $\pi\sin^2\left(\frac{\pi}{n}\right)$

(b) $\frac{\pi}{2}\left(1+\cos\frac{2\pi}{n}\right)$

(c) $\pi\left(\cos\frac{2\pi}{n}+1\right)$

(d) $2\pi\left(\cos^2\frac{\pi}{n}\right)$

Comprehension passage (2)
(Questions No. 4-6)

If complex number $'z_1'$ satisfy $|z-2-2i|=\frac{z+\overline{z}}{2}$ and complex number $'z_2'$ satisfy $|z+4-2i|=2$, then answer the following questions.

4. Minimum value of $|z_1-z_2|$ is :

(a) 2 (b) 1 (c) 3 (d) 5

5. If magnitude of $arg(z_2)$ is minimum then $|z_2|$ is :

(a) $5\sqrt{2}$ (b) $4\sqrt{2}$

(c) 4 (d) $\sqrt{18}$

6. Maximum possible value of $|z_2|$ is :

(a) $1+\sqrt{5}$ (b) $2(1+\sqrt{5})$

(c) $3(\sqrt{5}+1)$ (d) $2(\sqrt{5}-1)$

Comprehension passage (3)
(Questions No. 7-9)

Let $P(z_1)$, $Q(z_2)$ and $R(z_3)$ represent the vertices of an isosceles triangle PQR on the argand plane , where $RQ=PR$ and $\angle QPR=\alpha$. If incentre of ΔPQR is given by $I(z_4)$, then answer the following questions.

7. The value of $\left\{\left(\frac{PR}{PQ}\right)\left(\frac{PQ}{PI}\right)^2\right\}$ is equal to :

(a) $\frac{(z_1-z_2)(z_1-z_3)}{(z_1-z_4)^2}$

(b) $\frac{(z_1-z_2)(z_3-z_2)}{(z_1-z_4)^2}$

(c) $\frac{(z_1-z_3)(z_2-z_3)}{(z_2-z_4)^2}$

(d) $\frac{(z_1-z_2)(z_3-z_1)}{(z_3-z_4)^2}$

8. The value of $\left\{(z_1-z_2)^2\tan\alpha.\tan\left(\frac{\alpha}{2}\right)\right\}$ is equal to :

(a) $(z_1+z_2-2z_3)(z_1+z_2-2z_4)$

(b) $(z_1+z_2-z_3)(z_1+z_2-z_4)$

(c) $(2z_3-z_1-z_2)(z_1+z_2-2z_4)$

(d) $(z_1+z_2+z_3)(z_2+z_3-z_4)$

9. The value of $\left\{(z_1 - z_4)^2 . \left(\dfrac{1+\cos\theta}{\cos\theta}\right)\right\}$ is equal to :

(a) $(z_2 - z_1)(z_3 - z_1)$

(b) $\dfrac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)}$

(c) $\dfrac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2}$

(d) $(z_2 - z_1)(z_3 - z_1)^2$

Questions with Integral Answer :
(Questions No. 10-14)

10. Let moving complex number $'z_o'$ lies on the curve C_1 on argand plane , where

$$arg\left(\frac{z_o - 1 - i\tan\left(\frac{7\pi}{8}\right)}{z_o - \tan\left(\frac{15\pi}{8}\right) - i}\right) = 2\tan^{-1}(\sqrt{2} - 1).$$

If the curve C_2 on argand plane is represented by $|z| = 2$, then area of the region bounded by the curves C_1 and C_2 is equal to ....

11. Let moving complex point $A\,(z_0)$ satisfy the condition $|z_o - 3 + 2i| + |z_o - 3 - 6i| = 10$, and complex points B , C are represented by $3 + 6i$ and $3 - 2i$ respectively. If the area of triangle ABC is maximum , then three times the in-radius of triangle ABC is ....

12. Let z be uni-modular complex number , then value of $\dfrac{arg(z^2 + \bar{z}.z^{1/3})}{arg(z^{1/3})}$, where $arg(z) \in \left(0 , \dfrac{3\pi}{8}\right)$, is equal to ....

13. Let $A(z_1)$, $B(z_2)$, $C(z_3)$ form a triangle ABC, where

$$\angle ABC = \angle ACB = \frac{1}{2}(\pi - \alpha).$$

If $\left\{\dfrac{(z_3 - z_2)^2}{(z_3 - z_1)(z_1 - z_2)}\right\}\text{cosec}^2\dfrac{\alpha}{2} = k$, then value of $'k'$ is equal to ....

14. Let z_1 , z_2 , z_3 be three distinct complex numbers , where $2|z_1| = |z_3| = 4$, $|z_2| = |z_1| + 1$ and $|2z_1 + 3z_2 + 4z_3| = 4$. If $|8z_2z_3 + 27z_3z_1 + 64z_1z_2|$ is equal to $'k'$, then value of $\dfrac{k}{16}$ is equal to ....

Matrix Matching Questions :
(Questions No. 15-16)

15. Match the following Columns (I) and (II).

Column (I)	**Column (II)**
(a) Let $\theta \in R$ and $'z'$ be any complex number such that $\lvert 2z\cos\theta + z^2 \rvert = 3$, then minimum value of $\lvert z \rvert$ is :	(p) 1
(b) Let $z = x + iy$, where x , $y \in I$. Area of the octagon whose vertices are the roots of the equation $(z\bar{z})\lvert z^2 - \bar{z}^2 \rvert = 1200$ is :	(q) 27
(c) Let z be complex number such that $(z + \bar{z})(4 + i) - (3 + i)(z - \bar{z}) + 26i = 0$, then value of $z\bar{z}$ is :	(r) 14
	(s) 62
(d) Let $\lvert z_1 \rvert = \lvert z_2 \rvert = \lvert z_3 \rvert = 3$, then minimum value of $\lvert z_1 + z_2 \rvert^2 + \lvert z_2 + z_3 \rvert^2 + \lvert z_3 + z_1 \rvert^2$ is :	(t) 17

16. Match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) The roots of the equation $z^4+z^3+z+1=0$ on the complex plane are represented by the vertices of :	(p) an ellipse
(b) If variable complex number 'z' satisfy the condition $\lvert z-\bar{z}\rvert+\lvert z+\bar{z}\rvert=4$, then locus of z is given by :	(q) a square
	(r) a trapezium
(c) The roots of the equation $z^4+z^3+z^2+z+1=0$ on the complex plane are represented by the vertices of :	
	(s) a hexagon
(d) The roots of the equation $z^6+z^4-z^2-1=0$ on the complex plane are represented by the vertices of :	(t) an equilateral triangle

ANSWERS Exercise No. (1)

1. (d)
2. (b)
3. (b)
4. (b)
5. (d)
6. (b)
7. (a)
8. (a)
9. (d)
10. (c)
11. (a)
12. (d)
13. (b)
14. (d)
15. (c)
16. (c)
17. (c)
18. (b)
19. (d)
20. (c)
21. (a)
22. (d)
23. (c)
24. (b)
25. (d)
26. (b ,d)
27. (c , d)
28. (b , c)
29. (a , b)
30. (b , c , d)
31. (d)
32. (b)
33. (b)
34. (b)
35. (d)

ANSWERS Exercise No. (2)

1. (c)
2. (b)
3. (b)
4. (c)
5. (c)
6. (b)
7. (a)
8. (c)
9. (a)
10. (2)
11. (4)
12. (2)
13. (4)
14. (6)

15. (a) $\to$ p
(b) $\to$ s
(c) $\to$ t
(d) $\to$ q

16. (a) $\to$ t
(b) $\to$ q
(c) $\to$ r
(d) $\to$ q

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Chapter 4

Binomial Theorem

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-20)

1. Maximum value of the term independent of x in the expansion of $\left(x\sin\alpha+\dfrac{\cos\alpha}{x}\right)^{10}$, where $\alpha\in R$, is :

(a) $\dfrac{10!}{(5!)^2}$ (b) $\dfrac{10!}{32(5!)^2}$

(c) $\dfrac{10!}{1024(5!)^2}$ (d) ${}^{10}C_8$

2. Sum of the series , ${}^{20}C_0+{}^{20}C_1+{}^{20}C_2+\ldots\ldots\ldots\ldots+{}^{20}C_{10}$ is equal to :

(a) $2^{20}+{}^{20}C_{10}$ (b) $2^{19}+{}^{20}C_{10}$

(c) $2^{19}-\dfrac{1}{2}\cdot{}^{20}C_{10}$ (d) $2^{19}+{}^{19}C_9$

3. Coefficient of x^5 in the expansion of the product $(1+2x)^6(1-x)^7$ is :

(a) 172 (b) 171

(c) 170 (d) 160

4. If the binomial coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1 : 7 : 42 , then value of 'n' is :

(a) 32 (b) 65

(c) 55 (d) 50

5. Coefficient of x^5 in $\left\{(1+x)^{21}+(1+x)^{22}+\ldots\ldots.+(1+x)^{30}\right\}$ is :

(a) ${}^{31}C_6-{}^{21}C_5$ (b) ${}^{31}C_6-{}^{21}C_6$

(c) ${}^{32}C_5-{}^{20}C_4$ (d) ${}^{32}C_6+{}^{20}C_5$

6. Let ${}^{16}C_r=a_r$, then sum of the series , $3a_0^2-7a_1^2+11a_2^2-15a_3^2+\ldots\ldots.+67a_{16}^2$, is equal to :

(a) $-35\,a_8$ (b) $70\,a_8$

(c) $35\,a_8$ (d) $-70\,a_8$

7. Let $(1+x)^n=\sum_{r=0}^{n}{}^nC_r\,x^r$, then value of

$$\sum_{r=0}^{n}(-1)^r\cdot{}^nC_r\cdot\frac{1+r\ln 10}{(1+\ln 10^n)^r}$$ is equal to :

(a) 1 (b) 2

(c) 0 (d) -1

8. Coefficient of x^4 in expansion of $(1+x+x^2+x^3)^{11}$ is :

(a) 605 (b) 810

(c) 990 (d) 1020

9. If ${}^nC_r=\begin{pmatrix}n\\r\end{pmatrix}$, then $\sum_{r=2}^{n+1}\begin{pmatrix}n+2\\r-1\end{pmatrix}$ is equal to :

(a) $2^{n+2}-2$ (b) $2^{n+2}-n+1$

(c) $4(2^n-1)-n$ (d) $4(2^n+1)-2n$

10. $\sum_{r=0}^{10}(-1)^r\cdot{}^{10}C_r\left(\dfrac{1}{2^r}+\dfrac{3^r}{2^{2r}}+\dfrac{7^r}{2^{3r}}+\ldots\ldots\ldots.\infty\right)$ is equal to :

(a) $\dfrac{1}{255}$ (b) $\dfrac{1}{1023}$

(c) $\dfrac{1}{511}$ (d) $\dfrac{1}{2047}$

11. The value of $\sum\limits_{0\le i<j\le n}\sum j\cdot{}^nC_i$ is equal to :

(a) $n(3n+1)2^{n-3}$ (b) $n(n+3).2^{n-3}$

(c) $(n+3).2^{n-3}$ (d) $n.2^{n-3}$

12. Let $n\in N$ and $(1+x+x^2)^n=\sum_{r=0}^{2n}a_rx^r$; then value of $\sum_{r=0}^{2n}(-1)^r.a_r^2$ is equal to :

(a) a_n^2 (b) $3a_n$

(c) a_n (d) $2a_n^2$

13. Let $n \in I^+ - \{1, 2\}$ and the digits at the unit's place and ten's place of 3^n are 9 and 0 respectively, then $(n-2)$ must be divisible by :

(a) 16 (b) 6

(c) 10 (d) 18

14. Let T_r denotes the r^{th} term in the expansion of $(1+x)^n$ and T_n is the only term which is numerically greatest exactly for three natural values of 'x', then 'n' can be:

(a) 5 (b) 10

(c) 7 (d) 8

15. Let $n_1 + n_2 = 40$, where $n_1,\ n_2 \in N$ and the value of $\sum_{r=0}^{n} {}^{n_1}C_{n-r}.{}^{n_2}C_r$ is maximum, then value of 'n' must be :

(a) 25 (b) 15

(c) 20 (d) 22

16. Value of $\sum_{\alpha=0}^{n} {}^nC_\alpha(\sin\alpha x)$ is equal to :

(a) $2^n.\cos^n\frac{x}{2}.\sin\frac{nx}{2}$ (b) $2^n.\sin^n\frac{x}{2}.\cos\frac{nx}{2}$

(c) $2^{n+1}.\cos^n\frac{x}{2}.\sin\frac{nx}{2}$ (d) $2^{n+1}.\sin^n\frac{x}{2}.\cos\frac{nx}{2}$

17. If [.] represents the greatest integer function and $\alpha = (\sqrt{3}+2)^n$, then value of $\alpha[\alpha]+\alpha-\alpha^2$ is equal to :

(a) 0 (b) 1

(c) 2 (d) –1

18. For natural number m, n if $(1-y)^m(1+y)^n = 1+a_1y+a_3y^2+.....$, and $a_1 = a_2 = 10$ then (m, n) ordered pair is :

(a) (35 , 45) (b) (20 , 45)

(c) (35 , 20) (d) (45 , 35)

19. The coefficient of x^8 in $\left\{\sum_{r=0}^{\infty}(r+1)x^r\right\}^{-5}$, where $|x|<1$, is equal to :

(a) – 50

(b) – 45

(c) 50

(d) 45

20. Let T_β be the term which is independent of 'α' in the binomial expansion of $\left(\frac{\alpha-1}{\alpha-\alpha^{1/2}}-\frac{\alpha+1}{\alpha^{2/3}-\alpha^{1/3}+1}\right)^{10}$, then T_β is equal to :

(a) 300 (b) 210

(c) 420 (d) 500

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let $a_n = \frac{(10)^{3n}}{n!}$, $n \in N$, and the value of a_n is greatest , then :

(a) $n = 998$

(b) $n = 999$

(c) $n = 1000$

(d) $n = 1001$

22. Let $n \in I^+$, $(5+3\sqrt{3})^{2n+1} = \alpha+\beta$, where α is an integer and $\beta \in (0, 1)$, then :

(a) $(\alpha+\beta)^2$ is divisible by 2^{2n+1}

(b) $\alpha\beta = 2(4)^n$

(c) α is divisible by 10

(d) α is an odd integer

23. Let $A_n = \sum_{r=0}^{n} {}^nC_r.\cos\left(\frac{2r\pi}{n}\right)$ & $B_n = \sum_{r=0}^{n-1} {}^{n-1}C_r.\cos\left(\frac{2r\pi}{n}\right)$, where ${}^nC_r = \frac{n!}{r!.(n-r)!}$, then which of the following statements are correct :

(a) $A_n = 2B_{n+2}$ (b) $A_n = 2B_n$

(c) $B_8 = -\frac{27}{2}$ (d) $A_6 = -27$

24. If $S = \sum_{m=1}^{n+1}\frac{(1+x)^{n+m-1}}{2^{m-1}}$, where $x \in (-1, 1)$, then the correct statements are :

(a) coefficient of x^n in S is $2^{n+1}-2^{n-1}$

(b) $\lim_{p\to\infty}\frac{2^n(S)}{1+x+x^2+.......+x^p} = (1+x)^n-(1+x)^{2n+1}$

(c) coefficient of x^n in S is 2^n

(d) value of $\sum_{r=0}^{n} {}^{n+r}C_r\left(\frac{1}{2}\right)^{n+r}$ is 1

25. Let T_r denotes the r^{th} term in the binomial expansion of $(1+x)^n$, where T_{n-1} and T_n are equal for at least one integral value of x, then value of 'n' can be :

(a) 11 (b) 7

(c) 12 (d) 8

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. Statement 1 : Total number of distinct terms in the expansion of $\left\{(x+y^2)^{13}+(x^2+y)^{14}\right\}$ is 28,

because

Statement 2 : Total number of common terms in the expansion of $(x+y^2)^{13}$ and $(x^2+y)^{14}$ are 2.

27. Statement 1 : If the binomial expansion of $(\sqrt{2}+\sqrt[3]{7})^n$ contains only two rational terms, then value of 'n' can be 10

because

Statement 2 : The applicable natural values of 'n' are 6, 8, 10, which are all even in nature.

28. Statement 1 : The coefficient of term containing x^o in the expansion of $\left(x^2+\frac{1}{x^2}+2\right)^{23}$ is ${}^{46}C_{23}$

because

Statement 2 : ${}^{n}C_{\frac{n}{2}}$ is maximum, if n is even natural number.

29. Let a, b, c denote the sides of a triangle ABC opposite to the vertices A, B and C respectively, then
Statement 1 :

Value of $\sum_{r=0}^{n} {}^{n}C_r (a)^r.(b)^{n-r}.\cos\left((n-r)A-r\,B\right)$ is equal to zero

because

Statement 2 : In any triangle ABC,
$(a\cos B+b\cos A)^n=c^n$ for all $n\in R$.

30. Statement 1 : If ${}^{50}C_{25}$ is divisible by $(18)^n$, where $n\in N$, then maximum value of n can be 2

because

Statement 2 : ${}^{2n}C_n=\frac{2^n}{n!}\left\{\prod_{r=1}^{n}(2r-1)\right\}$ for all $n\in N$.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f(x) = (1+x+x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$,
and $g(x) = b_0 + b_1x + b_2x^2 + b_3x^3 + ... + b_{2n}x^{2n}$,
where $b_k = 1 \ \forall \ k \ge n, n \in N$. Answer the following questions based on the given information.

1. If $f(x) = g(x+1)$, then value of a_n is equal to :

(a) ${}^{2n+2}C_{n+1}$ (b) ${}^{2n-1}C_n$

(c) ${}^{2n+1}C_n$ (d) ${}^{2n}C_n$

2. In $f(x)$, if n is even positive integer, then value of $\left\{(a_0 - a_2 + a_4 - a_6 + a_8 .....)^2 + (a_1 - a_3 + a_5 - a_7 .....)^2\right\}$ is equal to :

(a) 1 (b) 2

(c) 0 (d) 4

3. In $f(x)$, if n is positive integral multiple of 3, then $\sum_{r=0}^{n} (-1)^r . a_r . {}^nC_r$ is equal to :

(a) ${}^{3n}C_{n/3}$ (b) ${}^{n}C_{2n/3}$

(c) ${}^{2n}C_{n/3}$ (d) ${}^{n+1}C_{n/3}$

Comprehension passage (2)
(Questions No. 4-6)

Let $m, n \in N$ and ${}^nS_m = \sum_{r=1}^{n} (r)^m$, if

$$P(m,n) = m!\left\{\binom{m}{m} + \binom{m+1}{m} + \binom{m+2}{m} + ... + \binom{n+m-1}{m}\right\},$$

where $\binom{p}{q} = {}^pC_q$, then answer the following questions.

4. Value of $\lim_{n\to\infty}\left\{\frac{{}^nS_6}{n^7}\right\}$ is equal to :

(a) 0 (b) 1/7

(c) 1/6 (d) 1/14

5. Value of ${}^nS_2 + {}^nS_1$ is equal to :

(a) $\frac{1}{2}P(2, n)$

(b) $P(2, n)$

(c) $\frac{1}{3}P(2, n)$

(d) $\frac{1}{6}P(2, n)$

6. Value of ${}^nS_3 + 3\,{}^nS_2$ is equal to :

(a) $P(3, n) - 2P(2, n)$

(b) $P(3, n) + 2P(1, n)$

(c) $P(3, n) - 2P(1, n)$

(d) $P(3, n) + 2P(2, n)$

Questions with Integral Answer :
(Questions No. 7-10)

7. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... C_nx^n$, where $n \in N$, and $\sum_{r=1}^{n}\left\{r^3\left(\frac{C_r}{C_{r-1}}\right)^2\right\} = 540$, then value of n is equal to ....

8. Let the binomial coefficients of the 3rd, 4th, 5th and 6th terms in the expansion of $(1+x)^{100}$ be a, b, c and d respectively. If α, β are relatively prime numbers and $\frac{b^2 - ac}{c^2 - bd} = \frac{\alpha a}{\beta c}$, then value of $(\alpha - \beta)$ is equal to ....

9. Let $n \in N$ and ${}^{n+1}C_{n-2} \le 100 + {}^{n-1}C_{n-2}$, then number of possible values of 'n' is equal to ....

10. If $\left\{\frac{1}{2}\,{}^{30}C_1 - \frac{2}{3}\,{}^{30}C_2 + \frac{3}{4}\,{}^{30}C_3 \,........ - \frac{30}{31}\,{}^{30}C_{30}\right\}$ is equal to $(10\lambda + 1)^{-1}$, then value of 'λ' is equal to ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) $\frac{^{20}C_0}{3} - \frac{^{20}C_1}{4} + \frac{^{20}C_2}{5}$ .... upto 21 terms.	(p) 465
(b) $\frac{^{30}C_1}{^{30}C_0} + 2\frac{^{30}C_2}{^{30}C_1} + 3\frac{^{30}C_3}{^{30}C_2} + ...$ upto 30 terms.	(q) 0
(c) $^{10}C_0^2 + 2\,^{10}C_1^2 + 3\,^{10}C_2^2 + ...$ upto 11 terms.	(r) $\frac{6}{5}\left(\frac{(19!)}{(9!)^2}\right)$
(d) $\sum_{r=1}^{10} \frac{(-1)^r\,^{10}C_r}{(4r+1)}$	(s) $\frac{1}{11\,.\,21\,.\,23}$
	(t) $\int_0^1 (1-x^4)^{10}\,dx$

12. Match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) If the sixth term in the binomial expansion of $\left(3^{log_3\sqrt{9^{x-1}+7}} + \frac{1}{5^{(1/5)log_5(3^{x-1}+1)}}\right)^7$ is 84, then values of $'x'$ can be	(p) 5
	(q) 1
	(r) 4
(b) The second last digit of number 7^{283} is equal to	
(c) The coefficient of x^{10} in the expansion of $(1 + x^2 - x^3)^8$ is not divisible by	(s) 3
(d) The positive integer which is greater than $(1+0.00001)^{10^5}$ can be	(t) 2

ANSWERS Exercise No. (1)

1. (b) **2.** (d) **3.** (b) **4.** (c) **5.** (b)

6. (c) **7.** (c) **8.** (c) **9.** (c) **10.** (b)

11. (a) **12.** (c) **13.** (c) **14.** (c) **15.** (c)

16. (a) **17.** (b) **18.** (a) **19.** (d) **20.** (b)

21. (b , c) **22.** (a , c) **23.** (b , d) **24.** (c , d) **25.** (a , b)

26. (a) **27.** (c) **28.** (b) **29.** (d) **30.** (b)

ANSWERS Exercise No. (2)

1. (c) **2.** (a) **3.** (b) **4.** (b) **5.** (b)

6. (c) **7.** (8) **8.** (2) **9.** (7) **10.** (3)

11. (a) → s
(b) → p
(c) → r
(d) → t

12. (a) → q , t
(b) → r
(c) → p , s
(d) → p , r , s

Chapter 5

Permutation & Combination

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-20)

1. The letters of the word 'GHAJINI' are permuted and all the permutations are arranged in a alphabetical order as in an English dictionary , then total number of words that appear after the word 'GHAJINI' is given by :

(a) 2093 (b) 2009
(c) 2092 (d) 2091

2. If John is allowed to select at most $(n + 1)$ chocolates from a collection of $(2n + 2)$ distinct chocolates , then total number of ways by which John can select at least two chocolates are given by :

(a) $(4)^n + 4.\,{}^{2n+1}C_n - 2n + 1$

(b) $2(4)^n + 4.\,{}^{2n+1}C_n - 2n + 3$

(c) $2(4)^n - {}^{2n+1}C_n - 2n - 3$

(d) $2(4)^n + {}^{2n+1}C_n - 2n - 3$

3. The coefficient of x^{1502} in the expansion of $\left\{(1+x+x^2)^{2007}.(1-x)^{2008}\right\}$ is

(a) ${}^{2007}C_{501} - {}^{2006}C_{500}$

(b) ${}^{2006}C_{500} - {}^{2006}C_{501}$

(c) ${}^{2007}C_{498} - {}^{2006}C_{499}$

(d) ${}^{2007}C_{501} - {}^{2007}C_{1506}$

4. X and Y are any 2 five digits numbers , total number of ways of forming X and Y with repetition , so that these numbers can be added without using the carrying operation at any stage , is equal to :

(a) $45(55)^4$ (b) $36(55)^4$
(c) $(55)^5$ (d) $51(55)^4$

5. A team of four students is to be selected from a total of 12 students , total number of ways in which team can be selected if two particular students refuse to be together and other two particular students wish to be together only , is equal to :

(a) 226 (b) 182
(c) 220 (d) 300

6. If the L.C.M. of $'\alpha'$ and $'\beta'$ is $p^2 q^4 r^3$, where p, q, r are prime numbers and $\alpha, \beta \in I^+$, then the number of ordered pairs (α, β) are :

(a) 225 (b) 420 (c) 315 (d) 192

7. Total number of non-negative integral solutions of $18 < x_1 + x_2 + x_3 \le 20$, is given by :

(a) 1245 (b) 685
(c) 1150 (d) 441

8. If Mr. and Mrs. Rustamji arrange a dinner party of 10 guests and they are having fixed seats opposite one another on the circular dinning table , then total number of arrangements on the table , if Mr. and Mrs. Batliwala among the guests don't wish to sit together , are given by :

(a) 148 (8!) (b) 888 (8!)
(c) 74 (8!) (d) 164 (8!)

9. If 10 identical balls are to be placed in identical boxes , then the total number of ways by which this placement is possible , if no box remains empty , is given by :

(a) 2^{10} (b) 11 (c) 9 (d) 5

10. Total number of ways by which the word 'HAPPYNEWYEAR' can by arranged so that all vowels appear together and all consonants appear together , is given by :

(a) 12(7!) (b) 6(8!)
(c) 8 (7!) (d) 3 (8!)

11. The number of seven digit integers , with sum of the digits equal to 10 and formed by using the digits 1 , 2 and 3 only , is :

(a) 55 (b) 66 (c) 77 (d) 88

12. Let $\vec{r}$ be a variable vector and $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ such that scalar values $(\vec{r}.\hat{i})$, $(\vec{r}.\hat{j})$ and $(\vec{r}.\hat{k})$ are positive integers. If $\vec{r}.\vec{a}$ is not greater than 10 , then total numbers of possible $\vec{r}$ are given by :

(a) 80 (b) 120 (c) 240 (d) 100

13. Let three lines L_1 , L_2 , L_3 be given by $2x + 3y = 2$, $4x + 6y = 5$ and $6x + 9y = 10$ respectively. If line L_r contains 2^r different points on it , where $r \in \{1 , 2 , 3\}$, then maximum number of triangles which can be formed with vertices at the given points on the lines , are given by :

(a) 320 (b) 304

(c) 364 (d) 360

14. Let function $'f'$ be defined from set A to set B , where $A = B = \{1 , 2 , 3 , 4\}$. If $f(x) \neq x$, where $x \in A$, then total number of functions which are surjective is given by :

(a) 12 (b) 10

(c) 9 (d) 8

15. Total number of five digit numbers that can be formed , having the property that every succeeding digit is greater than the preceding digit , is equal to :

(a) ${}^9P_5$ (b) ${}^9C_4$

(c) ${}^{10}C_5$ (d) ${}^{10}P_5$

16. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2 , 5 and 7. The smallest value of n for which this is possible , is :

(a) 6 (b) 7

(c) 8 (d) 9

17. Consider n boxes which are numbered by n consecutive natural numbers starting with the number m. If the box with labelled number k , $k \geq m$, contains k distinct books , then total number of ways by which m books can be selected from any one of the boxes , are :

(a) ${}^nC_{m+1}$ (b) ${}^{n+m}C_m$

(c) ${}^nC_{m+1}$ (d) ${}^{n+m}C_{n-1}$

18. Total number of triplets (x , y , z) which can be formed , selecting x , y , z from the set $\{1 , 2 , 3 , 4 , ... 100\}$ such that $x \leq y < z$, is equal to :

(a) ${}^{100}C_3$ (b) ${}^{101}C_3$

(c) ${}^{102}C_3$ (d) ${}^{100}C_2$

19. Total number of ways in which a group of 10 boys and 2 girls can be arranged in a row such that exactly 3 boys sit in between 2 girls , is equal to :

(a) 1440(8!) (b) 720(8!)

(c) 10(9!) (d) 180(8!)

20. Total number of ways of selecting two numbers from the set of $\{1 , 2 , 3 , 4 , ... , 3n\}$ so that their sum is divisible by 3 is equal to :

(a) $3n^2 - n$ (b) $\dfrac{3n^2 - n}{2}$

(c) $\dfrac{2n^2 - n}{2}$ (d) $2n^2 - n$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Total number of four letters words that can be formed from the letters of the word 'DPSRKPURAM' , is given by

(a) ${}^{10}C_4.(4!)$

(b) 2190

(c) Coefficient of x^4 in $\left\{4!.\left(1 + x + x^2\right)\left(1 + x\right)^6\right\}$

(d) Coefficient of x^4 in $\left\{3!.\left(1 + x\right)^6\left(1 + (x+1)^2\right)^2\right\}$

22. Consider seven digit number $x_1x_2x_3x_4 .... x_7$, where $x_1 , x_2 , ... x_7 \neq 0$, having the property that x_4 is the greatest digit and digits towards the left and right of x_4 are in decreasing order , then total number of such numbers in which all digits are distinct is given by :

(a) ${}^9C_7.{}^6C_3$ (b) ${}^9C_2 . {}^6C_4$

(c) $3. {}^9C_7 . {}^5C_1$ (d) $2. {}^9C_2 . {}^5C_2$

23. Consider $xyz = 24$, where $x , y , z \in I$, then

(a) Total number of positive integral solutions for x , y , z are 81

(b) Total number of integral solutions for x , y, z are 90

(c) Total number of positive integral solutions for x , y , z are 30

(d) Total number of integral solutions for x , y , z are 120

24. If ${}^nC_{r+1} = (m^2 - 8).{}^{n-1}C_r$; then possible value of $'m'$ can be :

(a) 4 (b) 2

(c) 3 (d) –5

25. Let 10 different books are to be distributed among four students A , B , C and D. If A and B get 2 books each C and D get 3 books each , then total number of ways of distribution are equal to :

(a) ${}^{10}C_4$ (b) 25200

(c) 12600 (d) $\dfrac{10!}{(2!)^2\,(3!)^2}$

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. **Statement 1 :** If $n, m \in I^+$, then $N = \dfrac{(mn)!}{(n!)^m.(m!)}$ is always an integral value

because

Statement 2 : $'N'$ represents the total numbers of ways of equal distribution of (mn) distinct objects among $'m'$ persons.

27. **Statement 1 :** From a group of 5 teachers and 5 students, if a team of 5 persons is to be formed having at least two teachers then total number of ways be which team can be formed is given by $\left({}^5C_2\right).\left({}^8C_3\right)$ {i.e., selection of 2 teachers from 5 and 3 more persons from remaining 8}

because

Statement II : The team may have 5 teachers, or 4 teachers and 1 student, or 3 teachers and 2 students, or 2 teachers and 3 students.

28. **Statement 1 :** Total number of polynomials of the form $x^3 + ax^2 + bx + c$ which are divisible by $x^2 + 1$, where $a, b, c \in \{1, 2, 3, ...10\}$ must be 10

because

Statement 2 : value of $'b'$ can be selected in 10 ways from the set of first 10 natural number and $a = c = 1$.

29. **Statement 1 :** If $a, b \in N$ and $x = 7^a.5^b$, where x and $7x$ is having 12 and 15 positive divisors respectively, then the number of positive divisors of $5x$ is 16

because

Statement 2 : Sum of all the positive divisors of $(\alpha)^a.(\beta)^b$, where $a, b \in N$, is equal to $\dfrac{(1-\alpha^{a+1})(1-\beta^{b+1})}{1-\alpha-\beta+\alpha\beta}$, provided α and β are the prime numbers.

30. **Statement 1 :** Let $A_1, A_2 \dots, A_{30}$ be thirty sets each with five elements and $B_1, B_2, \dots, B_n$ be n sets each with three elements such that $\bigcup_{i=1}^{30} A_i = \bigcup_{i=1}^{n} B_i = S$. If each element of S belongs to exactly ten of the A_i's and exactly nine of the B_j's, then the value of n is 45

because

Statement 2 : $n\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} n(A_i)$, where $n(A)$ represent the number of elements of set A.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1 - 3)

Consider the letters of the word 'MATHEMATICS' , some of them are identical and some are distinct. Letters are classified as repeating and non-repeating , such as $\{M , A , T\}$ is repeating set of letters and $\{H, E, I, C, S\}$ is non-repeating set of letters , answer the following questions based on given information.

1. Total numbers of words , taking all letters at a time , such that at least one repeating letter is at odd position in each word is given by

(a) $\frac{9!}{8}$ (b) $\frac{11!}{8}$

(c) $\frac{9!}{4}$ (d) $\frac{11!}{8}-\frac{9!}{4}$

2. Total number or words , taking all letters at a time , in which no vowel is together , is given by

(a) $\frac{7!}{(2!)^2}.{}^8C_4\left(\frac{4!}{2!}\right)$ (b) $\frac{7!}{2!}.{}^8C_4.\left(\frac{4!}{2!}\right)$

(c) $7!.{}^8C_4\left(\frac{4!}{2!}\right)$ (d) $\frac{7!}{8}.{}^8C_4.\left(\frac{4!}{2!}\right)$

3. Total number of words , taking all letters at a time , such that each word contains both M's together and both T's together but both A's are not together , is given by

(a) $\left({}^8C_2\right).7!$ (b) $\frac{11!}{8}-\frac{10!}{4}$

(c) 6(6!) (d) 9(7!)

Comprehension passage (2)
(Questions No. 4 - 6)

Let B_1 , B_2 and B_3 are three different boxes which contains y_1 , y_2 and y_3 distinct balls respectively , where $y_i \geq 1\ \forall\ i=\{1 , 2 , 3\}$, $\sum_{i=1}^{3} y_i = 20$ and $y_2 = y_1 + 2$. If total number of ways by which John can select exactly 2 balls from the boxes is 'N' and he is not allowed to select two balls from the same box , then answer the following questions

4. If $y_3 = 14$, then value of N is equal to :

(a) 90 (b) 112

(c) 140 (d) 92

5. If N assumes its maximum value , then which one of the following is correct :

(a) $y_1 = y_3 = 5$ (b) $y_1 = y_3 = 8$

(c) $y_2 = 8$ (d) $y_2 = 6$

6. Maximum value of N is equal to :

(a) 131 (b) 140

(c) 132 (d) 130

Comprehension passage (3)
(Questions No. 7 - 9)

Let $A = \{ 1 , 2 , 3 , 4 , ... , n \}$ be the set of first n natural numbers , where $S \subseteq A$. If the number of elements in set S is represented by $\eta(S)$ and the least number in the set S is denoted by S_{min} , then answer the following questions.

7. If any of the subset S of set A is having $\eta(S) = r$, where $1 \leq r \leq n$, then maximum value of S_{min} which can occur is equal to :

(a) r (b) $n-r$

(c) $n-r+1$ (d) $r+1$

8. The number of subsets $'S'$ with $S_{min} = m$ and $\eta(S) = r$, is equal to :

(a) $m\left({}^{n-m}C_{r-1}\right)$ (b) ${}^{n-m}C_r$

(c) ${}^nC_{r-1}$ (d) ${}^{n-m}C_{r-1}$

9. Let $\eta(S) = r$ and $S_{min} = m$, where $r < n - m$, then sum of all the S_{min} for possible subsets $'S'$ is equal to :

(a) $m\left({}^{n-m}C_{r-1}\right)$

(b) $n\left({}^{n-m}C_{r-1}\right)$

(c) $(n+1)\,{}^{n-m}C_{r-1} - r\left({}^{n-m+1}C_r\right)$

(d) $m\left({}^{n-m}C_{r-1}\right) + n\left({}^{n-m+1}C_r\right)$

Questions with Integral Answer :
(Questions No. 10-14)

10. Let $'N'$ triangles can be formed by joining the vertices of a regular decagon in which no two consecutive vertices are selected , then value of $\left\{\frac{N}{10}\right\}$ is equal to ....

11. Let in ${}^{\alpha}C_{\beta}$ number of ways four tickets can be selected from 35 tickets numbered from 1 to 35 so that no two consecutive numbered tickets are selected , then the value of $\left\{\frac{\alpha}{\beta}\right\}$ is equal to ....

12. Let all the letters of the word SACHHABACHHA be arranged in a matrix of order 4×3 , and at least one of the row of matrix is having all the identical elements. If the total number of arrangements are $'N'$, then least prime number dividing the number $'N'$ is equal to ....

13. Let $P(n)$ denotes the sum of the even digits of the number $'n'$, for example : $P(8592) = 8+2 = 10$, then value of of $\dfrac{\left\{\sum_{r=1}^{100} P(r)\right\}}{100}$ is equal to ....

14. Let 16 people are to be arranged around a regular octagonal frame such that people can either sit at the corner or at the mid of the side. If the number of ways in which the arrangement is possible is $\lambda(15!)$, then value of $'\lambda'$ is equal to ....

Matrix Matching Questions :
(Questions No. 15-16)

15. Consider a set $'A'$ containing 8 different elements from which a subset $'P'$ is chosen and the set A is reconstructed by replacing the elements of P. From set A if another subset Q is chosen , then match the following columns for the number of ways of choosing P and Q in column (II) with the conditions in column (I)

Column (I)	Column (II)
(a) $P\cap Q$ contains exactly one element	(p) 6561
(b) Q is subset of P	(q) 24
(c) $P\cup Q$ contains exactly one element	(r) 256
(d) $P\cup Q = A$	(s) 17496
	(t) 2187

16. Consider all possible permutations of the letters of the word ENDEANOEL. Match the statements in column I with the statements in column II .

Column (I)	Column (II)
(a) The number of permutations containing the word ENDEA	(p) 120
(b) The number of permutations in which the letter E occurs in the first and the last positions	(q) 240
	(r) 840
(c) The number of permutations in which none of the letters D , L , N occurs in the last five positions	(s) 2520
(d) The number of permutations in which the letters A , E , O occur only in odd positions	(t) 420

ANSWERS Exercise No. (1)

1. (c)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (a)
6. (c)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (d)
11. (c)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (b)
16. (b)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (b)
21. (b , d)	**22.** (a , d)	**23.** (c , d)	**24.** (a , c , d)	**25.** (b , d)
26. (c)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (b)

ANSWERS Exercise No. (2)

1. (b)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (c)
6. (c)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (5)
11. (8)	**12.** (2)	**13.** (2)	**14.** (2)	
15. (a) $\rightarrow$ s (b) $\rightarrow$ p (c) $\rightarrow$ q (d) $\rightarrow$ p	**16.** (a) $\rightarrow$ p (b) $\rightarrow$ s (c) $\rightarrow$ q (d) $\rightarrow$ q			

Chapter 6

Probability

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-25)

1. Let A, B, C be pair-wise independent events, where $P(A \cap B \cap C) = 0$ and $P(C) > 0$, then $P\left(\frac{\overline{A} \cap \overline{B}}{C}\right)$ is equal to :

(a) $P(\overline{A}) + P(\overline{B})$
(b) $P(\overline{A}) - P(\overline{B})$
(c) $P(\overline{A}) - P(B)$
(d) $P(A) - P(\overline{B})$

2. If three identical dice are rolled , then probability that the same number appears on each of them is :

(a) $\frac{1}{6}$
(b) $\frac{1}{36}$
(c) $\frac{3}{28}$
(d) $\frac{1}{18}$

3. If A, B, C are three mutually independent events, where $P(A \cup B \cup C) = 3P(A \cup B \cap \overline{C}) = \frac{1}{2}$ and $P(A \cap C) = P(\overline{A} \cap B \cap C) = \frac{1}{12}$, then $P(\overline{A} \cap C \cap \overline{B})$ is equal to :

(a) $\frac{1}{12}$
(b) $\frac{5}{6}$
(c) $\frac{1}{6}$
(d) $\frac{1}{24}$

4. An unbiased die is thrown and the number shown on the die is put for $'p'$ in the equation $x^2 + px + 2 = 0$, probability of the equation to have real roots is :

(a) $\frac{1}{3}$
(b) $\frac{1}{2}$
(c) $\frac{2}{3}$
(d) $\frac{1}{4}$

5. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is at least 0.95 is

(a) 4
(b) 5
(c) 6
(d) 7

6. Let $'A'$ and $'B'$ be two events such that $P(A) = 0.70$, $P(B) = 0.40$ and $P(A \cap \overline{B}) = 0.5$, then $P\left(\frac{B}{A \cup \overline{B}}\right)$ is equal to :

(a) 0.20
(b) 0.25
(c) 0.40
(d) 0.895

7. Three numbers are chosen at random without replacement from $\{1, 2, 3, \ldots, 10\}$. Probability that the minimum of the chosen number is 3 or their maximum is 7, is given by :

(a) 3/10
(b) 11/40
(c) 11/50
(d) 27/40

8. If $a, b, c, d \in \{0, 1\}$, then the probability that system of equations $ax + by = 2$; $cx + dy = 4$ is having unique solution is given by :

(a) $\frac{5}{8}$
(b) $\frac{3}{8}$
(c) 1
(d) $\frac{1}{2}$

9. For a student to qualify, he must pass at least two out of the three exams. The probability that he will pass the first exam is $\frac{1}{2}$, if he fails in one of the exams then the probability of his passing in the next exam is $\frac{1}{4}$ otherwise it remains the same. The probability that student will pass the exam is :

(a) $\frac{4}{5}$
(b) $\frac{3}{8}$
(c) $\frac{1}{4}$
(d) $\frac{3}{4}$

10. Let eight players $P_1, P_2, P_3, \ldots\ldots\ldots P_8$ be paired randomly in each round for a knock-out tournament. If the player P_i wins if $i > j$, then the probability that player P_6 reaches the final round is :

(a) $\frac{2}{35}$
(b) $\frac{8}{35}$
(c) $\frac{10}{17}$
(d) none of these

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11. Let John appears in the exams of physics , chemistry and mathematics and his respective probability of passing the exams is p , c and m. If John has 80% chance of passing in at least one of the three exams , 55% chance of passing in at least two exams , and 35% chance of passing in exactly two of the exams , then $p + c + m$ is equal to :

(a) $\frac{31}{20}$ (b) $\frac{18}{31}$

(c) $\frac{17}{20}$ (d) $\frac{45}{32}$

12. Let one hundred identical coins , each with probability $'p'$ of showing up head are tossed once. If $0 < p < 1$ and the probability that head turns up on 50 coins is equal to the probability that head turns up on 51 coins, then the value of $'p'$ is :

(a) $\frac{50}{101}$ (b) $\frac{49}{101}$

(c) $\frac{51}{101}$ (d) $\frac{52}{101}$

13. In a set of four bulbs it is known that exactly two of them are defective. If the bulbs are tested one by one in random order till both the defective bulbs are identified , then the probability that only two tests are needed is given by :

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

14. Let 3 faces of an unbiased die are red , 2 faces are yellow and 1 face is green. If the die is tossed three times , then the probability that the colors red , yellow and green appear in the first , second and the third tosses respectively is :

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$

(c) $\frac{7}{36}$ (d) $\frac{1}{9}$

15. Let one Indian and four American men and their wives are to be seated randomly around a circular table. If each American man is seated adjacent to his wife , then the probability that Indian man is also seated adjacent to his wife is given by :

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) $\frac{1}{2}$

16. Let $'A'$ and $'B'$ be two independent events. The probability that both A and B happen is $\frac{1}{12}$ and the probability that neither A nor B happen is $\frac{1}{2}$, then $\{3P(A) - 4P(B)\}$ may be

(a) 1 or 0 (b) $\frac{7}{12}$ or 0

(c) 0 or $-\frac{7}{12}$ (d) $-\frac{7}{12}$ or 1

17. An urn contains 2 white and 2 black balls , a ball is drawn at random , if it is white it is not replaced into the urn , otherwise it is dropped along with one another ball of same color. The process is repeated , probability that the third drawn ball is black , is :

(a) $\frac{31}{60}$ (b) $\frac{41}{60}$

(c) $\frac{29}{30}$ (d) none of these

18. An experiment has ten equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes , then number of outcomes that B must have so that A and B are independent , is :

(a) 2 , 4 or 8 (b) 3 , 6 or 9

(c) 4 or 8 (d) 5 or 10

19. A fair die is tossed repeatedly until a six is obtained , if $'k'$ denotes the number of tosses required , then the conditional probability that $'k'$ is not less than six when it is given that $'k'$ is greater than 3 , is equal to :

(a) $\frac{5}{36}$ (b) $\frac{125}{216}$

(c) $\frac{25}{36}$ (d) $\frac{25}{216}$

20. A box contain 15 coins , 8 of which are fair and the rest are biased. The probability of getting a head on fair coin and biased coin is $\frac{1}{2}$ and $\frac{2}{3}$ respectively. If a coin is drawn randomly from the box and tossed twice , first time it shows head and the second time it shows tail , then the probability that the coin drawn is fair , is given by :

(a) $\frac{5}{8}$ (b) $\frac{9}{16}$

(c) $\frac{3}{8}$ (d) $\frac{5}{16}$

21. A person goes to office either by car , scooter , bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late , if he takes car , scooter , bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. If it is given that he reached office in time then the probability that he travelled by car is :

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

22. Let $'K'$ be the integral values of x for which the inequation $x^2-9x+18<0$ holds. If three fair dice are rolled together , then the probability that the sum of the numbers appearing on the dice is K, is given by :

(a) $\frac{41}{216}$ (b) $\frac{5}{24}$

(c) $\frac{1}{24}$ (d) $\frac{31}{216}$

23. Let set $'S'$ contains all the matrices of 3×3 order in which all the entries are either 0 or 1. If a matrix is selected randomly from set $'S'$ and it is found that it contains exactly five of the entries as 1 , then the probability that the matrix is symmetric , is given by :

(a) $\frac{63}{256}$ (b) $\frac{3}{128}$

(c) $\frac{2}{21}$ (d) $\frac{7}{512}$

24. Let two positive real numbers $'x'$ and $'y'$ are chosen randomly , where $x\in[0,1]$ and $y\in[0,1]$. The probability that $x+y\le1$, given that $x^2+y^2\ge\frac{1}{4}$, is :

(a) $\frac{8+\pi}{16+\pi}$ (b) $\frac{\pi-8}{\pi-16}$

(c) $\frac{4-\pi}{8+\pi}$ (d) $\frac{\pi+2}{16-\pi}$

25. Let a natural number $'N'$ be selected at random from the set of first hundred natural numbers. The probability that $N+\frac{225}{N}$ is not greater than 30 is given by :

(a) 0.01 (b) 0.05

(c) 0.25 (d) 0.025

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. For two events A and B , if $P\left(\frac{B}{A}\right)=\frac{1}{2}$, $P(A)=P\left(\frac{A}{B}\right)=\frac{1}{4}$, then the correct statements are :

(a) $P(A\cap B)=\frac{3}{8}$ (b) $P(\bar{A}\cup\bar{B})=\frac{7}{8}$

(c) $P\left(\frac{\bar{A}}{\bar{B}}\right)=\frac{3}{4}$ (d) $P\left(\frac{\bar{B}}{\bar{A}}\right)=\frac{1}{2}$

27. Let A , B , C be three independent events , where $3P(A)=2P(B)=4P(C)=1$, then :

(a) probability of occurrence of exactly 2 of the three events is $\frac{1}{4}$.

(b) probability of occurrence of at least one of the three events is $\frac{3}{4}$.

(c) probability of occurrence of all the three events is $\frac{1}{24}$.

(d) probability of occurrence of exactly one of the three events is $\frac{11}{24}$.

28. Let a bag contain 15 balls in which the balls can have either black colour or white colour. If B_n is the event that bag contains exactly n black balls and its probability is proportional to n^2 , and E is the event of getting a black ball when a ball is drawn randomly from the bag , then :

(a) $\sum_{n=0}^{15}P(B_n)=1$

(b) $P(E)=\frac{24}{31}$

(c) $P\left(\frac{B_5}{E}\right)=\frac{5}{376}$

(d) $P\left(\frac{B_5}{E}\right)=\frac{5}{576}$

29. Let the events $'A'$ and $'B'$ be mutually exclusive and exhaustive in nature , then :

(a) $P(A)\le P(\bar{B})$

(b) $P(\bar{A}\cap\bar{B})=0$

(c) $P(A\cup B)=P(A)+P(B)$

(d) $P(A\cup B)=1-P(\bar{A})P(\bar{B})$

30. There are four boxes B_1, B_2, B_3 and B_4. Box B_i contain i cards and on each card a distinct number is printed, the printed number varies from 1 to i for box B_i. If a box is selected randomly, then probability of occurrence of box B_i is given by $\left(\frac{i}{10}\right)$ and if a card is drawn randomly from it then E_i represents the event of occurrence of number i on the card, then :

(a) value of $P(E_1)$ is $\frac{2}{5}$

(b) inverse probability $P\left(\frac{B_3}{E_2}\right)$ is $\frac{1}{3}$

(c) conditional probability $P\left(\frac{E_3}{B_2}\right)$ is zero

(d) value of $P(E_3)$ is $\frac{1}{4}$

Assertion Reasoning questions :
(Questions No. 31-35)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

31. Statement 1 : Let any two digit number is raised with power $4K + 2$, where $K \in N$, then the probability that unit's place digit of the resultant number is natural multiple of 3 is 1/3

because

Statement 2 : If any two digit number is raised with power $4K + 2$, $K \in N$, then digit at units place can be $0, 1, 4, 5, 6, 9$.

32. Statement 1 : Let 'A' and 'B' be two dependent events and if $\{P(A \cup B)\}^2 = P(\bar{B})$, then least value of $P(A \cup B)$ is $2\sin\left(\frac{\pi}{10}\right)$,

because

Statement 2 : $P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$, where $P(\bar{B}) \neq 0$

33. Statement 1 : In a binomial probability distribution $B(n, p = 1/4)$, if the probability of at least one success is not less than 0.90, then value of 'n' can be $\log_{\frac{2}{\sqrt{3}}} 12$

because

Statement 2 : In the given binomial probability distribution 'n' is greater than or equal to $\log_{\frac{4}{3}} 10$

34. Statement 1 : Let A and B be any two events of a random experiment, where $P(A) = \frac{4}{5}$ and $P(B) = \frac{1}{3}$, then the value of $P(A \cap B)$ lies in $\left[\frac{2}{15}, \frac{1}{3}\right]$

because

Statement 2 : For any two events A and B, $\max\{P(A), P(B)\} \leq P(A \cup B) \leq 1$ and $P(A \cap B) \leq \min\{P(A), P(B)\}$.

35. Statement 1 : Let an ellipse of eccentricity $\frac{4}{5}$ be inscribed in a circle and a point within the circle be chosen randomly, then the probability that the point lies outside the ellipse is $\frac{2}{5}$

because

Statement 2 : The area of an ellipse of eccentricity 'e' is given by $\pi a^2\sqrt{1-e^2}$ square units, where 'a' represents the radius of auxiliary circle of the ellipse.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

For a biased coin , let the probability of getting head be $\frac{2}{3}$ and that of tail be $\frac{1}{3}$. If A_n denotes the event of tossing the coin till the difference of the number of heads and tails become $'n'$, then answer the following questions.

1. If $n = 2$, then the probability that the experiment ends with more number of heads than tails , is equal to :

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$

(c) $\frac{5}{9}$ (d) $\frac{4}{9}$

2. If it is given that the experiment ends with a head for $n = 2$, then the probability that the experiment ends in minimum number of throws , is equal to :

(a) $\frac{3}{5}$ (b) $\frac{4}{9}$ (c) $\frac{3}{8}$ (d) $\frac{5}{9}$

3. If E is the event that the last two throws show either two consecutive heads or tails , then the value of $P\left(\frac{E}{A_n}\right)$ is equal to :

(a) 1 (b) $1-\left(\frac{5}{9}\right)^n$

(c) $1-\left(\frac{4}{9}\right)^n$ (d) 0

Comprehension passage (2)
(Questions No. 4-6)

Consider a bag containing six different balls of three different colours. If it is known that the colour of the balls can be white , green or red , then answer the following questions.

4. The probability that the bag contains 2 balls of each colour is :

(a) $\frac{1}{10}$ (b) $\frac{1}{7}$ (c) $\frac{1}{9}$ (d) $\frac{1}{8}$

5. If three balls are picked up at random from the bag and all the balls are found to be of different colour , then the probability that bag contained 4 white balls , is :

(a) $\frac{7}{25}$ (b) $\frac{1}{7}$

(c) $\frac{1}{14}$ (d) $\frac{1}{10}$

6. If three balls are picked up at random and found to be one of each colour , then the probability that bag contained equal number of white and green balls is equal to :

(a) $\frac{3}{14}$ (b) $\frac{1}{10}$

(c) $\frac{7}{25}$ (d) $\frac{2}{7}$

Comprehension passage (3)
(Questions No. 7-9)

A fair die is tossed repeatedly until a six is obtained. If X denote the number of tosses required , then answer the following questions.

7. The probability that $X = 3$ equals

(a) $\frac{25}{216}$ (b) $\frac{25}{36}$

(c) $\frac{5}{36}$ (d) $\frac{125}{216}$

8. The probability that $X \geq 3$ equals

(a) $\frac{125}{216}$ (b) $\frac{25}{36}$

(c) $\frac{5}{36}$ (d) $\frac{25}{216}$

9. The conditional probability that $X \geq 6$ given $X > 3$ equals

(a) $\frac{125}{216}$ (b) $\frac{25}{216}$

(c) $\frac{5}{36}$ (d) $\frac{25}{36}$

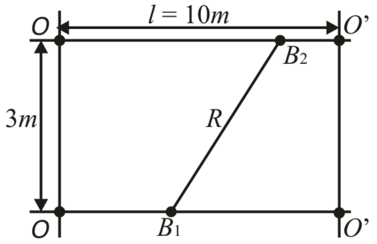
Questions with Integral Answer : (Questions No. 10-14)

10. If the papers of 4 students can be checked by any one of the 7 teachers. If the probability that all the 4 papers are checked by exactly 2 teachers is P, then the value $49P$ is equal to ....

11. A bag contain 3 black and 3 white balls , from the bag John randomly pick three balls and then drop 3 balls of red colour into the bag. If now John randomly pick three balls from the bag and the probability of getting all the three balls of different colour is p, then value of $\frac{100}{3}p$ is ....

12. Let a cubical die has four blank faces , one face marked with 2 , another face marked with 3 , if the die is rolled and the probability of getting a sum of 6 in 3 throws is p, then value of $\frac{432}{13}p$ is equal to ....

13. There are two parallel telephone lines of length $l = 10m$ which are $3m$ apart as showin figure. It is known that there is a break in each of them , the location of the break being unknown , if the probability that the distance *'R'* between the breaks is not larger than $5m$ is p, then $\frac{25}{2}p$ is ....



14. A person while dialing a telephone number forgets the last three digits of the number but remembers that exactly two of them are same. He dials the number randomly , if the probability that he dialed the correct number is P, then value of $(1080P)$ is ....

Matrix Matching Questions : (Questions No. 15-16)

15. Consider a cube having the vertex points A, B, C, D, E, F, G, and H. If randaonly three corner points are selected to form a triangle then match the following columns for the probability of the nature of triangle.

Column (I)	**Column (II)**
(a) Probability that the triangle is scalene	(p) $\frac{6}{7}$
(b) Probability that the triangle is right-angled	(q) $\frac{4}{7}$
(c) Probability that the triangle is isosceles with exactly	(r) $\frac{1}{7}$
two equal sides	(s) $\frac{3}{14}$
(d) Probability that the triangle is equailateral	(t) $\frac{3}{7}$

16. Five unbiased cubical dice are rolled simultaneously. Let m and n be the smallest and the largest number appearing on the upper faces of the dice , then match the probabilitiy given in the column II corresponding to the events given in the column I :

Column (I)	**Column (II)**
(a) $m = 3$	(p) $\left(\frac{2}{3}\right)^5$
(b) $n = 4$	(q) $\left(\frac{2}{3}\right)^5 + \left(\frac{1}{3}\right)^5 - \left(\frac{1}{2}\right)^4$
(c) $2 \le m \le 4$	(r) $\left(\frac{5}{6}\right)^5 - \left(\frac{1}{3}\right)^5$
(d) $m = 2$ and $n = 5$	(s) $\left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$

ANSWERS Exercise No. (1)

1. (c)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (b)
6. (b)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (d)
11. (a)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (c)
16. (c)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (b)
21. (a)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (a)
26. (b , c , d)	**27.** (a , b , c , d)	**28.** (a , b , d)	**29.** (a , b , c)	**30.** (a , b , c)
31. (d)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (b)

ANSWERS Exercise No. (2)

1. (b)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (c)
6. (a)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (6)
11. (9)	**12.** (2)	**13.** (8)	**14.** (4)	
15. (a) → t (b) → p (c) → p (d) → r	**16.** (a) → s (b) → s (c) → r (d) → q			

Chapter 7

Matrices

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-15)

1. Let $A=\left[a_{ij}\right]_{3\times3}$; $a_{ij}=\sin^3(i-j)$, then

(a) det $(A)=\sin 1$ (b) det $(A)=0$

(c) det $(A)>0$ (d) det $(A)<0$

2. Let $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, then $A+A^T=I$ if the values of $'\alpha'$ belong to :

(a) $2n\pi\pm\dfrac{\pi}{6}\ ;\ n\in I$ (b) $(2n+1)\pi\pm\dfrac{\pi}{3}\ ;\ n\in I$

(c) $(2n+1)\pi\pm\dfrac{2\pi}{3}\ ;\ n\in I$ (d) $2n\pi\pm\dfrac{2\pi}{3}\ ;\ n\in I$

3. Let $A=\left[a_{ij}\right]_{3\times3}$, $B=\left[b_{ij}\right]_{3\times3}$ and $C=\left[c_{ij}\right]_{3\times3}$ be three matrices , where det(A) = 2 and b_{ij} , c_{ij} are the corresponding cofactors of a_{ij} and b_{ij} respectively , then $det(2AB^TC)$ is equal to :

(a) $\sum\limits_{r=1}^{10}{}^{10}C_r$ (b) $\sum\limits_{r=1}^{10}{}^{11}C_r$

(c) $\sum\limits_{r=1}^{11}{}^{10}C_{r-1}$ (d) $\sum\limits_{r=1}^{11}{}^{11}C_{r+1}$

4. Let $A=\left[a_{ij}\right]_{10\times10}$ be a matrix for which $a_{ij}=2i^3+ij-2i^2j+\left[\dfrac{i+j}{4}\right]-\sin^2(i-j)$, where [.] represents the greatest integer function , then $trace(A)$ is equal to :

(a) 420 (b) 400

(c) 410 (d) 500

5. Let *'S'* be the set of all 3×3 symmetric matrices for which all the entries are either 1 or 2 , if five of these entries are 2 and four of them are 1 , then $n(S)$ is equal to : ($n(S)$ represents the cardinal number of S)

(a) 10 (b) 12

(c) 20 (d) 18

6. If A and B are two square matrices of order $n\times n$ and $AB=B$, $BA=A$, then $A^2+B^2=2I$ holds true for the condition :

(a) $|A|=|B|=0$ (b) $|A|=|B|\neq0$

(c) $|A|\neq|B|\neq0$ (d) $|A|$ and $|B|$ are non-zero

7. Let $A=\left[a_{ij}\right]_{3\times3}$; where $a_{ij}=\begin{cases}\min\{i\ ,\ j\}\ ;\ i\neq j\\ \left[\dfrac{2i+j}{2}\right]\ ;\ i=j\end{cases}$

and [.] represents the greatest integer function , then $det\{adj(adj(A))\}$ is equal to :

(a) 5 (b) 25

(c) 625 (d) 125

8. Total number of matrices that can be formed using all the seven different one digit numbers such that no digit is repeated in any matrix , is given by :

(a) 7! (b) $(7)^7$

(c) 2(7!) (d) 7(7!)

9. Let $A=\left[a_{ij}\right]_{3\times3}$ and $B=\left[b_{ij}\right]_{3\times3}$ be two matrices and $\alpha,\beta,\in\{1,2,3\}$, then which one of the following is always true :

(a) $\sum\limits_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}=\sum\limits_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}$

(b) $\sum\limits_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}\neq\sum\limits_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}$

(c) $\sum\limits_{\alpha=1}^{3}\left(\sum\limits_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}\right)=\sum\limits_{\alpha=1}^{3}\left(\sum\limits_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}\right)$

(d) $\sum\limits_{\alpha=1}^{3}\left(\sum\limits_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}\right)\neq\sum\limits_{\alpha=1}^{3}\left(\sum\limits_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}\right)$

10. Let $'\omega'$ be the non-real cube root of unity , where $A=\begin{bmatrix}\omega&0&0\\0&\omega&0\\0&0&\omega\end{bmatrix}$, then A^{2010} is equal to :

(a) A (b) $-A$

(c) 0 (d) I

11. Let $A=\begin{bmatrix}\cos(\pi/6) & \sin(\pi/6)\\ -\sin(\pi/6) & \cos(\pi/6)\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$;

where $C=ABA^T$, then $A^TC^{2010}A$ is equal to :

(a) $\begin{bmatrix}1 & 2010\\ 0 & 1\end{bmatrix}$

(b) $\begin{bmatrix}\sqrt{3}/2 & -1\\ 2010 & 1\end{bmatrix}$

(c) $\begin{bmatrix}\sqrt{3}/2 & 2010\\ 1 & -1\end{bmatrix}$

(d) $\begin{bmatrix}1 & \sqrt{3}/2\\ 0 & 2010\end{bmatrix}$

12. Let $\alpha_k=k(^8C_k)$ and $\beta_k=(2-k)\,^8C_k$, and $A_k=\begin{bmatrix}\alpha_k & 0\\ 0 & \beta_k\end{bmatrix}$. If $\sum\limits_{k=1}^{7}A_k=\begin{bmatrix}p & 0\\ 0 & q\end{bmatrix}$; then value of $(p+q)$ is equal to :

(a) 1020 (b) 508

(c) 204 (d) 420

13. Let $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & -2 & 4\end{bmatrix}$ and $6A^{-1}=A^2+pA+qI$, then $(2p+q)$ is equal to :

(a) 0 (b) -1

(c) 1 (d) 2

14. If matrix $A=\begin{bmatrix}a & b & c\\ b & c & a\\ c & a & b\end{bmatrix}$, where $a, b, c \in \mathbf{C}$, $abc=1$

and $AA^T=I$, then $a^3+b^3+c^3$ is equal to :

(a) 0 (b) $a+b+c$

(c) 3 (d) $3+a+b+c$

15. Let $A=\left[a_{ij}\right]_{3\times 3}$ represents a matrix and $(-1)^{i+j}a_{ij}+(-1)^{j+k}a_{jk}+(-1)^{k+i}a_{ki}=0$ for all i, j, k belongs to $\{1, 2, 3\}$, then $'A'$ is :

(a) symmetric matrix.

(b) singular matrix.

(c) non-singular matrix.

(d) orthogonal matrix.

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. Let $\theta \in [0, 2\pi)$, $\phi \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and

$$A=\begin{bmatrix}\sin\phi & \sin\theta & \cos\theta\\ -\sin\theta & -\sin\phi & 1\\ \cos\theta & 1 & \sin\phi\end{bmatrix}, \text{ then}$$

(a) det (A) is independent from θ.

(b) det (A) is independent from ϕ.

(c) det $(A) \in \left[\frac{-3\sqrt{3}}{8}, \frac{-1}{8}\right]$.

(d) det $(A) \in [-1, 1]$.

17. Let $A=\left[a_{ij}\right]_{3\times 3}$, where $a_{ij}=\begin{cases}\min\{i, j\}, & i \neq j\\ \left[\dfrac{i+2j}{10}\right] & ; i=j\end{cases}$.

If a_{ij} represents the element of i^{th} row and j^{th} column in matrix $'A'$, then : ([.] represents G.I.F.).

(a) det $(A)=0$

(b) det $(A)=4$

(c) A is symmetric matrix

(d) $Tr(A)=0$

18. Let $A(\theta)=\begin{bmatrix}\sin\theta & i\cos\theta\\ i\cos\theta & \sin\theta\end{bmatrix}$, where $i^2=-1$, then

(a) $A(\theta)$ is invertible $\forall\ \theta \in R$.

(b) Inverse of $A(\theta)=A(-\theta)$.

(c) Inverse of $A(\theta)=A(\pi-\theta)$.

(d) $A(\theta)+A(\pi+\theta)=O_{2\times 2}$.

19. Let $P=\begin{pmatrix}\dfrac{\sqrt{3}}{2} & \dfrac{1}{2}\\ \dfrac{-1}{2} & \dfrac{\sqrt{3}}{2}\end{pmatrix}$ and $A=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$, then

(a) $(A^{-1})^n=\begin{bmatrix}1 & -n\\ 0 & 1\end{bmatrix}$

(b) Matrices A and P both are orthogonal matrix

(c) If $A^n=I+nB$, then det $(B)=0$

(d) $det\{adj\,(adj\,(2AP))\}=4$

20. Let matrix $'A'$ be singular matrix , and $\theta \in [0\ ,\ \pi]$.

If $A = \begin{bmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{bmatrix}$, then

possible values of $'\theta'$ can be :

(a) $\dfrac{7\pi}{24}$ (b) $\dfrac{11\pi}{24}$

(c) $\dfrac{23\pi}{24}$ (d) $\dfrac{19\pi}{24}$

Assertion Reasoning questions : (Questions No. 21-25)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. Statement 1 : Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ and $A^2 = 3A - 2I$, then $A^8 = 255A - 256\,I_2$

because

Statement 2 : $A^n = \begin{bmatrix} 1 & 0 \\ 2^n - 1 & 2^n \end{bmatrix}$

22. Let A and B be two square matrices of order 3 , and $'O'$ represents the null matrix of order 3×3.

Statement 1 : If $AB = 0$, and A is non-singular matrix , then matrix B is necessarily a singular matrix

because

Statement 2 : Product of two equal order square matrices can only be zero matrix if both the matrices are not non-singular matrices.

23. Let A be a 2×2 matrix with real entries , and satisfy the condition $A^2 = I$, where $'I'$ is unit matrix of order 2.

Statement 1 : If $A \neq I$ and $A \neq -I$, then $det(A) = -1$

because

Statement 2 : If $\pm A \neq I$, then $Tr(A) \neq 0$

24. Statement 1 : Let $A^5 = 0$ and $A^n \neq I$ for all $n \in \{1\ ,2\ ,\ 3\ ,\ 4\}$, then $(I - A)^{-1} = A^4 + A^3 + A^2 + A + I$

because

Statement 2 : $1 + x + x^2 + x^3 + x^4 = \left(\dfrac{1-x^5}{1-x}\right)$, where $x \neq 1$.

25. Let A and B be square matrices of order 3 , where $A = \left[a_{ij}\right]_{3\times3}$; $a_{ij} = \sin^3(i-j)$.

Statement 1 : If $n = 7!$, then $B^T A^n B$ is skew-symmetric matrix

because

Statement 2 : determinant value of skew-symmetric matrix of odd order is always zero.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $'S'$ be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. If five of these entries are 1 and four of them are 0 , then answer the following questions.

1. The number of matrices in $'S'$ is :

(a) 12 (b) 6

(c) 9 (d) 3

2. The number of matrices A in $'S'$ for which the system of linear equation $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ has a unique solution, is :

(a) less than 4

(b) at least 4 but less than 7

(c) at least 7 but less than 10

(d) at least 10

3. The number of matrices A in $'S'$ for which the system of linear equation $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ is inconsistent , is

(a) 0 (b) more than 2

(c) 2 (d) 1

Comprehension passage (2)
(Questions No. 4-6)

For a given square matrix $'A'$, if $AA^T=A^TA=I$ holds true , then matrix is termed as orthogonal matrix. If $a , b , c\in R$ and matrix $'P'$ is orthogonal , where

$P=\begin{bmatrix}0 & a & a\\2b & b & -b\\c & -c & c\end{bmatrix}$, then answer the following questions :

4. If square matrices of order 2 is formed with the entries 0 , a , b and c , then maximum number of matrices which can be formed without repetition of the entries , is equal to :

(a) 840 (b) 24 (c) 256 (d) 192

5. Let $\alpha , \beta , \gamma\in R$ and matrix $Q=\begin{bmatrix}\alpha a^2 & 0 & 0\\0 & \beta b^2 & 0\\0 & 0 & \gamma c^2\end{bmatrix}$.

If $'Q'$ is orthogonal matrix then maximum number of ordered triplets (α , β , γ) which are possible is given by :

(a) 1 (b) 8

(c) 2 (d) 6

6. If $k=\dfrac{1}{(abc)^2}$; then number of positive integral solutions for the equation $x_1 . x_2 . x_3=k$, is equal to :

(a) 18 (b) 20

(c) 36 (d) 72

Comprehension passage (3)
(Questions No. 7-9)

Let $A=\begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\3 & 2 & 1\end{bmatrix}$, and R_1 , R_2 , R_3 be the row matrices satisfying the relations , $R_1A=\begin{bmatrix}1 & 0 & 0\end{bmatrix}$, $R_2A=\begin{bmatrix}2 & 3 & 0\end{bmatrix}$ and $R_3A=\begin{bmatrix}2 & 3 & 1\end{bmatrix}$. If B is square matrix of order 3 with rows R_1 , R_2 , R_3 , then answer the following questions.

7. The value of $det(B)$ is equal to :

(a) –3 (b) 3

(c) 0 (d) 1

8. Let $C=(2A^{100}.B^3)-(A^{99}.B^4)$, then value of $det(C)$ is equal to :

(a) 27 (b) –27

(c) 100 (d) –100

9. Sum of all the elements of matrix B^{-1} is equal to :

(a) 8 (b) 0

(c) 5 (d) 10

Questions with Integral Answer :
(Questions No. 10-13)

10. Let matrix $A=\begin{bmatrix}1 & 1 & 3\\5 & 2 & 6\\-2 & -1 & -3\end{bmatrix}$, then the least positive integer $'K'$ for which A^K becomes null matrix , is equal to ....

11. Let $A=\left[a_{ij}\right]_{4\times 4}$, where $|A| = 2$ and $B=\left[b_{ij}\right]_{4\times 4}$. If b_{ij} is the cofactor of a_{ij}, and $AB^T = C$, then sum of diagonal elements of matrix $'C'$ is equal to ....

12. Let a, x, $y \in R$, where $x+y=0$, and the system of equations is given by :

$$\begin{bmatrix} 2x^2 & -2ay^2 \\ x^2+axy & xy \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} (a+1)^2 \\ 1 \end{bmatrix}$$

If the system has at least one solution, then number of possible integral value(s) of $'a'$ is/are ....

13. Let a, x, $y \in R$, and matrices A, B and C be defined as $A=\begin{bmatrix} 2^{|x|}+|x| & y \\ x^2 & -y^2 \end{bmatrix}$, $B=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $C=\begin{bmatrix} x^2+a \\ 1 \end{bmatrix}$. If the matrix equations $AB = C$ is having only one solution, then total number of possible value(s) of $'a'$ is/are ....

Matrix Matching Questions :
(Questions No. 14-15)

14. Let $A=\begin{bmatrix} 1 & 4 & 5 \\ \alpha & 8 & 8\alpha-6 \\ 1+\alpha^2 & 8\alpha+4 & 2\alpha+21 \end{bmatrix}$, then match columns (I) and (II) for the values of α and the rank of matrix $'A'$.

Column (I)	**Column (II)**
(a) If $\alpha=2$, then rank of matrix A is :	(p) 1
(b) If $\alpha=-1$, then rank of matrix A is :	(q) 2
(c) If $\alpha\in R-\{2\}$, then rank of matrix A can be :	(r) 3
(d) If $\alpha=4$, then rank of matrix A is :	(s) 0

15. Match columns (I) and (II)

Column (I)	**Column (II)**
(a) Let $A=\left[a_{ij}\right]_{3\times3}$ and $B=\left[k^{i-j}a_{ij}\right]_{3\times3}$; if $k_1\|A\|+k_2\|B\|=0$; where $\|A\|\neq 0$, then (k_1+k_2) is	(p) 0
(b) Maximum value of third order determinant if each of its entries are either 1 or –1, is	(q) 4
(c) If $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix}=\begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\beta \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma$ is equal to :	(r) 1
(d) $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}=Ax+B$ where A and B are determinant of 3×3, then $(A + 2B)$ is equal to	(s) 2

ANSWERS Exercise No. (1)

1. (b)
2. (c)
3. (c)
4. (c)
5. (b)
6. (d)
7. (c)
8. (c)
9. (c)
10. (d)
11. (a)
12. (b)
13. (b)
14. (d)
15. (b)
16. (a , c)
17. (b , c , d)
18. (a , c , d)
19. (a , c , d)
20. (a , b , c , d)
21. (d)
22. (a)
23. (c)
24. (b)
25. (d)

ANSWERS Exercise No. (2)

1. (a)
2. (b)
3. (b)
4. (d)
5. (b)
6. (c)
7. (b)
8. (b)
9. (c)
10. (3)
11. (8)
12. (3)
13. (1)

14. (a) → p
(b) → q
(c) → q , r
(d) → r

15. (a) → p
(b) → q
(c) → r
(d) → p

Chapter
8

Determinants

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-20)

1. If system of equations : $4x+5y-z=0$, $x-y-4z=0$ and $(K+1)x+(2K-1)y+(K-4)z=0$ have non-trivial solution , then :

(a) $K=3$ (b) $K=0$

(c) $K=3$ or 0 (d) $K \in R$

2. Let (x, y, z) be points with integer co-ordinates satisfying the system of homogeneous equation :

$$3x-y-z=0$$
$$-3x+z=0$$
$$-3x+2y+z=0.$$

Then the number of such points for which $x^2+y^2+z^2 \le 100$ are :

(a) 6 (b) 5

(c) 10 (d) 7

3. If $\theta \in [0, 2\pi)$ and $A=\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$;

then det (A) lies in the interval :

(a) $[2, 4]$ (b) $[2, 3]$

(c) $[1, 4]$ (d) $(2, 4)$

4. The existence of unique solution for the system of equations , $x+y+z=p$, $5x-y+qz=10$ and $2x+3y-z=6$ depends on :

(a) 'p' only. (b) 'q' only.

(c) 'p' and 'q' both. (d) neither 'p' nor 'q'.

5. Let $f(x)=\begin{vmatrix} 2 & 1 & 0 \\ -3 & 2 & -1 \\ x|x| & \tan^{-1}x & \sin\pi[x] \end{vmatrix}$, where [.] represents the greatest integer function , then $\int_{-2}^{2} f(x)dx$ is :

(a) $2\cos^2 1$ (b) $\sin^2 2+\sec 1$

(c) $1+\cos 2-2\sin^2 1$ (d) $\cos 2+1-2\cos^2 1$

6. Let $f(x)$, $g(x)$ and $h(x)$ be cubic functions of x and

$$\phi(x)=\begin{vmatrix} f'(x) & f''(x) & f'''(x) \\ g'(x) & g''(x) & g'''(x) \\ h'(x) & h''(x) & h'''(x) \end{vmatrix}, \text{ then}$$

(a) $\phi''(x)=2$.

(b) graph of $\phi(x)$ is symmetric about origin.

(c) graph of $\phi(x)$ is symmetric about y-axis.

(d) $\phi(x)$ is polynomial of degree 3.

7. Let $P_K=a^K+b^K$, where $K \in N$ & $(a+b)=2ab=4$,

then value of $\begin{vmatrix} 3 & 1+P_1 & 1+P_2 \\ 1+P_1 & 1+P_2 & 1+P_3 \\ 1+P_2 & 1+P_3 & 1+P_4 \end{vmatrix}$ is equal to :

(a) 4 (b) 0 (c) 8 (d) 2

8. Let α, β and γ be internal angles of a triangle ,

then minimum value of $\begin{vmatrix} -2 & \cos\gamma & \cos\beta \\ \cos\gamma & -1 & \cos\alpha \\ \cos\beta & \cos\alpha & -1 \end{vmatrix}$ is equal to :

(a) 0 (b) 1 (c) -1 (d) 2

9. Let α, β, γ and δ be the positive real roots of the equation $x^4-12x^3+px^2+qx+81=0$, where

$p, q \in R^+$, then value of $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is equal to :

(a) $-\dfrac{5}{2}$ (b) $-\dfrac{9}{2}$

(c) $-\dfrac{3}{2}$ (d) none of these

10. Let set 'S' consists of all the determinants of order 3×3 with entries zero or one only and set 'P' is subset of 'S' consisting of all the determinants with value 1. If set 'Q' is subset of 'S' consisting of all the determinants with value -1, then :

(a) $n(S)=n(P)+n(Q)$ (b) $n(P)=2n(Q)$

(c) $n(P)=n(Q)$ (d) $P \cup Q=S$

11. Let $'M'$ be a 3×3 matrix, where $MM^T = I$ and det $(M) = 1$, then :

(a) $\det(M - I) \neq 0$.

(b) det $(M - I)$ is always zero.

(c) det $(M + 2I) = 0$.

(d) det $(M + I)$ is always zero.

12. Let $$px^4 + qx^3 + rx^2 + sx + t = \begin{vmatrix} x-3 & x+4 & 3x \\ x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \end{vmatrix}$$ be an identity in x, where p, q, r, s and t are constants, then $(q + s)$ is equal to :

(a) 52 (b) 51 (c) 50 (d) 102

13. Let A, B and C be the angles of a triangle, where A, $B, C \neq \frac{\pi}{2}$, then the value of $$\begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix}$$ is equal to :

(a) 0 (b) –1 (c) 2 (d) –2

14. Let $\vec{a}_r = x_r\hat{i} + y_r\hat{j} + z_r\hat{k}$, where $r = 1, 2, 3$ be three vectors and $|\vec{a}_r| = r$, $\vec{a}_1.\vec{a}_2 = \vec{a}_2.\vec{a}_3 = \vec{a}_3.\vec{a}_1 = 1$, then value of $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$ is equal to :

(a) ±4 (b) $\pm2\sqrt{6}$

(c) $\pm6\sqrt{2}$ (d) $\pm\sqrt{6}$

15. Let $f(x) = \begin{vmatrix} 1+\alpha & 1+\alpha x & 1+\alpha x^2 \\ 1+\beta & 1+\beta x & 1+\beta x^2 \\ 1+\gamma & 1+\gamma x & 1+\gamma x^2 \end{vmatrix}$; then $f(x)$ is independent of :

(a) α and β (b) β and γ

(c) α and γ (d) α, β and γ

16. If a homogenous system of equations is represented by: $ax + by + cz = 0$, $bx + cy + az = 0$ and $cx + ay + bz = 0$, and infinite ordered triplets (x, y, z) are possible without any linear constraint, then

(a) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$

(b) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

(c) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

(d) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$

17. If a determinant is chosen at random from the set of all determinants of order 2×2 with elements zero or one only, then the probability that the value of determinant chosen is non-negative is equal to :

(a) $\frac{3}{16}$ (b) $\frac{5}{8}$

(c) $\frac{3}{8}$ (d) $\frac{13}{16}$

18. Let α, β, γ be non-zero real numbers, then system of equations in x, y and z, $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} - \frac{z^2}{\gamma^2} = 1$, $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} = 1$ and $\frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} = 1 + \frac{x^2}{\alpha^2}$ has :

(a) no solution.

(b) unique solution.

(c) infinitely many solutions.

(d) finitely many solutions.

19. The number of values of $'K'$ for which the system of equations

$(2K + 1)x + (3K + 1)y + K + 2 = 0$ and

$(5K + 1)x + (7K + 1)y + 4K + 2 = 0$

is consistent and indeterminate is given by :

(a) 0 (b) 1

(c) 2 (d) infinite

20. If the system of equations ; $2x + y - 3 = 0$, $6x + ky - 4 = 0$ and $6x + 3y - 10 = 0$ is consistent, then

(a) $k = 1$ (b) $k = 3$

(c) $k = 1$ or 3 (d) $k \in \phi$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let a, b, c be non-zero real numbers and function $f(x)$ is given by $\begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$, then $f(x)$ is divisible by :

(a) x^4 (b) x^6

(c) $x^2 - a^2 - b^2 - c^2$ (d) $x^2 + a^2 + b^2 + c^2$

22. System of equations : $x + 3y + 2z = 6$, $x + \lambda y + 2z = 7$, $x + 3y + 2z = \mu$ has :

(a) Infinitely many solutions if $\lambda = 4$, $\mu = 6$.

(b) No solution if $\lambda = 5$, $\mu = 7$.

(c) Unique solution if $\lambda = 5$, $\mu = 7$.

(d) No solution if $\lambda = 3$, $\mu = 5$.

23. Consider the system of linear equations in x, y, z :

$$2x+7y+7z=0$$
$$(\sin 3\theta)x-y+z=0$$
$$(\cos 2\theta)x+4y+3z=0$$

If the system has non-trivial solutions , then angle $'\theta'$ can be :

(a) $\frac{25\pi}{6}$ (b) $\frac{17\pi}{6}$

(c) 4π (d) $\frac{7\pi}{6}$

24. Let determinant $'D'$ is having all the elements as either 1 or –1. If the product of all the elements of any row or any column of $'D'$ is negative , then it is represented by $'D_N'$. If the order of $'D'$ is 3×3 , then :

(a) minimum value of D_N is –2.

(b) minimum value of D_N is –4.

(c) total number of D_N is 16.

(d) total number of D_N is 32.

25. Let $f(x)$ be real valued polynomial function , and

$$\begin{vmatrix} 1 & x & x \\ x & 2 & x \\ x & x & 3 \end{vmatrix} = x f'(x)-f(x), \text{ then}$$

(a) $\int_1^3 f(x)dx=0$

(b) $\int_{-1}^1 f(x)dx=0$

(c) $y=f(x+2)$ is odd function

(d) $y=|f(x)|$ is symmetrical about line $x-2=0$

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. Statement 1 : Let $\alpha_K=\cos\frac{2K\pi}{9}+i\sin\frac{2K\pi}{9}$ for all $K\in W$, then value of determinant $\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_4 & \alpha_5 & \alpha_6 \\ \alpha_7 & \alpha_8 & \alpha_9 \end{vmatrix}$ is zero

because

Statement 2 : $\sum_{K=1}^{9}\alpha_K=0$

27. Let $f(x)$, $g(x)$ and $h(x)$ be the polynomial functions of degree 3 , 4 and 5 respectively , where

$$\phi(x)=\begin{vmatrix} f'(\theta) & g'(\theta) & h'(\theta) \\ f'(x) & g'(x) & h'(x) \\ f''(\theta) & g''(\theta) & h''(\theta) \end{vmatrix} \text{ and } \theta\in R.$$

Statement 1 : $\phi(x)$ is divisible by $(x-\theta)^2$

because

Statement 2 : $\phi(\theta)=\phi'(\theta)=0$

28. Let $S=\{\Delta_1, \Delta_2, \Delta_3, \ldots, \Delta_n\}$ be the set of 3×3 determinants that can be formed with the distinct non-zero real numbers $a_1, a_2, a_3, \ldots a_9$, where repeatition of elements is not permissible , then

Statement 1 : $\sum_{i=1}^{n}\Delta_i=0$

because

Statement 2 : $n=\prod_{i=1}^{9} i$

29. Let $f(x)=\begin{vmatrix} 1 & 1 & 1 \\ \sin 2x & \sin 4x & \sin 6x \\ \cos^2 2x & \cos^2 4x & \cos^2 6x \end{vmatrix}$

Statement 1 : If $x\in\left(0,\frac{\pi}{2}\right)$, then number of solutions of the equation $f(x)=0$ are five

because

Statement 2 : $|3\sin\pi x|-x=0$ is having five solutions if $x\in R^+$.

30. Let $'A_r'$ represents the number of positive integral solutions of $x+y+z=r$, where $r \in N-\{1, 2\}$,

and $\Delta = \begin{vmatrix} A_r & A_{r+1} & A_{r+2} \\ A_{r+1} & A_{r+2} & A_{r+3} \\ A_{r+2} & A_{r+3} & A_{r+4} \end{vmatrix}$.

Statement 1 : Value of $\Delta = 0$

because

Statement 2 : In a determinant if any two rows or any two columns are identical , then determinant value is zero.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $y = f(x)$ be quadratic function , and

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}.$$

If a , b and c are distinct real numbers , and maximum value of $f(x)$ occurs at point $'V'$, then answer the following questions.

1. Let $A = [a_{ij}]_{3\times 3}$, $a_{ij} = f\left(\frac{i+2j}{3}\right)$ $\forall\ i = j$ and $a_{ij} = 0$ for all $i \neq j$, then det (A) is equal to :

(a) 0 (b) 2 (c) –1 (d) –3

2. Let $'A'$ is the point of intersection of $y = f(x)$ with x-axis and point $B(\alpha\ ,\ f(\alpha))$ is such that chord AB subtends a right angle at $'V'$, then area (in square units) enclosed by $f(x)$ with chord AB is :

(a) $\frac{250}{3}$ (b) $\frac{125}{3}$

(c) $\frac{75}{4}$ (d) $\frac{301}{3}$

3. Let $g(x) = \frac{1}{f(x)}$ $\forall\ x \in (-2\ ,\ 2)$, then total number of points of discontinuity for $y = [g(x)]$ in $[-\sqrt{3}\ ,\ \sqrt{3}]$ are given by : ([.] represents G.I.F.)

(a) 4 (b) 6 (c) 8 (d) 2

Comprehension passage (2)
(Questions No. 4-6)

Consider the matrices , $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$,

$C_1 = \begin{bmatrix} -4 \\ 1 \\ 4 \end{bmatrix}$, $C_2 = \begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix}$ and $C_3 = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$.

Let matrix $'B_1'$ of order 3×3 is formed with the column vectors of the matrices C_1 , C_2 and C_3 , and $B_{n+1} = adj(B_n)$, $n \in N$, then answer the following questions :

4. Matrix addition for $B_2 + B_3 + B_4 + \ldots + B_{100}$ is equal to :

(a) $100\,B_1$ (b) $99\,B_1$

(c) $99\,I_3$ (d) $98\,I_3$

5. Let $M = AB_1^2 + A^2B_2^3 + A^3B_3^4 + \ldots A^{100}B_{100}^{101}$, then det (M) is equal to :

(a) 100 (b) –100

(c) 0 (d) 1000

6. For a variable matrix X. the matrix equation $AX = C_2$ will have :

(a) Unique solution
(b) No solution
(c) Finitely many solutions
(d) Infinitely many solutions

Questions with Integral Answer :
(Questions No. 7-10)

7. If a , b , $c \in I^+$ and $\begin{vmatrix} 1+a^3 & a^2b & a^2c \\ ab^2 & 1+b^3 & b^2c \\ ac^2 & bc^2 & 1+c^3 \end{vmatrix} = 11$, then total number of possible triplets of $(a\ ,\ b\ ,\ c)$ is/are..........

8. Let $f(x) = \begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$, and $I = \int_0^{\pi/2} \{ f(x) + f'(x) \}\, dx$, then the least integer just greater than $'I'$ is equal to ....

9. Let $U_n = \int_0^{\pi/2} \left(\frac{1-\cos 2nx}{1-\cos 2x}\right) dx$, then value of $\begin{vmatrix} U_1 & U_2 & U_3 \\ U_4 & U_5 & U_6 \\ U_7 & U_8 & U_9 \end{vmatrix}$ is equal to ....

10. Consider the system of equations :

$$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$$
$$x + (\cos\alpha)y + (\sin\alpha)z = 0$$
$$x - (\sin\alpha)y + (\cos\alpha)z = 0$$

If λ and α are real numbers , and the system of equations has non-trivial solutions , then number of integral values of λ which are possible for different values of α are ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Let $f\,(x)$ be polynomial function having local minima at $x = \frac{5}{2}$ and $f\,(0) = 2f\,(1) = 2$. If for all $x \in R\,,\ f\,'(x) = \begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2ax+2b & 2ax+2b+1 & 2ax+b \end{vmatrix}$ where $'a'$ and $'b'$ are some constants , then match the following column (I) and II.

Column (I)	**Column (II)**
(a) Value of $(a+b)$	(p) 1
(b) Value of $f(5)$	(q) 0
(c) Number of solutions for $4f(x) = \lvert x-1 \rvert$	(r) –1
(d) $\lim\limits_{x\to\infty}\left(\dfrac{f(x)}{x^2+1}\right)^{\frac{f(x)}{x}}$	(s) 2

12. Consider the system of equations :

$$Kx + y + z = 1$$
$$x + Ky + z = K$$
$$x + y + Kz = K^2$$

Match column (I) and (II) for the values of $'K'$ and the nature of solution for the system of equations.

Column (I)	**Column (II)**
(a) If $K = 1$, then system of equations have	(p) Unique solution.
(b) If $K \neq 1$, then system of equations may have	(q) Infinitely many solutions.
(c) If $K \in R - \{1\,,\,-2\}$, then system of equations have	(r) No solution.
(d) If $K \in \{1\,,\,-2\}$, then system of equations may have	(s) Finitely many solutions.

ANSWERS Exercise No. (1)

1. (d) **2.** (d) **3.** (a) **4.** (b) **5.** (d)

6. (c) **7.** (c) **8.** (c) **9.** (d) **10.** (c)

11. (b) **12.** (b) **13.** (c) **14.** (b) **15.** (d)

16. (d) **17.** (d) **18.** (d) **19.** (c) **20.** (d)

21. (a , d) **22.** (a , b , d) **23.** (a , b , c) **24.** (b , c) **25.** (a , c , d)

26. (b) **27.** (a) **28.** (b) **29.** (b) **30.** (d)

ANSWERS Exercise No. (2)

1. (a) **2.** (b) **3.** (b) **4.** (b) **5.** (c)

6. (b) **7.** (3) **8.** (4) **9.** (0) **10.** (3)

11. (a) $\rightarrow$ r
(b) $\rightarrow$ s
(c) $\rightarrow$ p
(d) $\rightarrow$ q

12. (a) $\rightarrow$ q
(b) $\rightarrow$ p , r
(c) $\rightarrow$ p
(d) $\rightarrow$ q , r

Chapter 9

Logarithm

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-25)

1. If $log_7 2 = m$, then $log_{49} 28$, is equal to :

(a) $2(2m+1)$
(b) $\frac{2m+1}{2}$
(c) $\frac{2}{2m+1}$
(d) $m+1$

2. If $ln\left(\frac{a+b}{2}\right) = \frac{1}{2}(ln\,a + ln\,b)$, where $a , b \in R^+$ then relation between a and b is :

(a) $a = b$
(b) $a = \frac{b}{2}$
(c) $a = 2b$
(d) $a = \frac{b}{3}$

3. The value of $(81)^{\frac{1}{\log_5 3}} + (27)^{\log_9 36} + (3)^{\frac{4}{\log_7 9}}$ is :

(a) 49
(b) 625
(c) 216
(d) 890

4. $7\,log\left(\frac{16}{15}\right) + 5\,log\left(\frac{25}{24}\right) + 3\,log\left(\frac{81}{80}\right)$ is equal to :

(a) 0
(b) 1
(c) $log\,2$
(d) $log\,3$

5. If $A = log_2\left\{log_2\left(log_4 256\right)\right\} + 2log_{\sqrt{2}} 2$, then A is :

(a) 2
(b) 3
(c) 5
(d) 7

6. If $x = log_a(bc)$, $y = log_b(ac)$, $z = log_c(ab)$, then which one of the following is equal to 1 ?

(a) $x + y + z$
(b) $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$
(c) $x\,y\,z$
(d) $(1+x)^{-2} + (1+y)^{-2} + (1+z)^{-2}$

7. If $a = log_{24} 12$, $b = log_{36} 24$, $c = log_{48} 36$, then value of $(1 + abc)$ is :

(a) $2ab$
(b) $2ac$
(c) $2bc$
(d) 0

8. If $a^x = b$, $b^y = c$, $c^z = a$, then value of (xyz) is :

(a) 0
(b) 1
(c) 2
(d) 3

9. $\sum_{r=1}^{89} log_3(\tan(r^o))$ is equal to :

(a) 3
(b) 1
(c) 2
(d) 0

10. $\sum_{r=1}^{n} \frac{1}{log_{2^r} a}$ is equal to :

(a) $\frac{n(n+1)}{2} log_a 2$
(b) $\frac{n(n+1)}{2} log_2 a$
(c) $\frac{(n+1)^n . n^2}{4} log_2 a$
(d) none of these

11. If $log_7\left\{log_5\sqrt{(x^2 + x + 5)}\right\} = 0$, then x is equal to :

(a) 2
(b) 3
(c) 4
(d) –2

12. The value of $(0.05)^{log_{\sqrt{20}}(0.1+.01+.001+\,.....\,\infty)}$ is :

(a) 81
(b) $\frac{1}{81}$
(c) 20
(d) 10

13. If $log_{12} 27 = a$, then $log_6 16$ is :

(a) $2\left(\frac{3-a}{3+a}\right)$
(b) $3\left(\frac{3-a}{3+a}\right)$
(c) $4\left(\frac{3-a}{3+a}\right)$
(d) $2\left(\frac{4-a}{4+a}\right)$

14. If $n = 2010!$, then $\frac{1}{log_2 n} + \frac{1}{log_3 n} + ... + \frac{1}{log_{2010} n}$ is equal to :

(a) -1
(b) 0
(c) 1
(d) 2

15. The number of solution(s) of $log_2(x+5) = 6 - x$ is/are :

(a) 2
(b) 0
(c) 3
(d) 1

16. If $log_{\cos x} \sin x \geq 2$, then the values of $\sin x$ lies in the interval :

(a) $\left[\frac{\sqrt{5}-1}{2}, 1\right]$
(b) $\left(0, \frac{\sqrt{5}-1}{2}\right]$
(c) $\left(0, \frac{1}{2}\right]$
(d) $\left[\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{4}\right]$

17. $log_{\frac{1}{\sqrt{2}}} (\sin x) > 0$, $x \in [0, 4\pi]$, then number of values of x which are integral multiples of $\frac{\pi}{4}$, is :

(a) 4
(b) 12
(c) 3
(d) 10

18. Set of real values of x satisfying the inequation $log_{0.5}(x^2 - 6x + 12) \geq -2$ is :

(a) $(-\infty, 2]$
(b) $[2, 4]$
(c) $[4, \infty)$
(d) none of these

19. Set of real x for which $2^{log_{\sqrt{2}}(x-1)} > (x+5)$ is :

(a) $(-\infty, -1) \cup (4, \infty)$
(b) $(4, \infty)$
(c) $(-1, 4)$
(d) $[1, 4) \cup (4, \infty)$

20. If $log_{0.2}\left(\frac{x+2}{x}\right) \leq 1$, then x belongs to :

(a) $\left(-\infty, -\frac{5}{2}\right] \cup (0, \infty)$

(b) $\left(\frac{5}{2}, \infty\right)$

(c) $(-\infty, -2) \cup (0, \infty)$

(d) none of these

21. If $\left(\frac{1}{2}\right)^{x^2-2x} < \frac{1}{4}$, then set of $'x'$ contains :

(a) $(-\infty, 0)$
(b) $(-\infty, 1)$
(c) $(1, \infty)$
(d) none of these

22. If $log_x\left(\frac{5x - x^2}{4}\right) \geq 0$, then exhaustive set of values of x is :

(a) $[0, 4]$
(b) $(0, 4] - \{1\}$
(c) $(0, 4)$
(d) none of these

23. The value of $\frac{log_2 24}{log_{96} 2} - \frac{log_2 192}{log_{12} 2}$ is :

(a) 3
(b) 0
(c) 2
(d) 1

24. If $log_3 2$, $log_3(2^x - 5)$ and $log_3\left(2^x - \frac{7}{2}\right)$ are in A.P. , then x is equal to :

(a) 2
(b) 3
(c) 4
(d) 8

25. If $log\, x^2 - log\, 2x = 3log\, 3 - log\, 6$, then x is :

(a) 10
(b) 9
(c) 1
(d) 2

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. If $(x)^{\frac{3}{4}(log_3 x)^2 + log_3 x - \frac{5}{4}} = \sqrt{3}$, then x has :

(a) one positive integral value.
(b) one irrational value.
(c) two positive rational values.
(d) no real value.

27. If $x = 9$ satisfy the equation $ln(x^2 + 15a^2) - ln(a-2) = ln\left(\frac{8ax}{a-2}\right)$, then

(a) value of $'a'$ is 3

(b) value of $'a'$ is $\frac{9}{5}$

(c) $x = 15$ is other solution

(d) $x = 12$ is other solution

28. Let $p = \dfrac{ln\,3}{ln\,20}$, then the correct statements are :

(a) p is a rational number

(b) p is an irrational number

(c) p lies in $\left(\dfrac{1}{3}, \dfrac{1}{2}\right)$

(d) p lies in $\left(\dfrac{1}{4}, \dfrac{1}{3}\right)$

29. Let set $'S'$ contain the values of x for which the equation $|x-1|^{(\log_{10} x)^2 - \log_{10} x^2} = |x-1|^3$ is satisfied , then :

(a) total number of elements in $'S'$ are 4

(b) set $'S'$ contains only one fractional number

(c) set $'S'$ contains only one irrational number

(d) total number of elements in $'S'$ are 2

30. If set $'S'$ contains all the real values of x for which $log_{(2x+3)}\, x^2 < 1$ is true , then set $'S'$ contain :

(a) $(log_2 5 , log_2 7)$

(b) $[log_3 4 , log_3 8]$

(c) $\left(-\dfrac{3}{2}, 1\right)$

(d) $(-1 , 0)$

Questions with Integral Answer :
(Questions No. 31-35)

31. Let $x \in (1, \infty)$ and $y \in (1, 16)$, where $xy = 16$. If x and y satisfy the relation $log_y x - log_x y = \dfrac{8}{3}$, then value of $(x-y)$ is equal to ....

32. If $a \in R^+ - \{1\}$, $\alpha = \dfrac{6}{5}(a)^{log_a x . log_{10} a . log_a 5}$ and $\beta = (3)^{log_{10}\left(\frac{x}{10}\right)} + (9)^{log_{100} x + log_4 2}$, where $\alpha - \beta = 0$, then value of $\sqrt{\dfrac{x}{4}}$ is equal to ....

33. If $M = \displaystyle\sum_{r=1}^{4} log_2\left(\sin\left(\frac{r\pi}{5}\right)\right)$, then value of $(2)^{M+4}$ is equal to ....

34. If $\dfrac{ln\,a}{(y-z)} = \dfrac{ln\,b}{(z-x)} = \dfrac{ln\,c}{(x-y)}$, then value of $\left\{(a)^{y^2+yz+z^2} . (b)^{z^2+zx+x^2} . (c)^{x^2+xy+y^2}\right\}$ is equal to ....

35. Total number of integral solution(s) of the equation $x + log_{10}(2^x + 1) = x\, log_{10} 5 + log_{10} 6$ is/are ....

ANSWERS

Exercise No. (1)

1. (b)
2. (a)
3. (d)
4. (c)
5. (c)
6. (b)
7. (c)
8. (b)
9. (d)
10. (a)
11. (c)
12. (a)
13. (c)
14. (c)
15. (d)
16. (b)
17. (a)
18. (b)
19. (b)
20. (a)
21. (d)
22. (d)
23. (a)
24. (b)
25. (b)
26. (a , b , c)
27. (a , c)
28. (b , c)
29. (a , b)
30. (a , b , d)
31. (6)
32. (5)
33. (5)
34. (1)
35. (1)

Chapter
10

Functions

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-40)

1. Which one of the following functions is an odd function ?

(a) $f(x) = log_e\left(\dfrac{x^4+x^2+1}{(x^2+x+1)^2}\right)$

(b) $f(x) = log_e\left(\dfrac{(x+1)(2-x)}{(x+1)(2+x)}\right)$

(c) $f(x)$, where $f(x) + f(y) = f(x+y).f(x-y)$ for all $x\,,\,y \in R$

(d) $f(x) = \dfrac{e^{-|x|}}{1+e^{-|x|}}$

2. Domain of function $f(x) = log_{(2x-1)}(x-1)$ is :

(a) $(1\,,\infty)$ (b) $\left(\dfrac{1}{2},\infty\right)$

(c) $(0\,,\infty)$ (d) $\left(-\dfrac{1}{2},0\right)\cup(0,\infty)$

3. If [.] represents the greatest integer function , then $\displaystyle\sum_{r=0}^{99}\left[\frac{3}{4}+\frac{r}{100}\right]$ is equal to :

(a) 30 (b) 70

(c) 75 (d) 100

4. Let $f(x) = \sin ax + \cos ax$ and $g(x) = |\sin x| + |\cos x|$ have equal fundamental period , then 'a' is :

(a) 1 (b) 2

(c) 3 (d) 4

5. Let $f(x) + f(1-x) = 2 \;\; \forall \;\; x \in R$ and $g(x) = f(x) - 1$, then $g(x)$ is symmetrical about :

(a) the line $x = \dfrac{1}{2}$ (b) the point (1 , 0)

(c) the line $x = 1$ (d) the point $\left(\dfrac{1}{2}\,,\,0\right)$

6. The values of 'a' and 'b' for which equation $\left|e^{|x-b|}-a\right| = 2$ has four distinct solutions , are :

(a) $a \in (3\,,\infty)\,;\, b = 0$ (b) $a \in (2\,,\infty)\,;\, b \in R$

(c) $a \in (3\,,\infty)\,;\, b \in R$ (d) $a \in (2\,,\infty)\,;\, b = 0$

7. Let $f : R \to R$ and $g : R \to R$ be functions defined as $f(x) = \begin{cases} 0\,;\; x \in Q \\ x\,;\; x \notin Q \end{cases}$ and $g(x) = \begin{cases} x\,;\; x \in Q \\ 0\;\,;\; x \notin Q \end{cases}$, then composition $f(x) - g(x)$ is :

(a) one-one onto (b) one-one into

(c) many-one onto (d) many-one into

8. If $0 < \lambda < 1$ and $f(x) = \left\{\dfrac{log_\lambda|x-2|}{|x|}\right\}^{\frac{1}{2}}$, then domain of $f(x)$ is :

(a) $[1\,,\,2)\cup(2\,,\,3)$ (b) $[1\,,\,2)\cup(2\,,\,3]$

(c) $(1\,,\,2)\cup(2\,,\,5)$ (d) none of these

9. The number of solutions of equation $6\,|\cos x| - x = 0$ in $[0\,,2\,\pi]$ are :

(a) 6 (b) 4

(c) 3 (d) 2

10. If $f : (3\,,6) \to (2\,,5)$ is a function defined as $f(x) = x - \left[\dfrac{x}{3}\right]$, where [.] represents the greatest integer function , then $f^{-1}(x)$ is given by :

(a) $x+1$ (b) $3x+2$

(c) $3x+1$ (d) none of these

11. If $x \in R^+$, then range of $f(x) = \dfrac{(1+x+x^2)(x^4+1)}{x^3}$ is :

(a) $\left[2\,,\dfrac{5}{3}\right)$ (b) $[6,\infty)$

(c) $\left[\dfrac{2}{3},\infty\right)$ (d) $\left[\dfrac{2}{3},\dfrac{5}{3}\right]$

12. If $3f(x)+2f\left(\frac{x+59}{x-1}\right)=10x+30\ \forall\ x\in R-\{1\}$, then $f(7)$ is equal to :

(a) 7 (b) 5

(c) 4 (d) 2

13. Let $f:[-2\ ,\ 2]\to R$ be an odd function defined as $f(x)=x^3+\tan x+\left[\frac{x^2+1}{\lambda}\right]$, then λ belongs to :

(a) $(5\ ,\ \infty)$ (b) $(7\ ,\ \infty)$

(c) R^- (d) R^+

14. If $f(x)=\sin 3x.\cos[3x]-\cos 3x.\sin[3x]$, where [.] represents the greatest integer function , then fundamental period of $f(x)$ is :

(a) 3 (b) $\frac{1}{3}$

(c) 6 (d) $\frac{1}{6}$

15. Let $f:R\to[0,\ \pi/2)$ be defined as $f(x)=\tan^{-1}(x^2+x+a)$ then set of values of *'a'* for which $f(x)$ is onto , is :

(a) $\left[\frac{1}{4},\infty\right)$ (b) $[4\ ,\ \infty)$

(c) $\left(\frac{1}{8},\infty\right)$ (d) none of these

16. If $f:R\to R$ be a function satisfying $f(2x+3)+f(2x+7)=2\ \forall\ x\in R$ then fundamental period of $f(x)$ is :

(a) 2 (b) 4

(c) 8 (d) 16

17. Interval of x satisfying the inequality $\frac{5}{2}\le|x-1|+|x-2|+|x-3|<6$ is given by :

(a) $\left(0\ ,\ \frac{1}{2}\right]\cup\left[\frac{3}{2}\ ,\ 4\right)$ (b) $\left(0\ ,\ 1\right]\cup\left[2\ ,\ 5\right)$

(c) $\left[-1\ ,\ \frac{3}{2}\right)\cup(4\ ,\ 5]$ (d) $\left(0\ ,\frac{3}{2}\right]\cup\left[\frac{5}{2}\ ,\ 4\right)$

18. Area enclosed by inequality $2\le|x+y|+|x-y|\le 4$ is :

(a) 12 sq. units (b) 5 sq. units

(c) 4 sq. units (d) 8 sq. units

19. Number of solutions of equation $e^{-|x|}=|1-|2-x||$ are :

(a) 3 (b) 4

(c) 5 (d) 2

20. The number of integral values of *'m'* for which function $f(x)=\frac{x^3}{3}+(m-1)x^2+(m+5)x+11$ is invertible , are :

(a) 4 (b) 10

(c) 6 (d) 8

21. If $3^{\log_a x}+3x^{\log_a 3}=2$, where $a\in R^+-\{1\}$, then value of x is :

(a) $a^{\log_2 3}$ (b) $a^{-\log_2 3}$

(c) $a^{-\log_3 2}$ (d) $2^{\log_3 a}$

22. If $x^4-18x^2+\lambda-2=0$ is having all four real roots , then *exhaustive set for* ' λ ' belongs to :

(a) [3 , 67] (b) [–1 , 61]

(c) [0 , 75] (d) [2 , 83]

23. Domain of function $f(x)=\sin^{-1}(x^2-5x+5)$ is :

(a) $[1\ ,\ 2]\cup[4\ ,\ 5]$ (b) $[1\ ,\ 2]\cup[3\ ,\ 4]$

(c) $[2\ ,\ 3]\cup[4\ ,\ 5]$ (d) $[1\ ,\ 2]\cup[3\ ,\ 5]$

24. Let $f(x)=\left\{\cos^2 x+\cos^2\left(\frac{\pi}{3}+x\right)-\cos x.\cos\left(\frac{\pi}{3}+x\right)\right\}$, then $f\left(\frac{\pi}{8}\right)$ is equal to :

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$

(c) $\frac{4}{5}$ (d) $\frac{2}{3}$

25. Domain of $f(x)=\sqrt{10[x]-21-[x]^2}$, where [.] is greatest integer function , is :

(a) [3 , 8) (b) [3 , 7]

(c) (2 , 7] (d) (2 , 8)

26. Let $f(x) = (\sin x + \sin 3x)\sin x$, then $\forall\ x \in R$, $f(x)$ is :

(a) positive
(b) non-positive
(c) negative
(d) non-negative

27. Number of solution(s) of the equation $x^2 - 4x + 5 - e^{-|x|} = 0$ is/are :

(a) 0
(b) 1
(c) 2
(d) 4

28. If $f : R \to R$ is defined by $f(x) = \dfrac{x^2+2x+3}{x^2+2x+2}$, then range of $f(x)$ is :

(a) $[1, 2]$
(b) $(1, 2]$
(c) $[1, 2)$
(d) $\left[1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, 2\right]$

29. Number of integral values of x which satisfy the inequality $\dfrac{x^4(x-1)^2(x+4)^3}{(x+2)^4(6-x)^5} \geq 0$ are :

(a) infinite
(b) 8
(c) 9
(d) 10

30. Let $f(x) = x^2 + (a-b)x + (1-a-b)$ cuts the x-axis at two distinct points for all values of b, where $a, b \in R$, then the interval of 'a' is :

(a) $[1, \infty)$
(b) $(1, \infty)$
(c) $(-\infty, 1)$
(d) $(-\infty, 1]$

31. If $\dfrac{(\ln x)^2 - 3\ln x + 3}{(\ln x - 1)} < 1$, then x belongs to :

(a) $(0, e)$
(b) $(1, e)$
(c) $(1, 2e)$
(d) $(0, 3e)$

32. Let $g(x) = 1 + x - [x]$ and $f(x) = sgn(x)$, where [.] is greatest integer function, then for all $x \in R$ $f(g(x))$ is :

(a) $f(x)$
(b) $g(x)$
(c) $[g(x)]$
(d) x

33. Let $f(x) = \begin{cases} x\ ; & \text{if } x \in Q \\ 1-x\ ; & \text{if } x \notin Q \end{cases}$, then $f(f(f(x)))$ is :

(a) 0
(b) $f(x)$
(c) x
(d) $1-x$

34. Let $f(x) = \dfrac{x}{e^x - 1} + \dfrac{x}{2} + 1$, then $f(x)$ is :

(a) even function
(b) odd function
(c) neither even nor odd function
(d) both even and odd function

35. Let $f(x) = \dfrac{x-[x]}{1+x-[x]}$, where [.] is greatest integer function, then range of $f(x)$ is :

(a) $\left[0, \frac{1}{2}\right]$
(b) $[0, 1)$
(c) $\left[0, \frac{1}{2}\right)$
(d) $[0, 1]$

36. If $f(x) = \dfrac{(K)^x}{(K)^x + \sqrt{K}}$; $K > 0$, then which one of the following statements is true :

(a) $f(x) + f(1-x) = 2$
(b) $f(x) + f(1-x) = 1$
(c) $f(x) + f(1+x) = 1$
(d) $f(x) = f(1-x)$

37. Let $f(x) = |x|$ and $g(x) = [x]$, where [.] represents the greatest integer function, then the inequality $g(f(x)) \leq f(g(x))$ is valid, if

(a) $x \in (-\infty, 0) \cup I$
(b) $x \in I$
(c) $x \in (-\infty, 0)$
(d) $x \in R$

38. Let $f(x) = \sin x - ax$ and $g(x) = \sin x - bx$, where $a < 0$, $b < 0$. If number of roots of $f(x) = 0$ is greater than number of roots of $g(x) = 0$, then :

(a) $a < b$
(b) $a > b$
(c) $ab = \dfrac{\pi}{6}$
(d) $a + b = 0$

39. Let $f(x) = \dfrac{x-3}{x+1}$, $x \neq -1$, then $f^{2010}(2009)$, where $f(f(f(x)))$ is represented by $f^3(x)$, is :

(a) 2010
(b) 2009
(c) 4013
(d) none of these

40. If $|f(x) + 6 - x^2| = |f(x)| + |4 - x^2| + 2$, then $f(x)$ is necessarily non-negative in :

(a) $[-2, 2]$
(b) $(-\infty, -2) \cup (2, \infty)$
(c) $\left[-\sqrt{6}, \sqrt{6}\right]$
(d) none of these

Multiple choice questions with MORE than ONE correct answer : (Questions No. 41-50)

41. Let $f: R \to R$ be a function defined as $f(x) = x^3 + k^2x^2 + 5x + 2\cos x$. If $f(x)$ is invertible function, then possible values of 'k' may lie in the interval :

(a) $(-\sqrt{2}\,,\sqrt{2})$ (b) $(2\,,\sqrt{5})$

(c) $(-1\,,1)$ (d) $(-e\,,-2)$

42. Let $f(x)$ be real valued function and $f(x+y) = f(x)f(a-y) + f(y)f(a-x)$ for all $x,\ y \in R$. If for some real 'a', $2f(0)-1=0$, then :

(a) $f(x)$ is even function.

(b) $f(x)$ is periodic function.

(c) $f(x) = \dfrac{1}{2}\ \ \forall\ \ x \in R$.

(d) $f''(x)$ is both even and odd function.

43. Let $f(x).f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right)\ \ \forall\ \ x \in R - \{0\}$, then function $f(x)$ may be :

(a) $f(x) = 1 \pm x^n$ (b) $f(x) = \dfrac{\pi}{2\tan^{-1}|x|}$

(c) $f(x) = 2$ (d) $f(x) = \dfrac{2}{1 + ln\, x^4}$

44. Let $f(x) = \begin{cases} 0 & ;\ \ |x| = \dfrac{1}{n} \\ \left[|x|\left[\dfrac{1}{|x|}\right]\right] & ;\ \ |x| \neq \dfrac{1}{n} \end{cases}$, where $n \in N$

and $[\alpha]$ represents the greatest integer just less than or equal to α, then which of the following statement(s) are true :

(a) $f(x)$ is odd function.

(b) $f(x)$ is not periodic.

(c) $sgn\,(f(x)) = 1\ \ \forall\ \ x \in R$.

(d) $f(x)$ is even function.

45. Let $f: R \to R$ be a function defined as $f(x) = 3 - 3x + 2|x+2| - |x-3|$, then :

(a) $f(x)$ is surjective function.

(b) number of integral solutions of the equation $f(x) - 4 = 0$ are six.

(c) number of real solutions of the equation $f(x) - 4 = 0$ are infinitely many.

(d) number of real solutions of the equation $f(x) - |4\sin \pi x| = 0$ are more than eight.

46. Let $f(x) = \begin{cases} -(x+3) & ;\ \ -2 \le x < -1 \\ x-1 & ;\ \ -1 \le x \le 4 \end{cases}$ and $g(x) = 1 - x\ \ \forall\ \ x \in [-1\,,2]$. If $h(x) = g(f(x))$, then :

(a) Range of $h(x)$ is $[-2\,,2]$.

(b) Domain of $h(x)$ is $[0\,,3]$.

(c) Domain of $h(x)$ is $[-2\,,3]$.

(d) Number of solutions of the equation $h(x) - 2\,sgn(x^2 + 2x + 8) = 0$ are two.

47. Let $A = \{x : [5\sin x] + [\cos x] + 6 = 0\,,\ x \in R\}$, where $[.]$ represents the greatest integer function. If $f(x) = \sqrt{3}\sin x + \cos x\ \ \forall\ \ x \in A$, then :

(a) value of $f(x)$ is less than $\tan\left(\dfrac{2\pi}{3}\right)$.

(b) value of $f(x)$ is less than $2\cos(\pi)$.

(c) value of $f(x)$ is more than $\dfrac{4-3\sqrt{3}}{5}$.

(d) value of $f(x)$ is more than $\dfrac{-3-4\sqrt{3}}{5}$.

48. Let $n \in N$ and [.] represents the greatest integer function, where $f:[0\,,\pi] \to \left[\dfrac{n^2+n}{2}\,,\dfrac{n^2+n+2}{2}\right]$ be defined as $f(x) = \sum\limits_{r=1}^{n}\left[r + \sin\left(\dfrac{x}{r}\right)\right]$, then :

(a) $f(x)$ is one-one function.

(b) $f(x)$ is onto function.

(c) $f(x)$ is into function.

(d) $f(x)$ is many-one function.

49. Let $\alpha\,,\beta\,,\gamma$ be non-zero real numbers and $f:[0\,,3] \to [0\,,3]$ be a function defined as $f(x) = \alpha x^2 + \beta x + \gamma$. If $f(x)$ is bijective function, then :

(a) value of γ is 0. (b) value of γ is 3.

(c) α is root of $\gamma x^2 + \beta x + \alpha = 0$.

(d) one of the possible values of 'α' can be $1/\pi$.

50. Consider the function

$f(x) = 3x^4 - 8kx^3 + 24(6-k)x^2 + 24$ for all $x \in R$. If the graph of function $f(x)$ is convex downwards , then possible values of 'k' can be :

(a) $\cos^{-1}(\cos 2)$ (b) $\cot^{-1}(\cot e)$

(c) $-2\tan 2$ (d) $-3\tan 1$

Assertion Reasoning questions :
(Questions No. 51-55)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

51. Let $f: R \to R$ and $g: R \to R$ be two bijective functions and both the functions are mirror images of one another about the line $y - 2 = 0$.

Statement 1 : If $h: R \to R$ be a function defined as $h(x) = f(x) + g(x)$, then $h(x)$ is many one onto function

because

Statement 2 : $h(2) = h(-2) = 4$.

52. Statement 1 : If $x \in (0\,,\,2\pi)$, then the equation $\tan x + \sec x = 2\cos x$ is having three distinct solutions

because

Statement 2 : graph of $y = 1 + \sin x$ and $y = 2\cos^2 x$ intersect each other at three distinct points in $(0\,,\,2\pi)$.

53. If $[x]$ represents the greatest integer function and

$f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, then

Statement 1 : Range of $f(x)$ is $\left\{\frac{\pi}{2}\,,\,\pi\right\}$

because

Statement 2 : $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ for all $x \in [-1\,,\,1]$.

54. Consider the function $\phi(x) = \log_2\left(\frac{\sqrt{9-x^2}}{2-x}\right)$ and

$f(x) = 3\sin\phi(x) + 4\cos\phi(x)$, then

Statement 1 : Range of $f(x)$ is $[-5\,,\,5]$

because

Statement 2 : If $\theta \in R$, then value of $(a\sin\theta + b\cos\theta)$ lies in $\left[-\sqrt{a^2+b^2}\,,\,\sqrt{a^2+b^2}\right]$.

55. Let function $f: N \to N$ be defined as $f(x) = x - (sgn(\cos 2))^x$, then

Statement 1 : $f(x)$ is bijective in nature

because

Statement 2 : $sgn(\cos x) = 1 \;\forall\; x \in \left(-\frac{\pi}{2}\,,\,\frac{\pi}{2}\right)$

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $A = \{(x\,,\ y) : \max\{|x+y|\,,|x-y|\} \geq 10\,;\ x\,,\ y \in R\}$ and $B = \{(x\,,\ y) : \max\{|x+y|\,,|x-y|\} \leq 20\,;\ x\,,\ y \in R\}$.
On the basis of given set of ordered pairs $(x\,,\,y)$ in the 2-dimensional plane , answer the following questions.

1. Area of the region which contain all the ordered pairs $(x\,,\,y)$ that belongs to the set of $A \cap B$ is equal to :

(a) 300 square units.
(b) 800 square units.
(c) 400 square units.
(d) 600 square units.

2. Let the ordered pair $(x\,,\,y)$ be termed as integral point if both x and y belong to the set of integers , then total number of integral points which belong to the set of $A \cap B$ are :

(a) 600 (b) 1000
(c) 660 (d) 860

3. Number of ordered pairs $(x\,,\ y)$ which satisfy the condition $|\,y\,| = 10\left\{\dfrac{x}{10}\right\}$ and belong to set $'A'$, where $\{\,\alpha\,\}$ represents the fractional part of α , are :

(a) 100 (b) 420
(c) finitely many (d) infinitely many.

Comprehension passage (2)
(Questions No. 4-6)

Let $f : A \rightarrow B$ be bijective function and its inverse exists , where the inverse function of $f\,(x)$ is given by $g : B \rightarrow A$. If the functions $y = f\,(x)$ and $y = g\,(x)$ are represented graphically by the continuous curves C_1 and C_2 respectively , then answer the following questions.

4. If the points (4 , 2) and (2 , 4) lie on the curve $'C_2'$ then minimum number(s) of solutions of the equation $f\,(x) - g\,(x) = 0$ is/are :

(a) 1 (b) 3
(c) 6 (d) 2

5. Let $f\,(x) = \int (\cos x - 2)dx$, where $f\,(0) = 0$, then which one of the following statements is true :

(a) C_1 and C_2 meet only at point (0 , 0).
(b) C_1 and C_2 meet at infinitely many points on the line $y - x = 0$.
(c) C_1 and C_2 meet at finitely many points on the line $y + 2x = 0$.
(d) All the points of intersection of C_1 and C_2 lie on the line $y + 2x = 0$.

6. Let $p \in A$ and $q \in B$, where $p - q \neq 0$. If point $(p\,,\,q)$ lies on C_1 but not on C_2 , then :

(a) C_1 and C_2 can't meet on the line $y - x = 0$.
(b) C_1 and C_2 don't meet each other.
(c) either C_1 and C_2 don't meet each other or they meet on the line $y - x = 0$.
(d) C_1 and C_2 meet on the line $y - x = 0$.

Comprehension passage (3)
(Questions No. 7-9)

Let $f : N \rightarrow N$ be a function defined by $f\,(x) = D_x$, where D_k represents the largest natural number which can be obtained by rearranging the digits of natural number k.
For example : $f\,(3217) = 7321$, $f\,(568) = 865$, $f\,(89) = 98$ .... etc.
On the basis of given definition of $f\,(x)$, answer the following questions.

7. Function $f\,(x)$ is :

(a) one-one and into.
(b) many-one and into.
(c) one-one and onto.
(d) many-one and onto.

8. If natural number $'n_0'$ divides $f\,(\,\alpha\,) - \alpha$ for every $\alpha \in N$, then maximum possible value of $'n_0'$ is equal to :

(a) 3 (b) 4
(c) 9 (d) 11

9. Let $f\,(\alpha) = 99852$, where $\alpha \in N$, then maximum number of possible distinct values of $'\alpha'$ are :

(a) more than 100. (b) less than 50.
(c) more than 55. (d) less than 30.

Comprehension passage (4)
(Questions No. 10-12)

Let $f : R \to R$ be a function defined as $f(x) = 3x^5 - 25x^3 + 60x + 5$, and

$$g(x) = \begin{cases} \max\{f(t)\,;\, -4 \le t \le x\} & ; \quad -4 \le x \le 0 \\ \min\{f(t)\,;\quad 0 < t \le x\} & ; \quad 0 < x \le 2 \\ f(x) - 16 & ; \quad x > 2 \end{cases}$$

On the basis of given definitions of $f(x)$ and $g(x)$, answer the following questions.

10. Total number of location(s) at which the graph of $y = g(x)$ breaks in $[-4\,,\,\infty)$ is/are :

(a) 2	(b) 1
(c) 0	(d) 4

11. If the equation $f(x) + \lambda = 0$ is having exactly three distinct real roots , then total number of possible integral values of $'\lambda'$ are :

(a) 20	(b) 21
(c) 40	(d) 42

12. If the equation $g(x) + \mu = 0$ is having infinitely many real solutions , then number of possible integral values of $'\mu'$ is/are :

(a) 0	(b) 1
(c) 2	(d) 3

Questions with Integral Answer :
(Questions No. 13-17)

13. Let $f(x) = \sin\left(\frac{5x}{n}\right).\cos(nx)$, where $n \in I$, and the period of $f(x)$ is 3π, then total number of possible values of $'n'$ is equal to ....

14. Total number of integral values of x in $\left[-\frac{3\pi}{2}\,,\,\frac{3\pi}{2}\right]$ for which the equation $\left|\sin^{-1}(\sin x) + x^4 - 17x^2 + 16\right| = \left|x^4 - 17x^2 + 16\right| + \left|\sin^{-1}(\sin x)\right|$ is satisfied , are ....

15. Let $n \in N$, and $f : N \to N$ be a function defined by $f(n) = \sum_{r=1}^{n} (r)!$. If $P(n)$ and $Q(n)$ are polynomials in n such that $f(n+2) = P(n)\,f(n+1) + Q(n)\,f(n)$ for all $n \in N$, then value of $P(10) + Q(6)$ is equal to ....

16. Let the equation $(P+1)(x^4 + x^2 + 1) - (P-1)(x^2 + x + 1)^2 = 0$ is having two distinct and real roots and $f(x) = \frac{1-x}{1+x}$, where $f(f(x)) + f\left(f\left(\frac{1}{x}\right)\right) = \alpha P$, then value of $'\alpha'$ is ......

17. Let $f(x)$ and $g(x)$ be even and odd functions respectively, where $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x)$, then value of $f(4)$ is equal to ....

Matrix Matching Questions :
(Questions No. 18-20)

18. Let $f(x) = \frac{|3-x| + |x+1|}{|1-x| + |x+3|}\ \forall\ x \in R$, and $[x]$ represents the greatest integer function of x, then match the conditions/expressions in column (I) with statement(s) in column (II).

Column (I)	**Column (II)**
(a) If $x \in (-\infty\,,\,-3)$, then $f(x)$ satisfies	(p) $0 \le [f(x)] \le 2$
(b) If $x \in [-1\,,\,1]$, then $f(x)$ satisfies	(q) $[f(x)] \ge 0$
(c) If $x \in [-4\,,\,-2]$, then $f(x)$ satisfies	(r) $[f(x)] = 0$
(d) If $x \in [2\,,\,\infty)$, then $f(x)$ satisfies	(s) $[f(x)] \ge 1$

19. Match the functions in column (I) with their corresponding range in column (II).

Column (I)	**Column (II)**
(a) $f(x)=\cos(\sin x)+\sin(\cos x)$ for all $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	(p) $[\cos 1\,,\,1]$
(b) $f(x)=\cos(\cos(\sin x))$ for all $x\in[0\,,\,\pi]$	(q) $[\cos 1\,,\,\cos(\cos 1)]$
(c) $f(x)=\cos(\cos x)$ all $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	(r) $[\cos(\cos 1)\,,\,\cos 1]$
(d) $f(x)=\cos(\sin\sqrt{2x-x^2})$ for all $x\in\left[0\,,\,\frac{3\pi}{8}\right]$	(s) $[\cos 1\,,\,1+\sin 1]$
	(t) $\left[\sin 1\,,\,1+\cos 1\right]$

20. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) Domain of $f(x)=\cos^{-1}\left(\dfrac{\tan^2 2x+\cot^2 2x}{2}\right)$ contain(s)	(p) $\frac{3\pi}{4}$
(b) Domain of $f(x)=\left\{log_3\sin^2(2x)\right\}^{1/2}$ contain(s)	(q) $\frac{\pi}{12}$
(c) Range of $f(x)=\tan^{-1}\left(\dfrac{x^2-x+1}{x^2+x+1}\right)$ contain(s)	(r) $\frac{\pi}{8}$
(d) If $[\,\alpha\,]$ represents the greatest integer function of α,	(s) $\frac{3\pi}{8}$
and $f(x)=\sqrt{[\cos^{-1}x]-[\sin^{-1}x]}$, then domain of $f(x)$ contain(s)	(t) $\frac{\pi}{4}$

ANSWERS Exercise No. (1)

1. (a) **2.** (a) **3.** (c) **4.** (d) **5.** (d)

6. (c) **7.** (a) **8.** (b) **9.** (c) **10.** (a)

11. (b) **12.** (c) **13.** (a) **14.** (b) **15.** (d)

16. (c) **17.** (d) **18.** (a) **19.** (b) **20.** (c)

21. (c) **22.** (d) **23.** (b) **24.** (a) **25.** (a)

26. (d) **27.** (a) **28.** (b) **29.** (c) **30.** (b)

31. (a) **32.** (c) **33.** (b) **34.** (a) **35.** (c)

36. (b) **37.** (d) **38.** (b) **39.** (b) **40.** (a)

41. (a , c) **42.** (a , b , c , d) **43.** (a , b , c , d) **44.** (a , d) **45.** (a ,b, c , d)

46. (a , d) **47.** (a , d) **48.** (c , d) **49.** (b , c , d) **50.** (a , b , d)

51. (d) **52.** (d) **53.** (d) **54.** (a) **55.** (b)

Objecti athematics Er. L.K.

ANSWERS Exercise No. (2)

1. (d) **2.** (c) **3.** (d) **4.** (b) **5.** (d)

6. (c) **7.** (b) **8.** (c) **9.** (c) **10.** (c)

11. (d) **12.** (c) **13.** (8) **14.** (7) **15.** (5)

16. (1) **17.** (0)

18. (a) → p , q , s
(b) → p , q , s
(c) → p , q , s
(d) → p , q , r

19. (a) → s
(b) → q
(c) → p
(d) → p

20. (a) → r
(b) → p , t
(c) → r , s , t
(d) → q , r , t

Chapter 11

Limits

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-15)

1. $\lim\limits_{x\to 0} \dfrac{\int_0^{x^2} t^2 . e^{-t^2} dt}{1-\cos(x^3)}$ is equal to :

(a) $-\dfrac{3}{2}$ (b) $\dfrac{2}{3}$

(c) $\dfrac{4}{3}$ (d) $\dfrac{1}{3}$

2. $\lim\limits_{n\to\infty} \left(\sin\dfrac{\pi}{2n} . \sin\dfrac{2\pi}{2n} ......\sin\dfrac{(n-1)\pi}{n} \right)^{1/n}$ is equal to :

(a) $\dfrac{1}{4}$ (b) $e^{4/\pi}$

(c) $e^{2/\pi}$ (d) $e^{\pi/8}$

3. Let $f(x)$ be differentiable and $f(1)=2$ and $f'(1)=4$,

then $\lim\limits_{x\to 1} \left(\dfrac{f(x)}{f(1)} \right)^{\frac{1}{x-1}}$ is equal to :

(a) 1 (b) e^2

(c) 0 (d) e^{-1}

4. $\lim\limits_{n\to\infty} \dfrac{(1+2^5+3^5+4^5+........+n^5)}{n^8}$ is :

(a) 0 (b) $\dfrac{1}{5}$

(c) $\dfrac{1}{6}$ (d) $\dfrac{1}{4}$

5. If $f(x)$ is differentiable and $f(0)=0$, such that $2f(x+y)+f(x-y)+3y^2 = 3f(x)-2xy$, then

$\lim\limits_{x\to 1} \dfrac{f(x)-1}{x-1}$ is equal to :

(a) –3 (b) 0

(c) –2 (d) 1

6. $\lim\limits_{x\to 0} \left(2\sin^2\dfrac{x}{2} \right)^{ln(\cos x)}$ is :

(a) 1 (b) 2

(c) $\dfrac{1}{2}$ (d) $\dfrac{1}{e}$

7. $\lim\limits_{x\to 0} (\cos x)^{\cot(x^2)}$ is :

(a) 1 (b) $\sqrt{e}$

(c) e^2 (d) $\dfrac{1}{\sqrt{e}}$

8. The value of $\lim\limits_{x\to\infty} \left(x+\dfrac{1}{x} \right) e^{1/x} - x$ is equal to :

(a) 1 (b) ∞

(c) 0 (d) none of these

9. Let $f(x)$ be real function and $g(x)$ is bounded function for all $x \in R^+$, then $\lim\limits_{n\to\infty} \dfrac{f(x).e^{nx}+g(x)}{1+e^{nx}}$ is :

(a) $f(x)$ (b) $g(x)$

(c) 0 (d) 1

10. If the graph of function $y = f(x)$ is having a unique tangent of finite slope at location $(a , 0)$, then $\lim\limits_{x\to a} \dfrac{\log_e(1+6f(x))}{3f(x)}$ is equal to :

(a) 0 (b) 1

(c) 2 (d) 1/3

11. Let $\lim\limits_{x\to 0} \dfrac{x(1+a\cos x)-b\sin x}{x^3} = 1$, then $(a + b)$ is :

(a) –3 (b) –2

(c) – 4 (d) –1

12. $\lim\limits_{n\to\infty} \frac{n}{3}\left\{\left(\frac{3}{n}+\frac{9}{n^2}\right)^2+\left(\frac{3}{n}+\frac{18}{n^2}\right)^2+\left(\frac{3}{n}+\frac{27}{n^2}\right)^2 ....+\left(\frac{3}{n}+\frac{9}{n}\right)^2\right\}$

is equal to :

(a) 62 (b) 63

(c) 64 (d) none of these

13. If normal to curve $y = f(x)$ at $x = 0$ is $3x - y + 3 = 0$,

then $\lim\limits_{x\to 0}\left\{\frac{x^2}{f(x^2)-5f(4x^2)+4f(7x^2)}\right\}$ is :

(a) $\frac{1}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{1}{3}$ (d) $-\frac{1}{2}$

14. Let $f:[-1,1]\to R$ and $f(0)=0, f'(0)=\lim\limits_{n\to\infty} n f\left(\frac{1}{n}\right)$,

where $0<\left|\lim\limits_{n\to\infty}\cos^{-1}\left(\frac{1}{n}\right)\right|<\frac{\pi}{2}$, then value of

$\lim\limits_{n\to\infty}\left\{\frac{2}{\pi}(n+1)\cos^{-1}\left(\frac{1}{n}\right)-n\right\}$ is equal to :

(a) $\frac{2}{\pi}$ (b) 0

(c) $1-\frac{2}{\pi}$ (d) $1+\frac{2}{\pi}$

15. $\lim\limits_{x\to 0}\frac{\sin(\pi(1-\sin^2 x))}{\tan^2 x}$ is equal to :

(a) π (b) $-\pi$

(c) $\frac{\pi}{2}$ (d) 1

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. Let $m, n \in I^+$ and $f(x)=\frac{(x-1)^{2m}}{log_e\left(\cos^n(x-1)\right)}$ for all $x\in(0, 2)$ .If $g(x)=e^{-|x-1|}\ \forall\ x\in R$ and

$\lim\limits_{x\to 1^+} f(x)=g'(1^+)$, then :

(a) $m + 2n = 5$ (b) $2m + n = 4$

(c) $m - n = 1$ (d) $2m - n = 0$

17. In which of the following case(s) , the limit doesn't exist ?

(a) $\lim\limits_{x\to 0}\frac{x}{\sqrt{\sec^2 x-1}}$ (b) $\lim\limits_{x\to 0}(\sin^3 x)^{\tan x}$

(c) $\lim\limits_{x\to\infty}\left(\frac{3x^2+1}{4x^2+x}\right)^{\frac{x^2+1}{2x}}$ (d) $\lim\limits_{x\to 0}(ln\ x^2)^{2x}$

18. Let $f(x)$ be differentiable function for all $x\in R^+$ and $f(1)=1$. If $\lim\limits_{\alpha\to x}\frac{\alpha^2 f(x)-x^2 f(\alpha)}{\alpha - x}=1$ for every $x>0$, then :

(a) $f(2)=\frac{17}{6}$

(b) $f(x)$ has local minima at $x=\frac{(2)^{1/3}}{2}$

(c) $f(x)$ is strictly increasing for all $x\ge 2$

(d) $f''(x)>0\ \forall\ x\in R^+$

19. Let $f(x)=\lim\limits_{k\to 0}\left(\frac{2x}{\pi}\cot^{-1}\left(\frac{x}{k^2}\right)\right)$, then

(a) $f(x)$ is increasing function for all $x\in R$.

(b) $f(x)$ is differentiable for all $x\in R-\{0\}$.

(c) $\int_{-1}^{\infty}[f(x)]dx=0$, where [.] represents greatest integer function.

(d) $f(|x|)$ is odd function.

20. If $\lim\limits_{x\to 1}(1+\alpha x+\beta x^2)^{\frac{\gamma}{x-1}}=\lim\limits_{x\to\infty}\left(\frac{x^2+4x}{x+x^2}\right)^{\frac{x^2}{x+1}}$, then :

(a) $\alpha+\beta=1$ (b) $\alpha+\beta=0$

(c) $\beta\gamma=4$ (d) $\beta\gamma=3$

Assertion Reasoning questions : (Questions No. 21-25)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. Statement 1 : Let $L = \lim\limits_{x\to -\infty}\left(\sqrt{4x^2+7x}+2x\right)$, then limiting value $'L'$ approaches to positive infinity

because

Statement 2 : The form of indeterminacy in $'L'$ is $\infty - \infty$ form.

22. Statement 1 : Let $a_1 = \sqrt{3}$ and $a_{n+1} = \dfrac{a_n}{1+\sqrt{1+a_n^2}}$, for all $n \in N$, then $\lim\limits_{n\to\infty} 2^n(a_n)$ is equal to $\dfrac{2\pi}{3}$

because

Statement 2 : Sequence $\{a_n\}$ for all $n \in N$ is converging in nature.

23. Let $S_n = \sum\limits_{r=0}^{n} \dfrac{r.2^r}{(r+2)!}$, then

Statement 1 : $\lim\limits_{n\to\infty} S_n = 1$

because

Statement 2 : $\lim\limits_{n\to\infty} \dfrac{2^{n+1}}{(n+2)!} = 0$

24. Statement 1 :

Let $L = \lim\limits_{n\to\infty}\left(\dfrac{1}{1+n^2}+\dfrac{2}{2+n^2}+.....+\dfrac{n}{n+n^2}\right)$, then value of limit $'L'$ is equal to $\dfrac{1}{2}$

because

Statement 2 : $\lim\limits_{n\to\infty} \dfrac{1}{n}\sum\limits_{r=1}^{n} f\left(\dfrac{r}{n}\right) = \int\limits_0^1 f(x)dx$

25. Statement 1 : Let $L = \lim\limits_{n\to\infty}\left\{(\sin 1)^n + (\cos 1)^n\right\}^{\frac{1}{n}}$, then value of $\sin^{-1}(L) = 1$

because

Statement 2 : $\lim\limits_{x\to 0} \dfrac{\sin\left(\cos^2\dfrac{x}{2}\right).\sin\left(x^2\right)}{\tan^2 x} = \sin 1$

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f(x)$ and $g(x)$ be continuous functions for all $x \in R$ and $f(0) = g(0) = 0$. If $\lim\limits_{x\to 0} \dfrac{g(x).\sin^2 x}{f(1-\cos x)} = \alpha$ and $\lim\limits_{x\to 0} \dfrac{f(x)}{x^2} = \beta$, then answer the following questions.

1. $\lim\limits_{x\to\infty} x^2 . g\left(\dfrac{1}{x}\right)$ is equal to :

(a) $\dfrac{\alpha}{2\beta}$ (b) $2\alpha\beta^2$

(c) $\dfrac{\alpha\beta}{4}$ (d) $\dfrac{4\alpha}{\beta}$

2. $\lim\limits_{x\to 0} \dfrac{g(\cos 2x - 1)}{x^4}$ is equal to :

(a) $\alpha\beta$ (b) $-\alpha\beta$

(c) $\dfrac{\beta}{\alpha}$ (d) $-\dfrac{\alpha}{\beta}$

3. $\lim\limits_{x\to 0} \left\{ \dfrac{f(x)g(x)}{\left(\sin(2x^2)\right)^2} \right\}$ is equal to :

(a) $\dfrac{\alpha^2\beta}{16}$ (b) $\dfrac{\alpha^2\beta}{4}$

(c) $\dfrac{\alpha\beta^2}{16}$ (d) $\dfrac{\alpha\beta^2}{4}$

Comprehension passage (2)
(Questions No. 4-6)

Let points *'A'* and *'B'* lies on the circle $C_1 : x^2 + y^2 - 1 = 0$, where $\angle AOB = \theta$, *'O'* being the origin. If tangents drawn at *'A'* and *'B'* to $'C_1'$ meet at *'P'*, and the tangent to $'C_1'$ drawn at the mid-point of arc $\overset{\frown}{AB}$ meet the lines *PA* and *PB* at *'C'* and *'D'* respectively , then answer the following questions.

4. If area of triangles *PAB* and *PCD* are $'A_1'$ and $'A_2'$ respectively , then $\lim\limits_{\theta\to 0^+} \left(\dfrac{A_1}{A_2}\right)$ is equal to :

(a) 2 (b) 4

(c) 1 (d) 6

5. If area of triangle *PAB* is $'A_1'$ and area enclosed by arc $\overset{\frown}{AB}$ with the chord *AB* is $'A_3'$, then $\lim\limits_{\theta\to 0^+} \left(\dfrac{A_1}{A_3}\right)$ is equal to :

(a) 3/2 (b) 5/2

(c) 2 (d) 1

6. If area of triangle *PCD* is $'A_2'$ and area enclosed by arc $\overset{\frown}{AB}$ with the chord *AB* is $'A_3'$, then $\lim\limits_{\theta\to 0^+} \left(\dfrac{A_2}{A_3}\right)$ is equal to :

(a) $\dfrac{3}{8}$ (b) $\dfrac{5}{4}$

(c) 1 (d) $\dfrac{1}{6}$

Questions with Integral Answer :
(Questions No. 7-10)

7. Let $f(x) = x\sin(\sin x) - \sin^2 x$ and $L = \lim\limits_{x\to 0} \dfrac{f(x)}{x^n}$. If limiting value *'L'* is non-zero and finite , then value of *'n'* must be equal to ....

8. Let $L = \lim\limits_{x\to 0} \left\{ \dfrac{\sin^3 x}{a\,x e^x - b\ln(1+x) + c\,x e^{-x}} \right\}$. If the value of *L* is 3/2 , then $(2b + a - c)$ is equal to ....

9. Let $S_n = \left(\sum\limits_{r=1}^{n} r\right) + 2\left(\sum\limits_{r=1}^{n-1} r\right) + 3\left(\sum\limits_{r=1}^{n-2} r\right) + ... + n$ and $\lim\limits_{n\to\infty} \dfrac{n^4}{S_n}$ is equal to *'L'* , then value of $\dfrac{L}{3}$ is equal to ....

10. Let $p(x)$ be a polynomial of degree 4 having the points of extremum at $x = 1$ and $x = 2$, where $\lim\limits_{x\to 0} \left(1 + \dfrac{p(x)}{x^2}\right) = 2.$ The value of $p(2)$ is ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Let $[x]$ represents the greatest integer which is just less than or equal to x, then match the following columns (I) and (II) .

Column (I)	Column (II)
(a) $\lim\limits_{x\to 0}\left(\left[\dfrac{\sin x}{x}\right]+\left[\dfrac{\tan x}{x}\right]\right)$	(p) 2
(b) $\lim\limits_{x\to 0}\left(\left[\dfrac{2x}{\sin x}\right]+\left[\dfrac{3\sin x}{x}\right]\right)$	(q) 0
(c) $\lim\limits_{x\to 0}\left([x^2+2]+[x^3+3]\right)$	(r) 1
(d) $\lim\limits_{x\to 0^+}\left(\dfrac{x}{2}\left[\dfrac{4}{x}\right]\right)$	(s) 4
	(t) limit doesn't exist

12. Let $L=\lim\limits_{x\to\infty}\left(\sqrt{x^4+ax^3+3x^2+bx+2}-\sqrt{x^4+2x^3-cx^2+3x-d}\right)$, then match the columns (I) and (II).

Column (I)	Column (II)
(a) If $L=4$, then value of $(c-a)$ is	(p) 1
(b) If $L=2$, then value of $'c'$ is	(q) 2
(c) If $L=6$, $b\in R^+$, then value of $(a+b)$ can be	(r) 3
	(s) 4
(d) If $L=3$, $d\in R^-$, then value of $(c+d)$ can be	(t) 0

ANSWERS Exercise No. (1)

1. (b)
2. (a)
3. (b)
4. (a)
5. (c)
6. (a)
7. (d)
8. (a)
9. (a)
10. (c)
11. (c)
12. (d)
13. (b)
14. (c)
15. (a)
16. (a , b , d)
17. (a , b)
18. (a , b , c , d)
19. (b , d)
20. (b , d)
21. (d)
22. (b)
23. (a)
24. (b)
25. (b)

ANSWERS Exercise No. (2)

1. (c)
2. (a)
3. (c)
4. (b)
5. (a)
6. (a)
7. (6)
8. (6)
9. (8)
10. (0)

11. (a) → r
(b) → s
(c) → t
(d) → p

12. (a) → r
(b) → p
(c) → r , s
(d) → p , q , t

Chapter 12 Continuity and Differentiability

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-20)

1. Let $f(x) = \min\{2, x^2 - 4x + 5, x^3 + 2\}$, then total number of points of non-differentiability is/are :

(a) 4 (b) 2

(c) 3 (d) 1

2. Total number of locations of non-differentiability for the function $f(x) = |x| + |\cos x| + \tan\left(\frac{\pi}{4} + x\right)$ in the interval $x \in (-1, 2)$ is/are :

(a) 3 (b) 1

(c) 2 (d) 4

3. If function $f : R \to R$ satisfy the condition

$$\frac{f(2x+2y) - f(2x-2y)}{f(2x+2y) + f(2x-2y)} = \frac{\cos x \sin y}{\sin x \cos y} \text{ and}$$

$f'(0) = \frac{1}{2}$, then :

(a) $f''(x) - f(x) = 0$ (b) $4f''(x) + f'(x) = 0$

(c) $4f''(x) + f(x) = 0$ (d) $4f'(x) + f''(x) = 0$

4. The number of points of non-differentiability of $f(x) = \max\{\sin x, \cos x, 0\}$ in $(0, 2n\pi)$, where $n \in N$, are given by :

(a) $4n$ (b) $2n$

(c) $6n$ (d) $3n$

5. Let $f(x) = \big||e^x - 1| - 1\big|$ then $f(x)$ is non-differentiable for x belongs to :

(a) $\{0, 2\}$ (b) $\{0, 1\}$

(c) $\{1, \ln 2\}$ (d) $\{0, \ln 2\}$

6. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$, then

$\lim\limits_{h \to 0} \dfrac{f(1-h) - f(1)}{h^3 + 3h}$, is equal to :

(a) $\frac{22}{3}$ (b) $\frac{53}{3}$

(c) $-\frac{53}{3}$ (d) $\frac{25}{3}$

7. Let $f(x) = x^3 + x$ and $g(x) = \begin{cases} f(|x|) & ; x \ge 0 \\ f(-|x|) & ; x < 0 \end{cases}$,

then :

(a) $g(x)$ is continuous $\forall\ x \in R$

(b) $g(x)$ is continuous $\forall\ x \in R^-$

(c) $g(x)$ is discontinuous $\forall\ x \in R^-$

(d) $g(x)$ is continuous $\forall\ x \in R^+$

8. Let $f(x) = \begin{cases} x^2 + 3x + a & ; x \le 1 \\ bx + 2 & ; x > 1 \end{cases}$ be a differentiable function for all $x \in R$, then $(a + 3b)$ is :

(a) 20 (b) 18

(c) 15 (d) 25

9. If $f(x) = \left|x^2 + a|x| + b\right|$ has exactly three points of non-differentiability, then

(a) $b \in R$, $a < 0$ (b) $a > 0, b = 0$

(c) $b = 0$, $a \in R$ (d) $a < 0, b = 0$

10. If $f(x) = [2x^3 - 5]$, [.] is greatest integer function, then total number of points in $(1, 2)$ where $f(x)$ is not continuous is/are :

(a) 10 (b) 12

(c) 13 (d) 15

11. Let $f(x) = \begin{cases} (\cos x - \sin x)^{\operatorname{cosec} x} & ; -\frac{\pi}{2} < x < 0 \\ a & ; x = 0 \\ \dfrac{e^{1/x} + e^{2/x} + e^{3/x}}{ae^{2/x} + be^{3/x}} & ; 0 < x < \frac{\pi}{2} \end{cases}$ be

continuous at location $x = 0$, then value of $(a + b)$ is :

(a) $e - \frac{1}{e}$ (b) $e + \frac{1}{e}$ (c) $e + \frac{2}{e}$ (d) $2e - \frac{1}{e}$

12. If $f:[-2a,2a]\to R$ is an odd-function such that left hand derivative at $x=a$ is zero and $f(x)=f(2a-x)$ for all $x\in(a,2a)$ then left hand derivative at $x=-a$ is :

(a) 1 (b) –1

(c) 0 (d) Data insufficient

13. If $\int_0^x t\,f(t)\,dt=\sin x-x\cos x-\dfrac{x^2}{2}$ for all $x\in R-\{0\}$,

then $f\left(\dfrac{\pi}{6}\right)$ is equal to :

(a) 0 (b) $\dfrac{1}{2}$

(c) $-\dfrac{1}{2}$ (d) $-\dfrac{1}{4}$

14. Let $f(x)=[\sin x]+[\sin 2x]\ \ \forall\ \ x\in(0\,,10)$, [.] is the greatest integer function , then $f(x)$ is discontinuous at :

(a) 8 points (b) 9 points

(c) 10 points (d) 11 points

15. If $f(x)=\begin{cases}\dfrac{x}{2x^2+|x|} & ;\ x\neq 0\\ 1 & ;\ x=0\end{cases}$, then $f(x)$ is :

(a) differentiable at $x=0$

(b) discontinuous at $x=0$

(c) continuous but not differentiable at $x=0$

(d) $f\,'(0^+)=-1$

16. Let function $y=f(x)$ be defined parametrically as $x=3t-|t|\,;\ y=2t^2+t\,|t|$ for all $t\in R$, then :

(a) $f(x)$ is continuous but non-differentiable at $x=0$.

(b) $f(x)$ is discontinuous at $x=0$.

(c) $f(x)$ is differentiable at $x=0$.

(d) $f\,'(0^+)=2$.

17. Let $f(x)=[x]^2-[x^2]$, where [.] represents the greatest integer function , then $f(x)$ is discontinuous at :

(a) $x\in I$ (b) $x\in I-\{0\}$

(c) $x\in I-\{0\,,1\}$ (d) $x\in I-\{1\}$

18. Let $f(x)=\sum_{r=0}^{n}a_r x^r$ and if $|\,f(x)\,|\le|\,e^{x-1}-1\,|$ for all $x\in[0\,,\infty)$, then value of $|\,na_n+(n-1)a_{n-1}+\ldots..+2a_2+a_1\,|$ is :

(a) less than one

(b) greater than one

(c) not less than one

(d) not greater than one

19. Let $f(x)=\begin{cases}\dfrac{x}{1+e^{1/x}} & ;x\neq 0\\ 0 & ;x=0\end{cases}$; then :

(a) $f(x)$ is discontinuous at $x=0$

(b) $f\,'(0^+)=1$

(c) $f\,'(0^-)=1$

(d) $f\,'(0^+)=f\,'(0^-)=1$

20. Let $f(x)$ be differentiable function with property $f(x+y)=f(x)+f(y)+xy$ and $\lim_{h\to 0}\dfrac{1}{h}f(h)=3$, then $f(x)$ is :

(a) linear function (b) $3x+x^2$

(c) $3x+\dfrac{x^2}{2}$ (d) x^3+3x

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let $f(x)$ be defined in [–2 , 2] by

$$f(x)=\begin{cases}\max\left\{\sqrt{4-x^2}\ ,\ \sqrt{1+x^2}\right\} & ;\ -2\le x\le 0\\ \min\left\{\sqrt{4-x^2}\ ,\ \sqrt{1+x^2}\right\} & ;\ 0<x\le 2\end{cases}\ ,\ \text{then}$$

(a) $f(x)$ is continuous at $x=\pm\sqrt{\dfrac{3}{2}}$ but non-differentiable

(b) $f(x)$ is discontinuous at $x=\pm\sqrt{\dfrac{3}{2}}\ ,\ 0$

(c) $f(x)$ is non-differentiable at $x=0$

(d) $f(x)$ is differentiable $\forall\ x\in(-2\,,\,2)$

22. Let $f:R\to R$ be defined by functional relationship $f\left(\dfrac{x+y}{3}\right)=\dfrac{2+f(x)+f(y)}{3}$ and $f\,'(0)=2$, then which of the following statements are correct ?

(a) $y=|\,f(x)\,|$ is continuous and non-differentiable at $x=-1$.

(b) $y=\sin(\,f(x))$ is differentiable for all real x.

(c) $\int_{-1}^{1}[\,f(x)]dx=2$, where [.] represents the greatest integer function.

(d) $\int_{1}^{2}f([x])dx=4$.

23. Let $f(x) = |\sin^{-1}(\sin x)| \;\; \forall \;\; x \in R$, then

(a) $f(x)$ is non-differentiable at $x = \dfrac{n\pi}{2}$; $n \in I$.

(b) Number of solutions of the equation $\dfrac{2}{\pi} f(x) - log_{3\pi} x = 0$ are five.

(c) $\displaystyle\int_0^{\pi} [f(x)]dx = \pi - 2$, where [.] represents the greatest integer function.

(d) $y = sgn(f(x))$ is continuous $\forall \; x \in R$.

24. Let [.] denotes the greatest integer function , and $f(x) = \dfrac{\sin\frac{\pi}{4}[x]}{[x]}$, then $f(x)$ is :

(a) continuous at $x = 2$. (b) discontinuous at $x = 2$.

(c) continuous at $x = 3/2$. (d) discontinuous at $x = 3/2$.

25. If $|c| \le \dfrac{1}{2}$ and $f(x)$ is differentiable function at $x = 0$

where $f(x) = \begin{cases} b\sin^{-1}\left(\dfrac{x+c}{2}\right) & ; \; -\dfrac{1}{2} < x < 0 \\ 1/2 & ; \; x = 0 \\ \dfrac{e^{ax/2}-1}{x} & ; \; 0 < x < \dfrac{1}{2} \end{cases}$, then

(a) $a = 2$
(b) $64b^2 + c^2 = 4$
(c) $a = 1$
(d) $16b^2 + c^2 = 64$

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. Statement 1 : Let $f(x)$ be discontinuous at $x = \pi$ and $\lim\limits_{x\to a} g(x) = \pi$, then $\lim\limits_{x\to a} f(g(x))$ can't be equal to $f\left(\lim\limits_{x\to a} g(x)\right)$.

because

Statement (2) : If $f(x)$ is continuous at $x = \pi$ and $\lim\limits_{x\to a} g(x) = \pi$, then $\lim\limits_{x\to a} f(g(x)) = f\left(\lim\limits_{x\to a} g(x)\right)$.

27. Let $g(x) = [x^2 - 3x + 4] \;\; \forall \;\; x \in R$, where [.] is greatest integer function , and $f(x) = \dfrac{\sin(\pi g(x))}{1+[x]^2}$ for all $x \in R$.

Statement 1 : $f(x)$ is discontinuous at infinitely many point locations

because

Statement 2 : $g(x)$ is discontinuous at infinitely many point locations.

28. Statement 1 : $f(x) = sgn(x)$, then $y = |f(x)|$ is not continuous at $x = 0$

because

Statement 2 : If $y = g(x)$ is discontinuous at location $x = a$, then $y = |g(x)|$ is also discontinuous at $x = a$.

29. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} \sqrt{x-2} \; ; & x \ge 2 \\ -\sqrt{2-x} \; ; & x < 2 \end{cases}$$

Statement 1 : $f(x)$ is non-differentiable at $x = 2$

because

Statement 2 : $f(x)$ is not having a unique tangent at $x = 2$.

30. Let $f(x) = \begin{cases} \max\{g(t) \, ; \, 0 \le t \le x\} & ; \; 0 \le x \le 4 \\ x^2 - 8x + 17 & ; \; x > 4 \end{cases}$ &

$g(x) = \sin x$ for all $x \in [0\,,\infty)$.

Statement 1 : $f(x)$ is differentiable for all $x \in [0\,,\infty)$

because

Statement 2 : $f(x)$ is continuous for all $x \in [0\,,\infty)$.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f : R \to R$ and $g : R \to R$ be the functions which are defined as $f(x) = max\left\{2x(1-x)\,,\, x^2\,,\,(x-1)^2\right\}$ and $g(x) = 2-|1-2x|$. On the basis of defined functions answer the following questions.

1. Total number of locations at which the function $h(x) = min\{\, f(x)\,, g(x)\}$ is non-differentiable is/are :

(a) 1 (b) 2

(c) 4 (d) 6

2. If α and β are the real roots of equation $f(x) - g(x) = 0$, then value of $(\alpha + \beta)$ is equal to :

(a) 2 (b) 1

(c) 0 (d) 3

3. If the equation $min\{\, f(x)\,,\, g(x)\} - \lambda = 0$ is having exactly four distinct real roots, then value of λ should not be :

(a) $\frac{4}{5}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{4}{9}$

Comprehension passage (2)
(Questions No. 4-6)

Let $\phi(x) = mid\{\, f(x)\,,\, g(x)\,,\, h(x)\}$ represents the function which is second in order when the values of three functions (viz : $f(x), g(x), h(x)$) are arranged in ascending or descending order at any given location of x. If $\phi(x) = mid\{\, x\,, x(4-x)^2\,, 4x\,\}$, then answer the following questions.

4. Exhaustive set of values of x at which the function $y = \phi(x)$ is non-differentiable , is given by :

(a) $\{0\,, 2\,, 3\,, 5\}$ (b) $\{2\,, 3\,, 4\,, 6\}$

(c) $\{3\,, 4\,, 5\,, 6\}$ (d) $\{2\,, 3\,, 5\,, 6\}$

5. Value of $\left\{\phi'(3^+) - \phi'(2^+) + \phi'(6^+)\right\}$ is equal to :

(a) 10 (b) 8

(c) 9 (d) 6

6. Value of $\int_3^5 \phi(x)dx$ is equal to :

(a) 10 (b) 8

(c) 12 (d) 6

Questions with Integral Answer :
(Questions No. 7-10)

7. Let $f(x) = \begin{cases} \cot^{-1}(x) & ; \ |x| \geq 1 \\ \dfrac{|x|-1}{2} + \dfrac{\pi}{4} & ; \ |x| < 1 \end{cases}$, then total number of locations which domain of $f'(x)$ doesn't contain is/are ....

8. Let $f(x) = \begin{cases} x + a & ; \ 0 \leq x < 2 \\ b - x & ; \ x \geq 2 \end{cases}$ and

$g(x) = \begin{cases} 1 + \tan x & ; \ 0 \leq x < \pi/4 \\ 3 - \cot x & ; \ \pi/4 \leq x < \pi \end{cases}$.

If $f\left(g(x)\right)$ is continuous at the location $x = \frac{\pi}{4}$, then value of $2(b-a)$ is equal to ....

9. Consider $f(x) = \begin{cases} a\sin x + b\cos x & ; \ x \leq 0 \\ \left(\dfrac{x+e^x}{2x+1}\right)^{1/x} & ; \ x > 0 \end{cases}$,

if $f(x)$ is continuous for all $x \in R$ and $f'(1) = f\left(-\frac{\pi}{2}\right)$, where $[x]$ represents the greatest integer less than or equal to x, then value of $[b] + [a]$ is equal to ....

10. Let $f : R^+ \to R^+$ be a differentiable function satisfying $f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x}$ $\forall\ x\,,\, y \in R^+$ also $f(1) = 0$, $f'(1) = 1$. If M be the greatest value of $f(x)$ then the value of $[M+3]$. , (where [.] denotes the greatest integer function) , is equal to ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Match the functions in columns (I) with their cosrespending properties in column (II).

Column (I)	Column (II)
(a) $f(x) = min\{x^3, x^2\}$	(p) continuous in $(-2, 2)$.
(b) $f(x) = min\{\|x\|, \|x-1\|, \|x+1\|\}$	(q) differentiable in $(-2, 2)$.
(c) $f(x) = \|2x+4\| - 2\|x-2\|$	(r) not differentiable at least at one point in $(-2, 2)$.
(d) $f(x) = \|\sin x\| + \|\cos x\|$	(s) increasing in $(-2, 2)$.

12. Let $f : R \to R$ be continuous quadratic function such that $f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2$. If $f(0) = 0$, then match the following columns (I) and (II).

Column (I)	Column (II)
(a) Value of $f'\left(\frac{9}{8}\right)$ is equal to	(p) 0
(b) Total number of points of non-differentiability for $y = \left\|1 - \|f(x) - 2\|\right\|$ is/are	(q) 2
(c) If $g(x) = min\left\{f(t)\,;\, 0 \le t \le x\right\}$, where $x \in [0, 4]$, then value of $g'(3)$ is	(r) 4
(d) Number of locations at which $y = \|f(x)\|$ is non-differentiable is/are	(s) 6

ANSWERS Exercise No. (1)

1. (b)
2. (a)
3. (c)
4. (d)
5. (d)
6. (b)
7. (a)
8. (b)
9. (d)
10. (c)
11. (b)
12. (c)
13. (c)
14. (b)
15. (b)
16. (c)
17. (d)
18. (d)
19. (c)
20. (c)
21. (a , c)
22. (a , b , d)
23. (a , b , c)
24. (b , c)
25. (b , c)
26. (d)
27. (d)
28. (c)
29. (c)
30. (b)

ANSWERS Exercise No. (2)

1. (c)
2. (b)
3. (b)
4. (d)
5. (c)
6. (b)
7. (3)
8. (8)
9. (0)
10. (3)

11. (a) $\to$ p , r , s
(b) $\to$ p , r
(c) $\to$ p , q , s
(d) $\to$ p , r

12. (a) $\to$ r
(b) $\to$ s
(c) $\to$ p
(d) $\to$ p

Chapter 13

Differentiation

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-15)

1. Let $y = e^{2x}$, then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is equal to :

(a) 1 (b) e^{-2x}
(c) $2e^{-2x}$ (d) $-2e^{-2x}$

2. Let $g(x)$ is reflection of $f(x)$ about the line mirror $y = x$ and $f'(x) = \frac{x^{12}}{1+x^2}$, if $g(3) = a$, then $g'(3)$ is :

(a) $\frac{1+a^2}{a}$ (b) $\frac{a^{12}}{1+a^2}$

(c) $\frac{1+a^2}{a^{12}}$ (d) $\frac{a}{1+a^2}$

3. Let $x = \tan\left(\frac{1}{a}\log_e y\right)$ and $(1+x^2)\frac{d^2y}{dx^2} = (\lambda - 2x)\frac{dy}{dx}$ then 'λ' is equal to :

(a) $2a$ (b) $3a$
(c) a (d) $-4a$

4. If $y = f(x)$ and $y\cos x + x\cos y = \pi$ for all $x \in R$, then $f''(0)$ is :

(a) π (b) $-\pi$
(c) 0 (d) 2π

5. If $y = f\left(\frac{2x-1}{1+x^2}\right)$ and $f'(x) = \sin^2 x$, then $\left.\frac{dy}{dx}\right|_{x=0}$ is :

(a) $\sin^2(1)$ (b) $-2\sin^2(1)$
(c) $1 - \cos 2$ (d) $1 + \cos(1)$

6. Second derivative of $a\sin^3 t$ w.r.t. $a\cos^3 t$ at $t = \frac{\pi}{4}$ is :

(a) $\frac{4\sqrt{2}}{3a}$ (b) 2

(c) $\frac{1}{2a}$ (d) $\frac{3\sqrt{2}}{4a}$

7. If $\int_0^y \cos t^2\,dt = \int_0^{x^2} \frac{\sin t}{t}\,dt$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{2\sin^2 x}{x\cos y^2}$ (b) $\frac{2\sin x^2}{x^2\cos y}$

(c) $\frac{\sin x^2}{x\cos y^2}$ (d) $\frac{2\sin x^2}{x\cos y^2}$

8. Let $y = \tan^{-1}\left(\frac{ln(e/x^2)}{ln(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\,ln\,x}{1-6\,ln\,x}\right)$; then $\frac{d^2y}{dx^2}$ is :

(a) 0 (b) 1
(c) 2 (d) –1

9. Let $f(x)$ be a polynomial function , then second derivative of $f(e^x)$ is :

(a) $e^{2x}f'(x) + e^x f''(e^x)$ (b) $e^x f''(x) + f''(e^x)$

(c) $f''(e^x) + e^x f'(e^x)$ (d) $e^x f'(e^x) + e^{2x} f''(e^x)$

10. If $xe^{xy} = y + \sin^2 x$, then $\left.\frac{dy}{dx}\right|_{x=0}$ is :

(a) –1 (b) 2
(c) 1 (d) 0

11. Let $f(x)$ be differentiable and $\int_0^{t^2} x\,f(x)\,dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ is :

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$

(c) $-\frac{5}{2}$ (d) 1

12. For an invertible function $y = f(x)$, value of

$$\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}} + \frac{\left\{1+\left(\frac{dx}{dy}\right)^2\right\}^{3/2}}{\frac{d^2x}{dy^2}} \text{ is :}$$

(a) 1 (b) 0

(c) -1 (d) $\sqrt{2}$

13. Let (α, β), where $\alpha, \beta \neq 0$, satisfy the equation $ax^2 + 2hxy + by^2 = 0$, then $\left.\frac{dy}{dx}\right|_{(\alpha,\beta)}$ is equal to :

(a) 1 (b) $\frac{\alpha}{\beta}$

(c) $\frac{\beta}{\alpha}$ (d) 0

14. Let $F(x) = \frac{1}{x^2}\int_4^x \left\{4z^2 - 2F'(z)\right\} dz$, then value of $F'(4)$ is equal to :

(a) $\frac{64}{9}$ (b) $\frac{32}{9}$

(c) $\frac{64}{3}$ (d) $\frac{32}{3}$

15. If $f(x) = (1+x)^n$, then the value of $f(0) + f'(0) + \frac{f''(0)}{2!} + \dots + \frac{f^n(0)}{n!}$ is

(a) n (b) 2^n

(c) 2^{n-1} (d) 0

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. Let $n \in N$ and $f(x)$ is twice differentiable positive function on $(0, \infty)$ such that $x f(x) - f(x+1) = 0$. If $f(x) = e^{g(x)}$, then :

(a) $g''(x+1) + g''(x) = \frac{1}{x^2}$

(b) $g''(x) - g''(x-1) = -\frac{1}{(x-1)^2}$

(c) $g''\left(n+\frac{1}{2}\right) - g''\left(n-\frac{1}{2}\right) = -\frac{4}{(1-2n)^2}$

(d) $g''\left(\frac{1}{2}\right) - g''\left(n+\frac{1}{2}\right) = 4\sum_{r=1}^{n}\frac{1}{(2r+1)^2}$

17. Let $f: R \to R$ be strictly increasing function for all $x \in R$ and $f''(x) - 2f'(x) + f(x) = 2e^x$, then which of the following may be correct :

(a) $|f(x)| = f(x) \;\; \forall \;\; x \in R$

(b) $f(5) = -8$

(c) $f(3) = 8$

(d) $|f(x)| = -f(x) \;\; \forall \;\; x \in R$

18. Let $p, q \in R$, and $f(x) = (x^2 - 6x + p)(x^2 - 8x + q)$. If exactly one real value of 'α' exists for which $f(\alpha) = f'(\alpha) = 0$ and $f''(\alpha) \neq 0$, then which of the following ordered pairs (p, q) are applicable :

(a) (9 , 16) (b) (9 , 15)

(c) (5 , 15) (d) (8 , 12)

19. Let $f(x) = \sin^{-1}(\sin x)$ and $g(x) = \cos^{-1}(\cos x)$ for all $x \in R$, then which of the following statements are correct :

(a) $f'(7) = g'(7) = 1$ (b) $f'(2) + g'(2) = 0$

(c) $f'(-4) = g'(-4) = -1$ (d) $f'(e) = g'(2e) = -1$

20. Let $f(x) = \cos^2(x+1) - \cos x \,.\, \cos(x+2)$ for all $x \in R$, then :

(a) $f'(x) = 0 \;\; \forall \;\; x \in R$

(b) $f''(x) \neq 0 \;\; \forall \;\; x \in R$

(c) $f'(x) \neq 0$ for some real values of x

(d) $f(x)$ is non-decreasing $\forall \;\; x \in R$

Assertion Reasoning questions :
(Questions No. 21-25)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. Let $f^n(x)$ denotes the n^{th} derivative of $f(x)$ and $f(x) = (x^2 - 1)^k$, where $k \in N$.

Statement 1 : If the equation $f^n(x) = 0$ is having 10 distinct real roots for exactly one value of $'n'$, then $'k'$ equals to 9

because

Statement 2 : A polynomial function of 'm' degree, where $m \in N$, vanishes after m^{th} derivative.

22. **Statement 1 :** Let $f(x) = \sin x - x\cos x$, then $f'(x) = x\sin x$. Both the functions $f(x)$ and $f'(x)$ are non-periodic

because

Statement 2 : The derivative of non-periodic differentiable function is non-periodic in nature.

23. **Statement 1 :** Let $f(x) = \cos^{-1}(4x^3 - 3x)$, then $f'\left(\frac{1}{4}\right) = \frac{4}{5}\sqrt{15}$

because

Statement 2 : $\cos^{-1}(4x^3 - 3x) = -3\cos^{-1}x$ for all $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

24. Let $f_n(x) = exp\left(f_{n-1}(x)\right)$ $\forall$ $n \in N$ and $f_0(x) - x = 0$, then

Statement 1 : $\frac{d}{dx}\left(f_n(x)\right) = \prod_{i=1}^{n} f_i(x)$

because

Statement 2 : $\prod_{i=1}^{n} f_i(x) = \exp\left(\sum_{i=1}^{n} f_{i-1}(x)\right)$

25. **Statement 1 :** Let $y = t^2$ and $x = t + 1$ $\forall$ $t \in R$, then $\frac{d^3y}{dx^3} = 0$ at $t = 0$

because

Statement 2 : $\frac{dy}{dx} = \frac{d^2y}{dx^2} = 0$ at $t = 0$

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Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f(x)$ be a cubic polynomial function for which $x^3 + f'(1)x^2 + f''(3)x - f(x) = 0$ holds true for all $x \in R$, then answer the following questions which are based on $f(x)$.

1. With reference to $f(x)$, the incorrect statement is :

(a) $f(0) + f(2) = -12$ (b) $f(0) + f(3) \neq -26$

(c) $f(1) + f(3) \neq -26$ (d) $f(1) + f(2) = -14$

2. Let $[x]$ represents the greatest integer which is just less than equal to x, and α, β, γ are the roots of $f(x) = 0$, where $\alpha < \beta < \gamma$, then value of $[\alpha] + 2[\beta] + 3[\gamma]$ is equal to :

(a) 18 (b) 15

(c) 20 (d) 12

3. If $g(x) = |f(x)|$, then total number of critical points for $y = g(x)$ are :

(a) 4 (b) 5

(c) 3 (d) 2

Comprehension passage (2)
(Questions No. 4-6)

Let $f(x) = x^4 - 8x^3 + 22x^2 - 24x \quad \forall \ x \in R$ and function $g(x)$ is defined as :

$$g(x) = \begin{cases} \min\{f(t) : 0 \le t \le x\} & ; \ 0 \le x < 2 \\ \max\{f(t) : 2 < t \le x\} & ; \ 2 \le x \le 5 \end{cases}$$

On the basis of given definition of $f(x)$ and $g(x)$ answer the following questions :

4. Function $g(x)$ in $(0, 5)$ is non-differentiable at :

(a) one point location. (b) two point locations.

(c) three point locations. (d) infinite point locations.

5. Value of $g'\left(\frac{3}{2}\right)$ is equal to :

(a) -2

(b) 1

(c) 0

(d) -4

6. In which one of the following intervals, $f(x) = g(x)$ holds true :

(a) $\left(\frac{1}{e}, e\right)$

(b) $[\cos 1, 2]$

(c) $[\sin 3, \sin 1]$

(d) $(1, \pi)$

Questions with Integral Answer :
(Questions No. 7-10)

7. If $x = \sec\theta - \cos\theta$, $y = (\sec\theta)^n - (\cos\theta)^n$, where $n \in N$, and $\left(\frac{dy}{dx}\right)^2 = n^2\left(\frac{y^2+\alpha}{x^2+\beta}\right)$, then value of $(\alpha - \beta)$ is equal to ....

8. Let $f(x) = -\frac{x^3}{3} + (\sin 6)x^2 - (\sin 4)(\sin 8)x$, and $f'(\sin 8) = K(\sin^2 1)(\sin 8)(\sin 6)$, then value of $'K'$ is equal to ....

9. Let $f(x) = \int_0^\infty \frac{e^{-xt}}{1+t^2}dt$, then value of $f''\left(\frac{1}{4}\right) + f\left(\frac{1}{4}\right)$ is equal to ....

10. Let the function $f(x)$ be defined as $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is equal to ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Let α , $\beta \in R$, where $\alpha \neq \beta$, and $f(x) = x^3 - 9x^2 + 24x + k \equiv (x-\alpha)^2(x-\beta)$ then match the following columns.

Column (I)	**Column (II)**
(a) Absolute value of the difference of the two possible values for 'k' is	(p) 0
(b) If $\alpha < \beta$, then 'α' is	(q) 2
(c) If $\alpha > \beta$, then 'α' is	(r) 4
(d) If $\alpha > \beta$, then 'β' is	(s) 1

12. Match the following columns for the function and their derivatives.

Column (I)	**Column (II)**
(a) If $f(x) = 2\tan^{-1}x$, then $f'(x)$ is :	(p) $\dfrac{2}{1+x^2}$; $\lvert x \rvert < 1$
	(q) $\dfrac{-2}{1+x^2}$; $x < 0$
(b) If $f(x) = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$, then $f'(x)$ is :	(r) $\dfrac{2}{1+x^2}$; $\lvert x \rvert \neq 1$
(c) If $f(x) = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$, then $f'(x)$ is :	(s) $\dfrac{-2}{1+x^2}$; $\lvert x \rvert > 1$
(d) If $f(x) = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$, then $f'(x)$ is :	(t) $\dfrac{2}{1+x^2}$; $x \in R$

ANSWERS Exercise No. (1)

1. (d)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (c)
6. (a)	**7.** (d)	**8.** (a)	**9.** (d)	**10.** (c)
11. (a)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (b)
16. (b , c)	**17.** (a , c)	**18.** (c , d)	**19.** (a , b , d)	**20.** (a , d)
21. (b)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (c)

ANSWERS Exercise No. (2)

1. (c)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (c)
6. (c)	**7.** (0)	**8.** (4)	**9.** (4)	**10.** (2)
11. (a) $\rightarrow$ r (b) $\rightarrow$ q (c) $\rightarrow$ r (d) $\rightarrow$ s	**12.** (a) $\rightarrow$ t (b) $\rightarrow$ p , r (c) $\rightarrow$ p , s (d) $\rightarrow$ q			

Chapter 14

Tangent and Normal

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-10)

1. Let *'P'* be a point on the curve $y = \dfrac{x}{1+x^2}$ and tangent drawn at P to the curve has greatest slope in magnitude , then point *'P'* is

(a) $\left(\sqrt{3}, \dfrac{\sqrt{3}}{4}\right)$ (b) $(0 , 0)$

(c) $\left(-\sqrt{3}, -\dfrac{\sqrt{3}}{4}\right)$ (d) $\left(1 , \dfrac{1}{2}\right)$

2. The equation of common tangent to the curves $y = 6 - x - x^2$ and $xy = x + 3$ is :

(a) $3x - y = 8$ (b) $3x + y = 10$

(c) $2x + y = 4$ (d) $3x + y = 7$

3. If $\alpha > 0$, then set of values of α for which $\alpha e^x - x = 0$ has real roots is :

(a) $\left(0 , \dfrac{1}{e}\right]$ (b) $\left[\dfrac{1}{e} , 1\right]$

(c) $\left[\dfrac{1}{e} , \infty\right)$ (d) $[0 , 1]$

4. If $|\, f(x_1) - f(x)_2 \,| \le (x_1 - x_2)^2 \;\; \forall \;\; x_1 , \; x_2 \in R$, then equation of tangent to the curve $y = f(x)$ at point $(2 , 8)$ is :

(a) $x - 8 = 0$ (b) $y - 2 = 0$

(c) $y - 8 = 0$ (d) $x - 2 = 0$

5. Any normal to the curve $x = a(\cos\theta + \theta\sin\theta)$; $y = a(\sin\theta - \theta\cos\theta)$ at any point $'\theta'$ is such that :

(a) it passes through $(0 , 0)$.

(b) it makes constant angle with x-axis.

(c) it is at a constant distance from $(0 , 0)$.

(d) none of these.

6. Angle of intersection between the curves given by $x^3 - 3xy^2 + 2 = 0$ and $y^3 - 3x^2y - 2 = 0$ is :

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{4}$

7. Equation of normal to curve $y = (1+x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$ is :

(a) $x + y + 1 = 0$

(b) $x - y + 1 = 0$

(c) $x + y - 1 = 0$

(d) $x + y = 0$

8. Let at point *'P'* on the curve $y^3 + 3x^2 = 12y$, the tangent is vertical , then *'P'* may be :

(a) $(0 , 0)$ (b) $\left(\pm\dfrac{4}{\sqrt{3}} , -2\right)$

(c) $\left(\pm\sqrt{\dfrac{11}{3}} , 1\right)$ (d) $\left(\pm\dfrac{4}{\sqrt{3}} , 2\right)$

9. Acute angle of intersection between the curves $y = |1 - x^2|$ and $y = |x^2 - 3|$ is given by :

(a) $\tan^{-1}\left(\dfrac{4\sqrt{3}}{7}\right)$ (b) $\sin^{-1}\left(\dfrac{3\sqrt{2}}{7}\right)$

(c) $\cos^{-1}\left(\dfrac{7}{9}\right)$ (d) $\cos^{-1}\left(\dfrac{7}{9\sqrt{2}}\right)$

10. If the tangent and normal to the curve $y = e^{-x}$ at point $P(0 , 1)$ intersects the x-axis at *'T'* and *'N'* respectively , then area (in sq. units) of equilateral triangle which is circumscribed by the incircle of ΔPTN is :

(a) $\dfrac{3\sqrt{3}}{2}(\sqrt{2}+1)^2$ (b) $\dfrac{3\sqrt{3}}{4}(\sqrt{2}-1)^2$

(c) $\dfrac{\sqrt{3}}{4}(\sqrt{2}+1)^2$ (d) $\dfrac{\sqrt{3}}{4}(\sqrt{2}-1)^2$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 11-15)

11. Let $x + 2y - k = 0$ be the tangent to the curve $y = \cos(x+y)$, $-2\pi \le x \le 2\pi$, then possible values of $'k'$ can be :

(a) $\pi/2$ (b) $-\pi/2$

(c) $3\pi/2$ (d) $-3\pi/2$

12. If a function is having horizontal tangent at origin then it holds the H-property , functions having H-property are :

(a) $y=\begin{cases} x\sin\frac{1}{x} & ;\ x\neq 0 \\ 0 & ;\ x=0 \end{cases}$ (b) $y=\begin{cases} x^2\sin\frac{1}{x} & ;\ x\neq 0 \\ 0 & ;\ x=0 \end{cases}$

(c) $y=x\,|x|$ (d) $y=\min\{x^2, |x|\}$

13. Let a curve in parametric form be represented by $x=3t^2$, $y=2t^3$ for all $t\in R$, then which of the following lines are tangent to curve at one point and normal at another point of curve ?

(a) $\sqrt{2}x+y-2\sqrt{2}=0$ (b) $\frac{x}{2}-\frac{\sqrt{2}}{4}y-1=0$

(c) $\frac{x}{2}+\frac{\sqrt{2}}{2}y-\sqrt{2}=0$ (d) $x-\sqrt{2}y+\sqrt{2}=0$

14. Let $f:R\to R$ and $g:R^+\to[0\ ,\infty)$ be the functions which are given by $f(x)=kx$ and $g(x)=|\log_e x|$. If the equation $f(x)-g(x)=0$ is having three distinct real roots , then possible values of $'k'$ can be :

(a) $\frac{1}{e^2}$ (b) $\frac{1}{\sqrt{e}}$

(c) $\frac{1}{e^3}$ (d) $\frac{1}{2+\sqrt{\pi}}$

15. Functions which are having vertical tangent at point $x=1$ are :

(a) $f(x)=sgn(x-1)$

(b) $f(x)=\sqrt[3]{x-1}$

(c) $f(x)=(x-1)^{2/3}$

(d) $f(x)=\begin{cases} \sqrt{x-1} & ;\ x\geq 1 \\ \sqrt{1-x} & ;\ x<1 \end{cases}$

Assertion Reasoning questions :
(Questions No. 16-20)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

16. Consider the curves $C_1: y^2=2x$ and $C_2: y=e^{-|x|}$.

Statement 1 : Curves $'C_1'$ and $'C_2'$ form an orthogonal pair of curves

because

Statement 2 : Curves $'C_1'$ and $'C_2'$ intersect each other at only one point location

17. Let $a\in(0\ ,\sqrt{2})$ and $b\in R^+$, where

$$D=(a-b)^2+\left(\sqrt{2-a^2}-\frac{9}{b}\right)^2$$

Statement 1 : For given conditions on $'a'$ and $'b'$, the minimum value of $'D'$ is 8

because

Statement 2 : The minimum distance between the curves $xy=9$ and $x^2+y^2=2$ is equal to $2\sqrt{2}$ units.

18. Statement 1 : Let $y=f(x)$ be polynomial function , and tangent at point $A(a\ ,f(a))$ is normal to the curve of $y=f(x)$ at point $B(b\ ,f(b))$, then at least one point $(c\ ,f(c))$ exists for which $f'(c)=0$, where $c\in(a\ ,b)$

because

Statement 2 : Product of the slopes of tangents to the curve $y=f(x)$ at $'A'$ and $'B'$ is equal to -1 if tangents are not parallel to the axes.

19. Consider the curves $C_1: y=x^2+x+1$ and $C_2: y=x^2-5x+6$.

Statement 1 : Equation of common tangent to the curves C_1 and C_2 is given by $9y+3x-4=0$

because

Statement 2 : Acute angle of intersection of the curves C_1 and C_2 is $\tan^{-1}\left(\frac{54}{71}\right)$.

20. Statement 1 : Length of subtangent at point $P\,(2\ ,2)$ for the curve $x^2y^3=32$ is equal to 3 units

because

Statement 2 : Length of subtangent at any point $(\alpha\ ,\beta)$ for the curve $x^2y^3=32$ is equal to $\left|\frac{3\beta}{\alpha}\right|$.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Consider the curve $C_1 : 5x^5 - 10x^3 + x + 2y + 6 = 0$. If the normal $'N'$ to curve C_1 at point $P(0\ ,-3)$ meets the curve again at two points Q and R, then answer the following questions.

1. Minimum area (in square units) of the circle passing through the points Q and R is equal to :

(a) 5π (b) 4π
(c) 8π (d) 2π

2. With reference to line of normal $'N'$, which of the following statement is correct ?

(a) line $'N'$ is tangential to curve C_1 at point Q only.

(b) line $'N'$ is tangential to curve C_1 at point R only.

(c) line $'N'$ is tangential to curve C_1 at both the points Q and R..

(d) line $'N'$ is not tangential to curve C_1 at either of the point Q and R.

3. Let the length of subtangents at the points Q and R for the curve C_1 be l_1 and l_2 respectively, where $OQ > OR$, $'O'$ being the origin, then $\frac{l_1}{l_2}$ is equal to :

(a) 4 (b) 1
(c) 2 (d) 5

Comprehension passage (2)
(Questions No. 4-6)

Let $f : R \to R$ be defined as $f(x) = ax^3 + bx^2 + cx + 27$, where the curve of $y = f(x)$ touches the x-axis at point $P(-3\ ,\ 0)$ and meets the y-axis at point Q. If $f'(0) = 9$, then answer the following questions.

4. If $f(\alpha) = f(\beta) = 0$ and $\alpha \neq \beta$, then value of $[\alpha] + [\beta]$ is equal to : ([.] represents the greatest integer function)

(a) 0 (b) 2
(c) 5 (d) 10

5. Area (in square units) of the triangle formed by normal at $(\alpha\ ,\ 0)$, where $\alpha \neq -3$, with the co-ordinate axes is equal to :

(a) $\frac{1}{2}$

(b) $\frac{1}{8}$

(c) 1

(d) $\frac{1}{4}$

6. Let $g(x) = f(x) + \lambda$, where $(g'(x))^2 + g''(x).g(x) = 0$ is having exactly four distinct real roots, then exhaustive set of values of $'\lambda'$ belong to :

(a) $(-27\ ,\ 8)$

(b) $(-24\ ,\ 4)$

(c) $(-32\ ,\ 0)$

(d) $(-20\ ,\ 32)$

Questions with Integral Answer :
(Questions No. 7-10)

7. Let tangent at $'t_1'$ point to the curve $C : y = 8t^3 - 1$, $x = 4t^2 + 3$ is normal at another point $'t_2'$ to the curve $'C'$, then value of $729(t_1)^6$ is equal to ....

8. Let any point $'P'$ lies on the curve $y^2(3 - x) = (x - 1)^3$, where the distance of $'P'$ from the origin is $'r_1'$ and the distance of tangent at $'P'$ from the origin is $'r_2'$. If point P is $(2\ ,\ 1)$, then value of $\left|\frac{(r_1^2 + 15)r_2^2}{r_1^2 - 1}\right|$ is equal to ....

9. Let l_1 and l_2 be the intercepts made on the x-axis and y-axis respectively by tangent at any point of the curve $x = a\cos^3\theta$; $y = b\sin^3\theta$, then the value of $\left\{\frac{l_1^2}{a^2} + \frac{l_2^2}{b^2}\right\}$ is ....

10. Let chord PQ of the curve $y + \lambda^2x^2 - 5\lambda x + 4 = 0$ be tangential to curve $y(1 - x) = 1$ at the point $R(2\ ,-1)$, if $PR = RQ$, then the least possible value of 4λ is equal to ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) If the angle between the curves $yx^2 = 1$ and $y = e^{2-2\lvert x\rvert}$ at point (1 , 1) is θ , then value of $\cos\theta$ is	(p) 3
(b) If the acute angle of intersection of the curves $x^2 = 4ay$ and $y = \dfrac{8a^3}{x^2+4a^2}$, $a \in R^+$, is $\tan^{-1}(\lambda)$, then 'λ' is equal to	(q) 2
	(r) 1
(c) The length of subtangent at any point on the curve $y = ae^{x/3}$ is equal to	(s) 5/4
(d) If the slope of tangent , if exists , varies at every point of the curve $y = \max\left\{e^x , 1+e^{-x} , k\right\}$, then 'k' can be	(t) 1/2

12. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) If the non-vertical common tangent of the curves $xy = -1$ and $y^2 = 8x$ is line 'L', then area (in square units) of the triangle formed by line 'L' with the co-ordinate axes is	(p) 1
(b) If the curves $y = 1 - \cos x$, $-\pi < x < \pi$ and $y = \dfrac{\sqrt{3}}{2}\lvert x\rvert + \lambda$ touch each other , then the number of possible values of 'λ' is/are	(q) 1/2
(c) The area (in square units) of triangle formed by normal at the point (1 , 0) to the curve $x = e^{\sin y}$ with coordinate axes is :	(r) 4
	(s) 2
(d) If the inequation $3 - x^2 > \lvert x - \lambda\rvert$ has at least one negative solution , then the possible values of 'λ' can be	(t) -4

ANSWERS Exercise No. (1)

1. (b)
2. (d)
3. (a)
4. (c)
5. (c)
6. (b)
7. (c)
8. (d)
9. (c)
10. (b)
11. (a , d)
12. (a , b , c , d)
13. (a , b)
14. (a , c , d)
15. (a , b)
16. (b)
17. (a)
18. (a)
19. (d)
20. (c)

ANSWERS Exercise No. (2)

1. (a)
2. (c)
3. (d)
4. (a)
5. (b)
6. (c)
7. (8)
8. (9)
9. (1)
10. (1)

11. (a) → r
(b) → p
(c) → p
(d) → r , s , t

12. (a) → s
(b) → s
(c) → q
(d) → p , q , s

Chapter 15

Rolle's Theorem & Mean Value Theorem

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-10)

1. The tangent to curve of $f(x) = (x+1)^2$ at the point $\left(\frac{\alpha+\beta}{2}, f\left(\frac{\alpha+\beta}{2}\right)\right)$ intersects the line joining $(\alpha, f(\alpha))$ and $(\beta, f(\beta))$; where $\alpha < \beta$ and $\alpha, \beta \in R$.

(a) on left of $x = \frac{\alpha+\beta}{2}$ (b) on right of $x = \frac{\alpha+\beta}{2}$

(c) at no point (d) at infinite points

2. If $f(x)$ and $g(x)$ are differentiable functions for all $x \in [0, 1]$ such that $f(0) = g(1) = 2$, $g(0) = 0$ and $f(1) = 6$, then there exists some value of $x \in (0, 1)$ for which :

(a) $f'(\alpha) = g'(\alpha)$ (b) $f'(\alpha) = 4g'(\alpha)$

(c) $f'(\alpha) = 2g'(\alpha)$ (d) $f'(\alpha) = 3g'(\alpha)$

3. If $4(b+3d) = 3(a+2c)$, then $ax^3 + bx^2 + cx + d = 0$ will have at least one real root in :

(a) $\left(-\frac{1}{2}, 0\right)$ (b) $(-1, 0)$

(c) $\left(-\frac{3}{2}, 0\right)$ (d) $(0, 1)$

4. If Rolle's theorem is applicable to the function $f(x) = \int_0^x e^{t^2}(t^2 - \alpha^2)\,dt$ on the interval $[0, 2]$, then $'\alpha'$ belongs to :

(a) $(-4, 4) - \{0\}$ (b) $(-3, 3) - \{0\}$

(c) $(-1, 1) - \{0\}$ (d) $(-2, 2) - \{0\}$

5. Let $f(x)$ be a differentiable function $\forall\ x \in R$ and $f(1) = -2$ and $f'(x) \geq 2\ \forall\ x \in [1, 6]$, then $f(6)$ is :

(a) more than 5 (b) not less than 5

(c) more than 8 (d) not less than 8

6. Let $f(x) = \begin{cases} x^{\alpha} \ln x & ; \ x > 0 \\ 0 & ; \ x = 0 \end{cases}$, then value of $'\alpha'$ for which Rolle's theorem is applicable in $[0, 1]$ is :

(a) $-\frac{2}{3}$ (b) $-\frac{1}{2}$

(c) 0 (d) 1/2

7. If $2a + 3b + 6c = 0$, then equation $ax^2 + bx + c = 0$ is having at least one root in the interval :

(a) $(1, 2)$ (b) $(-1, 0)$

(c) $(0, 1)$ (d) $(-1, 1/2)$

8. Let $f : [0, 8] \to R$ is differentiable function, then for $0 < \alpha, \beta < 2$, $\int_0^8 f(t)\,dt$ is equal to :

(a) $3\left(\alpha^3 f(\alpha^2) + \beta^3 f(\beta^2)\right)$.

(b) $3\left(\alpha^3 f(\alpha) + \beta^3 f(\beta)\right)$.

(c) $3\left(\alpha^2 f(\alpha^3) + \beta^2 f(\beta^3)\right)$.

(d) $3\left(\alpha^2 f(\alpha^2) + \beta^2 f(\beta^2)\right)$.

9. Let a, b, c be non-zero real numbers such that $\int_0^1 (1+\sin^4 x)(ax^2+bx+c)\,dx = \int_0^2 (1+\sin^4 x)(ax^2+bx+c)\,dx$, then quadratic equation $ax^2 + bx + c = 0$ has :

(a) exactly two real roots in $(0, 2)$.

(b) no root in $(0, 2)$.

(c) at least one root in $(0, 1)$.

(d) at least one root in $(1, 2)$.

10. If $a + b + 2c = 0$, where $ac \neq 0$, then the equation $ax^2 + bx + c = 0$ has

(a) at least one root in $(0, 1)$
(b) at least one root in $(-1, 0)$
(c) exactly one root in $(0, 1)$
(d) exactly one root in $(-1, 0)$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 11-15)

11. Let $f(x)=\sin\pi[x^2+1]+(x)^{\frac{1}{\ln x}}$ for all $x\in[2\,,4]$, where [x] denotes the integral part of x , then which of the following statements are not correct ?

(a) Rolle's theorem can't be applied to $f(x)$.

(b) Lagrange's Mean value theorem can be applied to $f(x)$.

(c) Rolle's theorem can be applied to $f(x)$.

(d) Lagrange's Mean value theorem can't be applied to $f(x)$.

12. Let $f(x)=\min\{\ln(\tan x)\,,\ \ln(\cot x)\}$, then which of the following statements are correct :

(a) Lagrange's mean value theorem is applicable on $f(x)$ for $x\in\left[\frac{\pi}{8}\,,\frac{\pi}{4}\right]$.

(b) $f(x)$ is continuous for $x\in\left(0\,,\frac{\pi}{2}\right)$.

(c) Rolle's theorem is applicable on $f(x)$ for $x\in\left[\frac{\pi}{8}\,,\frac{3\pi}{8}\right]$.

(d) Rolle's theorem is not applicable on $f(x)$ for $x\in\left[\frac{\pi}{4}\,,\frac{3\pi}{8}\right]$.

13. Let $f(x)$ be thrice differentiable function and $f(1)=1$, $f(2)=8$ and $f(3)=27$, then which of the following statements are correct :

(a) $f'(x)=3x^2$ for at least two values in $x\in(1\,,3)$.

(b) $f''(x)=6x$ for at least one value in $x\in(1\,,3)$.

(c) $f'''(x)=6\ \forall\ x\in R$.

(d) $f'(x)=3x^2$ for at least one value in $x\in(2\,,3)$.

14. If $f(x)=ax^3+bx^2+11x-6$ satisfy the conditions of Rolle's theorem in [1 , 3] and $f'\left(2+\frac{1}{\sqrt{3}}\right)=0$, then values of 'a' and 'b' satisfy :

(a) $a-b=8$ (b) $4a-b=10$

(c) $\ln a=1+\operatorname{sgn}(b)$ (d) $ab=2$

15. Let $f(x)$ be a non-constant twice differentiable function defined on R such that $f(x)-f(4-x)=0$ and $f'(1)=0$, then :

(a) $f'(x)$ vanishes at least thrice in [0 , 4].

(b) $f''(x)$ vanishes at least twice in [0 , 4].

(c) $f'(x)$ vanishes at least once in [2 , 4].

(d) $f'''(x)$ vanishes at least once in [0 , 4].

Assertion Reasoning questions : (Questions No. 16-20)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

16. Statement 1 : If $f(x)$ and $g(x)$ are continuous and differentiable functions for all real x , then there exists some value of 'β' in $(\alpha\,,\gamma)$ such that

$$\frac{f'(\beta)}{f(\alpha)-f(\beta)}+\frac{g'(\beta)}{g(\gamma)-g(\beta)}=1$$

because

Statement 2 : $\big(f(\alpha)-f(x)\big)\big(g(\gamma)-g(x)\big)e^{2x}$ is continuous and differentiable function in R.

17. Statement 1 : Let functions $f(x)$ and $g(x)$ be continuous in $[a\,,b]$ and differentiable in $(a\,,b)$, then there exists at least one value $x=c$ in $(a\,,b)$ such that

$$\begin{vmatrix} f(a) & f(b)\\ g(a) & g(b)\end{vmatrix}=(b-a)\begin{vmatrix} f(a) & f'(c)\\ g(a) & g'(c)\end{vmatrix}$$

because

Statement 2 : Lagrange's mean value theorem is applicable for function $h(x)=f(a)\,g(x)-g(a)\,f(x)$ in $[a\,,b]$.

18. Statement 1 : Let $f(x)$ be twice differentiable function such that $f(1)=1$, $f(2)=4$ and $f(3)=9$, then $f''(x)=2$ for all $x\in(1\,,\ 3)$

because

Statement 2 : Function $h(x)=f(x)-x^2$ is continous and differentiable for all $x\in[1\,,3]$.

19. **Statement 1 :** Let $f:[0\,,4]\to R$ be differentiable function , then there exists some values of 'a' and 'b' in $(0\,,4)$ for which $(f(4))^2-(f(0))^2=8f'(a)\,f(b)$

because

Statement 2 : Rolle's theorem is applicable for $f(x)$ in $[0\,,4]$.

20. **Statement 1 :** Let $f(x)$ be twice differentiable function and $f''(x)<0\ \ \forall\ \ x\in[a\,,b]$, then there exists some $x_1\,,x_2$ in $(a\,,b)$ for which $f\left(\dfrac{x_1+x_2}{2}\right)<\dfrac{f(x_1)+f(x_2)}{2}$

because

Statement 2 : Lagrange's mean value theorem is applicable for $f(x)$ in $[a\,,b]$.

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 21-23)

Let $f(x)$ be thrice differentiable function such that $f(p)=f(t)=0\,,f(q)=f(s)=4$ and $f(r)=-1$, where $t>s>r>q>p$, then answer the following questions.

21. If $g(x)=f(x)\,.\,f''(x)+(f'(x))^2$, then minimum number of roots of $y=g(x)$ in the interval $x\in[p\,,t]$ are :

(a) 8 (b) 4

(c) 6 (d) 10

22. If $h(x)=f(x).\,f'''(x)+f'(x).\,f''(x)$, then minimum number of roots of $y=h(x)$ in the interval $x\in[q\,,t]$ is/are :

(a) 2 (b) 1

(c) 3 (d) 4

23. If $\phi(x)=\left(f''(x)\right)^2+f'(x).f'''(x)$, then minimum number of roots of $y=\phi(x)$ in the interval $x\in[p\,,s]$ is/are :

(a) 1 (b) 2

(c) 4 (d) 3

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ANSWERS

Exercise No. (1)

1. (c)
2. (c)
3. (b)
4. (d)
5. (d)
6. (d)
7. (c)
8. (c)
9. (d)
10. (c)
11. (a , d)
12. (a , b , d)
13. (a , b , d)
14. (b , c)
15. (a , b , c , d)
16. (b)
17. (a)
18. (d)
19. (c)
20. (d)
21. (c)
22. (c)
23. (b)

Chapter
16

Monotonocity

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-10)

1. Let $f(x)$ be non-zero function and $\int_0^x f(t)\,dt = f^2(x)-1$ $\forall\ x \in R$, then $f(x)$ is :

(a) constant function. (b) non-monotonous.
(c) strictly increasing. (d) non-decreasing.

2. If $\phi(x) = 3f\left(\frac{x^2}{3}\right) + f(3-x^2)$ $\forall\ x \in (-3, 4)$, where $f''(x) > 0$ $\forall\ x \in (-3, 4)$, then $\phi(x)$ is :

(a) increasing in $\left(-\frac{3}{2}, 4\right)$ (b) decreasing in $(-3, 3)$
(c) increasing in $\left(-\frac{3}{2}, 0\right)$ (d) decreasing in $(0, 3)$

3. Let $f(x) = \int_{x^2}^{x^2+1} e^{-t^2}\,dt$, then $f(x)$ increases for :

(a) $x \in (-2, \infty)$ (b) $x \in R$
(c) $x \in R^+$ (d) $x \in R^-$

4. Let $f(x)$ be twice differentiable function and $f''(x) < 0$ $\forall\ x \in R$, then $g(x) = f(\sin^2 x) + f(\cos^2 x)$, where $|x| \le \pi/2$, increases in :

(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{2}, 0\right]$
(c) $\left[0, \frac{\pi}{4}\right]$ (d) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

5. Let function $f(x)$ is defined for all real x and $f(0) = 1$, $f'(0) = -1$, $f(x) > 0$ $\forall\ x \in R$, then

(a) $f''(x) > 0$ $\forall\ x \in R$
(b) $f''(x) < -2$ $\forall\ x \in R$
(c) $-1 < f''(x) < 0$ $\forall\ x \in R$
(d) $-2 \le f''(x) \le -1$ $\forall\ x \in R$

6. Let the function $f : R \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be defined as $f(x) = \frac{\pi}{2} - 2\tan^{-1}(e^x)$, then $f(x)$ is :

(a) odd function and strictly increasing in $(0, \infty)$.
(b) odd function and strictly decreasing in $(-\infty, \infty)$.
(c) even function and strictly decreasing in $(-\infty, \infty)$.
(d) neither even nor odd but strictly increasing in $(-\infty, \infty)$.

7. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$ and $f(x) = (\sin\theta + \cos\theta)^x$, then $f(x)$ is :

(a) increasing for all $x \in R$.
(b) decreasing for all $x \in R$.
(c) strictly decreasing for all $x \in R$.
(d) non-increasing for all $x \in R$.

8. Let $f(x) = \frac{x^2}{2-2\cos^2 x}$ and $g(x) = \frac{x^2}{6x - 6\sin x}$, where $x \in (0, 1)$, then :

(a) both $f(x)$ and $g(x)$ are increasing.
(b) $f(x)$ is increasing and $g(x)$ is decreasing.
(c) $f(x)$ is decreasing and $g(x)$ is increasing.
(d) both $f(x)$ and $g(x)$ are decreasing.

9. If $f(x) = (k+2)x^3 - 3kx^2 + 9kx - 1$ is decreasing function for all $x \in R$, then exhaustive set of values of $'k'$ is given by

(a) $[-3, -2]$ (b) $(-\infty, -3]$
(c) $(-\infty, -3)$ (d) $[0, \infty)$

10. If $f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$ is increasing for all $x \in R$, then $'a'$ belongs to :

(a) R (b) $[0, \infty)$
(c) R^- (d) $[1, \infty)$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 11-15)

11. Let $f(x)$ and $g(x)$ be differentiable functions for all real values of x. If $f'(x) \le g'(x)$ and $f'(x) \ge g'(x)$ holds for all , $x \in (-\infty, 2)$ and $x \in (2, \infty)$ respectively , then which of the following statements are always true ?

(a) $f(x) \ge g(x)$ holds $\forall \ x \in R$ if $f(2) \ge g(2)$.

(b) $f(x) \le g(x)$ holds $\forall \ x \in R$ if $f(2) \le g(2)$.

(c) $f(x) \ge g(x)$ holds for some real x if $f(2) \le g(2)$.

(d) $f(x) < g(x)$ holds for some real x if $f(2) \ge g(2)$.

12. For function $f(x) = x\cos\left(\dfrac{1}{x}\right)$, $x \ge 1$,

(a) for at least one x in interval $[1, \infty)$, $f(x+2) - f(x) < 2$

(b) $\lim\limits_{x\to\infty} f'(x) = 1$

(c) for all x in the interval $[1, \infty)$, $f(x+2) - f(x) > 2$

(d) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

13. Let $'S'$ be the set of real values of x for which the inequality $f(1 - 5x) < 1 - f(x) - f^3(x)$ holds true. If $f(x) = 1 - x^3 - x$ for all real x , then set $'S'$ contains :

(a) $\left(-\dfrac{3}{2}, -\dfrac{1}{2}\right)$ (b) (e, ∞)

(c) $(\sqrt{2}, 2)$ (d) $(-\sqrt{3}, -\sqrt{2})$

14. Let $f(x) = \dfrac{x^3}{3} - 2x^2 - x\cot^{-1}x - ln\sqrt{1+x^2} \quad \forall \ x \in R.$

If $'S'$ denotes the exhaustive set of values of x for which $f(x)$ is strictly increasing , then set $'S'$ contains:

(a) $[-2, -1]$ (b) $[0, 2]$

(c) $[5, 10]$ (d) $[2, 3]$

15. Let $f(x)$ be monotonically increasing function for all $x \in R$ and $f''(x)$ is non-negative , then which of the following inequations hold true :

(a) $\dfrac{f(x_1) + f(x_2)}{2} > f\left(\dfrac{x_1 + x_2}{2}\right)$

(b) $\dfrac{f^{-1}(x_1) + f^{-1}(x_2)}{2} > f^{-1}\left(\dfrac{x_1 + x_2}{2}\right)$

(c) $\dfrac{f(x_1) + f(x_2)}{2} < f\left(\dfrac{x_1 + x_2}{2}\right)$

(d) $\dfrac{f^{-1}(x_1) + f^{-1}(x_2)}{2} < f^{-1}\left(\dfrac{x_1 + x_2}{2}\right)$

Assertion Reasoning questions : (Questions No. 16-20)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

16. Statement 1 : If $f : R \to R$ be defined as $f(x) = 2x + \sin x$, then function is injective in nature

because

Statement 2 : For a differentiable function in domain $'D'$, if $f'(x) > 0$, then function is injective in nature.

17. Consider the function $f(x) = \sqrt{|x|}$ for all $x \in R$.

Statement 1 : If $\alpha < \beta < 0$, then

$$\frac{f(\alpha) + f(\beta)}{2} < f\left(\frac{\alpha + \beta}{2}\right)$$

because

Statement 2 : for all $x \in R^-$, $f'(x)$ and $f''(x)$ are negative.

18. Consider the function

$f(x) = 2\sin^3 x - 3\sin^2 x + 12\sin x + 5$ for all $x \in R$.

Statement 1 : $f(x)$ is increasing in nature for all $x \in \left(0, \dfrac{\pi}{2}\right)$.

because

Statement 2 : $y = \sin x$ is increasing in nature for all $x \in \left(0, \dfrac{\pi}{2}\right)$

19. Let $f : R \to R$ be strictly increasing function such that $f''(x) > 0$ and the inverse of $f(x)$ exists , then

Statement 1 : $\dfrac{d^2(f^{-1}(x))}{dx^2} < 0 \;\; \forall \;\; x \in R$

because

Statement 2 : Inverse function of an increasing concave up graph is convex up graph.

20. Let $f(x)$ be twice differentiable function $\forall \;\; x \in (a\,,\,b)$.

Statement 1 : $f'(x)$ vanishes at most once in $(a\,,\,b)$ if $f''(x) < 0 \;\; \forall \;\; x \in (a\,,\,b)$

because

Statement 2 : $f'(x)$ vanishes at least once in $(a\,,\,b)$ if $f''(x) > 0 \;\; \forall \;\; x \in (a\,,\,b)$.

Matrix Matching Questions :
(Questions No. 21-22)

21. Match the following functions in column (I) with their monotonic behaviour in column (II).

Column (I)	**Column (II)**
(a) $f(x) = \int\limits_0^{x^2} e^t(t^2 - 5t + 4)\,dt.$	(p) increasing in $(2\,,\,\infty)$
(b) $f(x) = e^{-x} + x$	(q) decreasing in $(-1\,,\,0)$
(c) $f(x) = \lvert x^2 - 2x \rvert$	(r) decreasing in $(-\infty\,,\,-2)$
(d) $f(x) = xe^{x(1-x)}$	(s) increasing in $(0\,,\,1)$

22. Let $f(x)$ be differentiable function such that $f'(x) \le 2\alpha f(x) \;\; \forall \;\; x \in R$ where $\alpha \in R^+$ and $f(1) = 0$. If $f(x)$ is non-negative for all $x \ge 1$ and $f(x)$ is non-positive for all $x \le 1$, then match the following columns for the functioning values and their nature.

Column (I)	**Column (II)**
(a) $f(ln\ 2)$ is	(p) positive.
(b) $f(\sqrt{\pi})$ is	(q) non-negative.
(c) $f(\sqrt{e^2 + e})$ is	(r) negative.
(d) $f(\sin 4)$ is	(s) non-positive.
	(t) zero.

ANSWERS Exercise No. (1)

1. (c) **2.** (c) **3.** (d) **4.** (c) **5.** (a)

6. (b) **7.** (c) **8.** (b) **9.** (b) **10.** (b)

11. (a , c) **12.** (b , c , d) **13.** (a , b , d) **14.** (a ,c) **15.** (a , d)

16. (c) **17.** (a) **18.** (b) **19.** (a) **20.** (c)

21. (a) $\to$ p , q , r , s
(b) $\to$ p , q , r , s
(c) $\to$ p , q , r , s
(d) $\to$ r , s

22. (a) $\to$ q , s , t
(b) $\to$ q , s , t
(c) $\to$ q , s , t
(d) $\to$ q , s , t

Chapter
17

Maxima and Minima

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-15)

1. Let $f : R \to R$ be real valued function defined by $f(x) = \left| x^2 - 4|x| + 3 \right|$, then which one of the following option is incorrect :

(a) $f'(2) = f'(-2) = 0.$

(b) local maxima exists at $x = 0.$

(c) $f'(3)$ and $f'(1)$ don't exist.

(d) $x = 0$ is not a critical point.

2. Let $f(x) = \begin{cases} |x-2|-1 & ; \quad x \neq 2 \\ 1 & ; \quad x = 2 \end{cases}$, then

(a) $|f(x)|$ is discontinous at $x = 2.$

(b) $f(|x|)$ is differentiable at $x = 0.$

(c) local maxima exists for $f(x)$ at $x = 2.$

(d) local minima exists for $\left| f(|x|) \right|$ at $x = 0.$

3. Minimum value of function $f(x) = \max\{x, x+1, 2-x\}$, is

(a) 1/2 (b) 3/2

(c) 0 (d) 1

4. Let $f(x) = \min\{1, \cos x, 1 - \sin x\} \ \forall \ x \in [-\pi, \pi]$, then $f(x)$ is :

(a) differentiable at $x = \dfrac{\pi}{2}$

(b) non-differentiable at $x = 0$

(c) having local maxima at $x = \dfrac{\pi}{2}$

(d) having local minima at $x = 0$

5. If $\alpha, \beta \in R$, then minimum value of

$(\alpha - \beta)^2 + \left(\sqrt{1-\alpha^2} - \sqrt{4-\beta^2} \right)^2$ is equal to :

(a) 14 (b) 6

(c) 1 (d) 4

6. A line segment of fixed length 'K' slides along the co-ordinate axes and meets the axes at $A(a, 0)$ and $B(0, b)$, then minimum value of $\left\{ \left(a + \dfrac{1}{a} \right)^2 + \left(b + \dfrac{1}{b} \right)^2 \right\}$ is given by :

(a) 8 (b) $K^2 + \dfrac{4}{K^2} - 4$

(c) $K^2 + \dfrac{4}{K^2} + 6$ (d) $K^2 + \dfrac{4}{K^2} + 4$

7. If $f(x) = |1 - x|$ and $g(x) = |x^2 - 2|$, then number of critical location(s) for composite function $f(g(x))$ is/are :

(a) 0 (b) 6

(c) 7 (d) 5

8. Let $f(x) = \begin{cases} (x+2)^3 & ; -3 < x \le -1 \\ x^{2/3} & ; -1 < x \le 2 \end{cases}$, then the local maxima exists at :

(a) $x = 0$ (b) $x = 1$

(c) $x = -1$ (d) $x = \dfrac{3}{2}$

9. Let 'P' be any point on the curve $x^2 + 3y^2 + 3xy = 1$ and 'O' being the origin, then minimum value of OP is :

(a) $\sqrt{\dfrac{2}{2+\sqrt{13}}}$ (b) $\sqrt{\dfrac{2}{2+\sqrt{3}}}$

(c) $\sqrt{\dfrac{2}{4+\sqrt{13}}}$ (d) $\sqrt{\dfrac{2}{\sqrt{3}}}$

10. If $f(x) = \begin{cases} 2 - |x^2 + 5x + 6| & ; \ x \neq -2 \\ a^2 + 1 & ; \ x = -2 \end{cases}$, then range of values of 'a' for which $f(x)$ has local maxima at $x = -2$ is given by :

(a) $a \in (-1, 1)$ (b) $a \in R/(-1, 1)$

(c) $a \in R/[-1, 1]$ (d) $a \in [-1, 1]$

11. Let function $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5\,dt$, then $f(x)$ has point of inflection at location x equals to :

(a) 1 (b) 2

(c) 0 (d) none of these

12. Function $f(x) = x + x^2 \tan x$ has :

(a) one local maxima point in $\left(0\,,\,\frac{\pi}{2}\right)$

(b) one local minima point in $\left(0\,,\,\frac{\pi}{2}\right)$

(c) no point of extremum in $\left(0\,,\,\frac{\pi}{2}\right)$

(d) one point of inflection in $\left(0\,,\,\frac{\pi}{2}\right)$

13. Let $x \in N$ and $f(x) = \left(\frac{x^2}{200 + x^3}\right)$, then maximum value of $f(x)$ is equal to :

(a) $\frac{64}{712}$ (b) $\frac{49}{543}$

(c) $\frac{57}{628}$ (d) $\frac{58}{625}$

14. Let $f(x) = (a-1)\,x + (a^2 - 3a + 2)\cos\frac{x}{2}$, then set of all values of $'a'$ for which $f(x)$ doesn't possess any critical point is :

(a) $[1\,,\,\infty)$

(b) $(-2\,,\,4)$

(c) $(1\,,\,3) \cup (3\,,\,5)$

(d) $(0\,,\,1) \cup (1\,,\,4)$

15. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x / x^2 + 20 \le 9x\,,\, x \in R\}$ is :

(a) 6 (b) 7

(c) 5 (d) 4

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. Let $f(x) = ax^3 + bx^2 + x + d$ has local extrema at $x = \alpha$ and $x = \beta$, where $\alpha\beta < 0$ and $f(\alpha)\,f(\beta) > 0$, then equation $f(x) = 0$ has only one root which is :

(a) positive if $a\,f(\alpha) > 0$

(b) negative if $a\,f(\alpha) > 0$

(c) positive if $a\,f(\beta) < 0$

(d) negative if $a\,f(\beta) < 0$

17. Let $f(x) = \frac{\tan x + \cot x}{2} - \left|\frac{\tan x - \cot x}{2}\right|$, then

(a) $f(x)$ is discontinuous at $x = \frac{n\pi}{2}\,;\, n \in I$

(b) $f(x)$ is non-differentiable at $x = \frac{n\pi}{4}\,;\, n \in I$

(c) $f(x)$ has local maxima at $x = (2n+1)\frac{n\pi}{4}\,;\, n \in I$

(d) $f(x)$ has local minima at $x = (2n+1)\frac{\pi}{4}\,;\, n \in I$

18. $f(x)$ is cubic polynomial which has local maxima at $x = -1$. If $f(2) = 18$, $f(1) = -1$ and $f'(x)$ has local minima at $x = 0$, then

(a) The distance between $(-1\,,\,2)$ and $(a\,,\,f(a))$, where $x = a$ is the point of local minima is $2\sqrt{2}$

(b) $f(x)$ is increasing for all $[1\,,\,2\sqrt{5}]$

(c) $f(x)$ has local minima at $x = 1$

(d) the value of $f(0)$ is 5

19. Let $f(x) = \begin{cases} 2 + |x^2 - 6x + 8|\,; & x \ne 4 \\ (a^2 - 2) & ; x = 4 \end{cases}$, then

(a) $f'(3) = 0$.

(b) at $x = 2$ local minima exists.

(c) at $x = 4$, local maxima exists if $a \in R - (-2\,,\,2)$.

(d) at $x = 4$, local minima exists if $a \in [-2\,,\,2]$.

20. Let $f(x)=\begin{cases}\left[\tan^2 x\right] & ; -\dfrac{\pi}{4}\le x\le\dfrac{\pi}{3}\\ 2+\left(x-\dfrac{\pi}{3}\right)^2 & ; x>\dfrac{\pi}{3}\end{cases}$, where [.] represents the step-function. For function $f\,(x)$ in $\left[-\dfrac{\pi}{4}\,,\infty\right)$, which of the following statement(s) is/are true :

(a) Total number of points of discontinuity are four.

(b) $x=\dfrac{\pi}{3}$ is the location of local maxima.

(c) Total number of points of discontinuity are three.

(d) $f'\left(\dfrac{\pi}{4}^{+}\right)=f'\left(\dfrac{\pi}{4}^{-}\right)$

Assertion Reasoning questions :
(Questions No. 21-25)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. Consider the function $f:R\to R$ defined as $f\,(x)=x^3-3x+3.$

Statement 1 : For function $f\,(x)$, x = 0 is not the location of point of inflection

because

Statement 2 : $x = 0$ is not the critical point for function $f(x)$.

22. Let $f\,(x)=\begin{cases}1-\sin 2x & ; \quad x\ne\pi/2\\ \quad 1 & ; \quad x=\pi/2\end{cases}$, then

Statement 1 : $y = f\,(x)$ is having local maximum value at $x=\dfrac{\pi}{2}$

because

Statement 2 : $y=|\,f\,(x)\,|$ is having local minimum value at $x=\dfrac{\pi}{2}$.

23. Let $f\,(x)=\dfrac{x^3}{3}-x\tan^{-1}x+\dfrac{1}{2}\,ln(1+x^2)$ for all $x\in R$

Statement 1 : $y = f\,(x)$ is having exactly one point of local maxima and one point of local minima

because

Statement 2 : $y = f\,(x)$ is having exactly one point of inflection which lies in $\left(0\,,\dfrac{1}{2}\right)$.

24. Consider $f\,(x)=\sin|\,x\,|\;\;\forall\;\; x\in[-2\pi\;,\,2\pi]$

Statement 1 : For $y=f\,(x)$, local maximum and local minimum values can be equal

because

Statement 2 : There exists exactly two points of inflection for $y=f\,(x)$.

25. **Statement 1 :** If $x\,,\,y\in R^{+}$ and satisfy the condition $x^2+y^2+99=4(3x+4y)$, then minimum value of $\log_3(x^2+y^2)$ is 4

because

Statement : maximum value of (x^2+y^2) is 121.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f(x)=\begin{cases} ax-b & ; \quad x<1 \\ x^2+bx+5 & ; \quad x\geq 1 \end{cases}$ be continuous and differentiable function $\forall \ x\in R.$ If tangent to the curve of $y=f(x)$ at $x=1$ cuts the coordinate axes at P and Q, then answer the following questions.

1. If $'O'$ represents the origin , then maximum area (in square units) of the rectangle which can be inscribed in the incircle of triangle OPQ is equal to :

(a) $\dfrac{32}{9+4\sqrt{2}}$ (b) $\dfrac{12}{5+2\sqrt{5}}$

(c) $\dfrac{9}{12+\sqrt{5}}$ (d) $\dfrac{16}{7+3\sqrt{5}}$

2. Total number of solutions of the equation $f(x)-\left|\sin\dfrac{\pi}{4}x\right|=0$ is/are :

(a) Infinitely many (b) 0

(c) 1 (d) finitely many

3. If $g(x)=|2-f(x)| \ \ \forall \ x\in R$, then total number of points of extremum for function $y=g(x)$ is/are :

(a) 2 (b) 1

(c) 4 (d) 3

Comprehension passage (2)
(Questions No. 4-6)

Let function $f:R\rightarrow R$ be defined as $f(x)=\left(\lambda-\dfrac{1}{\lambda}-x\right)\left(4-3x^2\right)$, where $'\lambda'$ is non-zero real parameter , then answer the following questions.

4. If $x=\alpha$ and $x=\beta$ are the locations for local maxima and local minima respectively , then minimum value of $\left(\alpha^2+\beta^2\right)$ is equal to :

(a) 4/9 (b) 8/9

(c) 2/27 (d) $16/27$

5. If $\lambda\in R^+$ and $f(\alpha)$, $f(\beta)$ are the values of local maxima and local minima respectively , then $f(\alpha)-f(\beta)$ is equal to :

(a) $\dfrac{2}{9}\left(\lambda-\dfrac{1}{\lambda}\right)^3$ (b) $\dfrac{4}{9}\left(\lambda+\dfrac{1}{\lambda}\right)^3$

(c) $\dfrac{2}{9}\left(\lambda+\dfrac{1}{\lambda}\right)^3$ (d) $\dfrac{4}{9}\left(\lambda-\dfrac{1}{\lambda}\right)^3$

6. If $\lambda=-1$ and $g(x)=\begin{cases} |f(x)| & ; \quad x\geq 0 \\ f(x)-1 & ; \quad x<0 \end{cases}$, then which one of the following statement is true :

(a) $x=\dfrac{2}{\sqrt{3}}$ is the location of local maxima.

(b) $x=0$ is the location of point of inflection.

(c) $x=0$ is the location of local minima.

(d) $x=-\dfrac{2}{3}$ is the location of local minima.

Comprehension passage (3)
(Questions No. 7-9)

Let the fixed points A , B , C and D lie on a straight line such that $AB=BC=CD=2$ units. The points A and C are joined by a semi-circle of radius 2 units , where $'P'$ is variable point on the semicircle such that $\angle PBD=\alpha$. If $'R'$ is the region bounded by the line segments AD , PD and the arc $\overset{\frown}{AP}$, then answer the following questions.

7. Maximum area (in square units) of the region $'R'$ is equal to :

(a) $\dfrac{3\pi}{2}+2\sqrt{2}$ (b) $2+\dfrac{5\pi}{3}$

(c) $\dfrac{4\pi}{3}+2\sqrt{3}$ (d) $\dfrac{4\pi}{3}+4\sqrt{3}$

8. Maximum perimeter of the region $'R'$ is equal to :

(a) $\left(4+\dfrac{2\pi}{3}+2\sqrt{2}\right)$ units.

(b) $\left(3+\dfrac{2\pi}{3}+4\sqrt{3}\right)$ units.

(c) $\left(8+\dfrac{2\pi}{3}+4\sqrt{2}\right)$ units.

(d) $\left(6+\dfrac{4\pi}{3}+2\sqrt{3}\right)$ units.

9. If the area of circle inscribed in the triangle PAB is maximum, then value of $\sin^{-1}\left(\frac{1}{2}\cos\frac{\alpha}{2}\right)$ is equal to :

(a) $\sin^{-1}\left(\frac{1}{3}\right)$ (b) $\sin^{-1}\left(\frac{1}{4}\right)$

(c) $\sin^{-1}\left(\frac{1}{10}\right)$ (d) $\sin^{-1}\left(\frac{1}{8}\right)$

Questions with Integral Answer :
(Questions No. 10-15)

10. In a triangle ABC, $AB = AC$ and the length of median from B to the side AC is 1 unit. If the area of triangle ABC is minimum, then value of $10(\cos A)$ is equal to ....

11. If the location of local minima of $f(x) = \lambda^2 x - x^3 + 1$ satisfies the inequality $\frac{x^2+2x+3}{x^2+5x+6} < 0$, then minimum positive integral value of $'\lambda'$ is equal to ....

12. Let area of triangle formed by x-axis, tangent and normal at point (t, t^2+1) on the curve $y = x^2+1$ be $'A'$ square units. If $t \in [1, 3]$, then minimum value of $'A'$ is equal to ....

13. If $a \in R^+$ and $f(x) = x^3 + 3(a-7)x^2 + 3(a^2-9)x - 2$ is having point of local maxima at $x = x_0$, where $x_0 \in R^+$, then the least possible integral value of $'a'$ is equal to ....

14. Let the perimeter of ΔABC be 12 units, where $AB = AC$. If the volume of solid generated by revolving the triangle ABC about its side BC is maximum, then length $(2\,AB)$ is equal to ....

15. Let a variable line through (1, 2) is having negative slope and meet the axes at P and Q. If $'O'$ is origin and area of triangle OPQ is $'A'$ square units, then minimum value of A is equal to ....

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Matrix Matching Questions :
(Questions No. 16-18)

16. Match the following Columns (I) and (II)

Column (I)	**Column (II)**
(a) If three sides of trapezium are of equal length 3/5 units and its area is maximum, then perimeter of trapezium is :	(p) 1
(b) If $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $f(x) = p\sin^2 x + \sin^3 x$ is having exactly one location of local minima, then value(s) of $'p'$ can be :	(q) 0
	(r) 2
(c) Number of points of inflection in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ for the function $f(x) = \cos^2 x$ is/are	(s) 3
(d) If $f(x) = \lvert 1-x\rvert + \lvert x-3\rvert \;\; \forall \;\; x \in [0, 5]$, then global minima exists at x equal to :	(t) –1/2

17. Let $f(x) = x^2 - bx + c$, where b is odd positive integer and $f(x) = 0$ is having two distinct roots which are prime numbers. If $b + c = 23$, then match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) Global minimum value of $f(x)$ in $[3, 8]$ is equal to :	(p) 0
(b) Global maximum value of $y = \lvert f(x) \rvert$ in $[0, 8]$ is equal to :	(q) 14
(c) Local maximum value of $y = f(\lvert x \rvert)$ is equal to :	(r) 9/2
(d) If $y = \lvert f(\lvert x \rvert) \rvert$, and $x = \alpha$ is the location for critical	(s) –25/4
points, then values of 'α' can be :	(t) –7

18. Match the functions of column (I) with their corresponding behaviour in column (II).

Column (I)	**Column (II)**
(a) If $f(x) = x^4 - 4x^3 + 2$, $x \in (-1, 4)$, then	(p) $f(x)$ has exactly one point of local maxima.
(b) If $f(x) = x^{2/3}(x-5)$, $x \in (-2, 4)$, then	(q) $f(x)$ has exactly one point of local minima.
(c) If $f(x) = \left(\dfrac{x}{1 + x\tan x}\right)^{-1}$, $x \in \left(0, \dfrac{\pi}{2}\right)$, then	(r) $f(x)$ has exactly one point of inflection.
(d) $f(x) = \dfrac{x^3}{3} - x\cot^{-1}x - \dfrac{1}{2}ln(1+x^2)$, $x \in \left(-\sqrt{\pi}, \sqrt{\pi}\right)$,	(s) $f(x)$ has no critical point.
then	(t) $f(x)$ has exactly two points of inflection.

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ANSWERS Exercise No. (1)

1. (d)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (c)
6. (d)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (b)
11. (c)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (d)
16. (b , c)	**17.** (a , b , c)	**18.** (b , c)	**19.** (a , b , d)	**20.** (a , b , d)
21. (d)	**22.** (c)	**23.** (b)	**24.** (b)	**25.** (b)

ANSWERS Exercise No. (2)

1. (d)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (b)
6. (b)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (8)
11. (4)	**12.** (5)	**13.** (4)	**14.** (9)	**15.** (4)
16. (a) → s (b) → p , t (c) → r (d) → p , r , s	**17.** (a) → s (b) → q (c) → q (d) → p , r , t	**18.** (a) → q , t (b) → p , q , r (c) → q (d) → p , q , r		

Chapter 18

Indefinite Integral

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-15)

1. Value of $\int \frac{(x(x^2-1)-2)}{x^2\sqrt{1+x+x^3}}dx$ is :

(a) $\frac{2}{x^2}\sqrt{x^3+x+1}+c$. (b) $-\frac{2}{x}\sqrt{x+x^3+1}+c$.

(c) $\frac{1}{x}\sqrt{x^3+x+1}+c$. (d) $\frac{2}{x}\sqrt{1+x+x^3}+c$.

2. Let $f'(x)=g(x)$ and $g'(x)=-f(x)$ $\forall$ $x\in R$ and $f(2)=f'(2)=4$, then $f^2(4)+g^2(4)$ is equal to :

(a) 32 (b) 8 (c) 16 (d) 64

3. If $\int f(x)dx=F(x)$, then $\int x^3 f(x^2)dx$ equals to :

(a) $\frac{1}{2}\left[x^2(F(x))^2-\int (F(x))^2dx\right]$

(b) $\frac{1}{2}\left[x^2F(x^2)-\int F(x^2)d(x^2)\right]$

(c) $\frac{1}{2}\left[x^2F(x)-\frac{1}{2}\int (F(x))^2dx\right]$

(d) $\frac{1}{2}\left[x^2F(x^2)+\int F(x^2)d(x^2)\right]$

4. Let $f(x)$ be strictly increasing function satisfying $f(0)=2$, $f'(0)=3$ and $f''(x)=f(x)$, then $f(4)$ is equal to :

(a) $\frac{5e^8+1}{2e^4}$ (b) $\frac{5e^8-1}{2e^4}$

(c) $\frac{2e^4}{5e^8-1}$ (d) $\frac{2e^4}{5e^8+1}$

5. If $f'(x)=\frac{(x^2+\sin^2 x)}{1+x^2}\sec^2 x$; $f(0)=0$, then $f(1)$ is equal to :

(a) $1-\frac{\pi}{4}$ (b) $\frac{\pi}{4}-1$

(c) $\tan 1-\frac{\pi}{4}$ (d) none of these

6. $\int \frac{e^x(1+e^{2x})dx}{e^{4x}-e^{2x}+1}$ is equal to :

(a) $\tan^{-1}(e^x+e^{-x})+c$

(b) $\tan^{-1}(e^x-e^{-x})+c$

(c) $\tan^{-1}(e^{-2x}+e^{2x})+c$

(d) $\tan^{-1}(e^{-x}-e^x)+c$

7. $\int \frac{(\sin x-\cos x)dx}{(\sin x+\cos x)\sqrt{\sin x\cos x+\sin^2 x\cos^2 x}}$ is equal to :

(a) $\cot^{-1}\left\{\sqrt{\sin^2 2x-\sin x}\right\}+c$

(b) $\cot^{-1}\left\{\sqrt{\sin^2 2x+2\sin 2x}\right\}+c$

(c) $\tan^{-1}\left\{\sqrt{\sin^2 2x+2\sin x}\right\}+c$

(d) $\tan^{-1}\left\{\sqrt{\sin^2 2x-\sin x}\right\}+c$

8. $\int \frac{dx}{x^n(1+x^n)^{1/n}}$ is equal to :

(a) $(1-n)\left(\frac{x^n}{x^n+1}\right)^{\frac{n-1}{n}}+c$

(b) $\frac{1}{(n-1)}\left(\frac{x^n}{1+x^n}\right)^{\frac{n-1}{n}}+c$

(c) $\frac{1}{(1-n)}\left(\frac{x^n}{x^n+1}\right)^{\frac{1-n}{n}}+c$

(d) $\frac{1}{(1+n)}\left(\frac{x^n+1}{x^n}\right)^{\frac{n-1}{n}}+c$

9. $\displaystyle\int \frac{(x+\sqrt{a^2+x^2})^n}{\sqrt{a^2+x^2}}\,dx$ is equal to :

(a) $\dfrac{(x+\sqrt{x^2+a^2})^n}{n}+C$

(b) $\dfrac{(x+\sqrt{x^2+a^2})^{n+1}}{(n+1)}+C$

(c) $\dfrac{(x+\sqrt{x^2+a^2})^{n-1}}{(n-1)}+C$

(d) none of these

10. $\displaystyle\int \frac{x^3-x}{x^6+1}\,dx$ is equal to :

(a) $\dfrac{1}{8}\,ln\left|\dfrac{x^4-x^2+1}{(1+x^2)^2}\right|+c$ (b) $\dfrac{1}{6}\,ln\left|\dfrac{x^4+x^2-1}{(1-x^2)^2}\right|+c$

(c) $\dfrac{1}{4}\,ln\left|\dfrac{x^4-x^2+1}{(1+x^2)^2}\right|+c$ (d) none of these

11. $\displaystyle\int \frac{(x+\sqrt[3]{x^2}+\sqrt[6]{x})}{x(1+\sqrt[3]{x})}\,dx$ is equal to :

(a) $\dfrac{3}{2}(x)^{2/3}+\tan^{-1}(x^6)+c$

(b) $\dfrac{3}{2}(x)^{2/3}+6\tan^{-1}(x^{1/6})+c$

(c) $\dfrac{3}{2}(x)^{2/3}+\tan^{-1}(x^{1/6})+c$

(d) none of these

12. If $\displaystyle\int \frac{(x^2+1)dx}{x^3-6x^2+11x-6}=ln\,|(x-1)^A.(x-2)^B.(x-3)^C\,|+k$ then $4(A+B+C)$ is :

(a) 0 (b) 2 (c) 5 (d) 4

13. If $\displaystyle\int \frac{dx}{x^2-2\pi x+1}=Kf(x)+c$ then $f(x)$ is

(a) logrithm function (b) inverse tangent function

(c) cosine function (d) tangent function

14. $\displaystyle\int \frac{2\theta+\sin 2\theta}{1+\cos 2\theta}\,d\theta$ is equal to :

(a) $\dfrac{\theta\sin^2\theta}{\cos\theta}+c$ (b) $\theta\cos^2\theta+c$

(c) $\dfrac{\theta\tan\theta}{\sec^2\theta}+c$ (d) $\dfrac{\theta\sin\theta}{\cos\theta}+c$

15. $\displaystyle\int \frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\left(\dfrac{1+x^2}{x}\right)}$ is equal to :

(a) $ln\left|\tan\left(x+\dfrac{1}{x}\right)\right|+c$ (b) $ln\left|\tan^{-1}\left(x-\dfrac{1}{x}\right)\right|+c$

(c) $ln\left|\tan^{-1}\left(x+\dfrac{1}{x}\right)\right|+c$ (d) $ln\left|\tan^{-1}\left(x-\dfrac{2}{x}\right)\right|+c$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. Let $y^2=x^2-x+1$ and $I_n=\displaystyle\int\frac{x^n}{y}dx$, if $\alpha I_3+\beta I_2+\gamma I_1=yx^2$, then :

(a) $\alpha+2\beta+\gamma=0$ (b) $\alpha-\beta=4$

(c) $\gamma-2\beta=8$ (d) $\alpha-\gamma=1$

17. Let $f(x)=\displaystyle\int\frac{e^x}{x}dx$ and

$\displaystyle\int\frac{e^{x-1}.2x}{x^2-5x+4}dx=\alpha f(x-4)+\beta f(x-1)+\gamma$, then :

(a) $ln\ 3\alpha=3$ (b) $4+3\beta=ln\ 3\alpha$

(c) $3\beta+2=0$ (d) $ln\ 3\alpha=3+ln\ 8$

18. Let $f(x)=\displaystyle\int\frac{x^3dx}{\sqrt{1+x^2}}$, where $f(\sqrt{2})=0$, then which of the following statements are incorrect ?

(a) $f(-1)=\dfrac{2}{\sqrt{3}}$ (b) $f(\sqrt{5})=\sqrt{6}$

(c) $f(0)=\dfrac{1}{3}$ (d) $f(1)=-\dfrac{\sqrt{2}}{3}$

19. Let $\displaystyle\int\sin(ln\,x)dx=f(x).\sin\left(g(x)-\frac{\pi}{4}\right)+c$, where $'c'$ is constant , $f\ (x)$ and $g\ (x)$ are two distinct functions , then :

(a) $\tan^{-1}\left(\dfrac{1}{f(1)}\right)=\dfrac{\pi}{4}$ (b) $\sin^{-1}\left(g(1)\right)=\dfrac{\pi}{4}$

(c) $\tan^{-1}\left(f(1)\,.\,g(1)\right)=0$ (d) $\tan^{-1}\left(\dfrac{1}{f(1)}-1\right)=\dfrac{\pi}{8}$

20. Let $\int \frac{x^4+1}{1+x^6}dx = f(x)+\frac{1}{3}f(g(x))+c$, where $g(x)$ is polynomial function and $'c'$ is constant value , then which of the following statements are true :

(a) $\tan\left(\frac{1}{3}f(g(1))\right)=2-\sqrt{3}$

(b) number of solutions of $g(x)-x=0$ are two.

(c) number of solution of $f(x)-x=0$ is one.

(d) $\sin(2f(\sqrt{2}))=\frac{2\sqrt{2}}{3}$

Assertion Reasoning questions :
(Questions No. 21-25)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. Let $f(x)=\sin^6 x+\cos^6 x \ \forall \ x\in R$, and $g(x)=\int\frac{dx}{f(x)}$, where $g\left(\frac{\pi}{4}\right)=0$.

Statement 1 : $\tan\left(g\left(\frac{3\pi}{8}\right)\right)=2$

because

Statement 2 : all possible values of $f(x)$ lies in [1/4 , 1].

22. Statement 1 : If $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, then

$$\int\left(ln\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)+x\sec x\right)dx = x\,ln\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)+c$$

because

Statement 2 : $\int(xf'(x)+f(x))\,dx = xf(x)+c$, where $'c'$ is integration constant.

23. Let $I_n=\int\tan^n x\,dx$, where $n\in W$ and integration constant is zero , then

Statement 1 : Summation of

$I_0+I_1+2(I_2+....+I_8)+I_9+I_{10}$ is equal to $\sum_{r=1}^{10}\frac{(\tan x)^r}{r}$

because

Statement 2 : $I_n+I_{n+2}=\frac{(\tan x)^{n+1}}{n+1} \quad \forall \quad n\in W$

24. Let $f:R\to R$ be defined as $f(x)=ax^2+bx+c$, where a , b , $c\in R$ and $a\neq 0$.

Statement 1 : If $f(x)=0$ is having non-real roots , then $\int\frac{dx}{f(x)}=\lambda\tan^{-1}(g(x))+\mu$, where λ , μ are constants and $g(x)$ is linear function of x

because

Statement 2 : $\tan(\tan^{-1}(g(x)))=g(x) \ \forall \ x\in R.$

25. Statement 1 : If $\int\frac{(e^{3x}+e^x)dx}{e^{4x}-e^{2x}+1}=\tan^{-1}(f(x))+c$, where $'c'$ is integration constant , then $\tan^{-1}(f(-x))=-\tan^{-1}(f(x))$

because

Statement 2 : $y=f(x)$ and $y=\tan^{-1}x$ are both odd functions.

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 26-28)

Consider the indefinite integral $I=\int\frac{(x^3-x-1)}{\sqrt{x^2+2x+2}}dx.$

If $I=f(x)\sqrt{x^2+2x+2}+\alpha\int\frac{dx}{\sqrt{x^2+2x+2}}$, where $f(x)$ is quadratic function and $'\alpha'$ is a constant , then answer the following questions.

26. Total number of critical points for $y=|f(x)|$ is/are :

(a) 1 (b) 2 (c) 3 (d) 0

27. Value of $\tan(\sin^{-1}(\alpha))$ is equal to :

(a) $\frac{1}{\sqrt{3}}$ (b) 1

(c) $\sqrt{3}$ (d) $\sqrt{2}-1$

28. Value of $\lim_{n\to\infty}\left\{\sum_{r=0}^{n}(-1)^r\ {}^nC_r\,(f(3))^r\right\}$ is :

(a) 1 (b) 0

(c) e (d) infinite

Comprehension passage (2)
(Questions No. 29-31)

Let $I_n=\int_0^{\pi/2} x\,.\,\sin^n x\,dx=\frac{1}{n^2}+f(n)\,I_{n-2}$, where $n\in N$, then answer the following questions.

29. Value of $f(4)$ is equal to :

(a) $\frac{2}{3}$ (b) $\frac{5}{4}$

(c) $\frac{3}{4}$ (d) $\frac{5}{3}$

30. Value of $8I_8-7I_6$ is equal to :

(a) $\frac{1}{7}$ (b) $\frac{1}{8}$

(c) $\frac{1}{49}$ (d) $\frac{1}{64}$

31. Value of $10I_{10}-\sum_{n=0}^{4}I_{2n}$ is equal to :

(a) $\frac{147}{120}$ (b) $\frac{159}{120}$

(c) $\frac{137}{120}$ (d) $\frac{149}{120}$

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Exercise No. (1)

1. (d) **2.** (a) **3.** (b) **4.** (b) **5.** (c)

6. (b) **7.** (b) **8.** (c) **9.** (a) **10.** (d)

11. (b) **12.** (d) **13.** (a) **14.** (d) **15.** (c)

16. (a , d) **17.** (c , d) **18.** (a , c) **19.** (c , d) **20.** (a , c , d)

21. (b) **22.** (a) **23.** (d) **24.** (b) **25.** (a)

26. (c) **27.** (a) **28.** (b) **29.** (c) **30.** (b)

31. (c)

Chapter 19

Definite Integral

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-30)

1. If $I_1 = \int_0^{\pi/2} ln(\sin x)dx$ and $I_2 = \int_{-\pi/4}^{\pi/4} ln(\sin x + \cos x)\,dx$, then :

(a) $I_1 = I_2$

(b) $I_1 = 2I_2$

(c) $I_2 = 2I_1$

(d) $I_2 = 4I_1$

2. Let $f : (0, \infty) \to R$ and $F(x) = \int_0^x f(t)\,dt$, if $F(x^2) = (1 + x)x^2$, then $f(16)$ is equal to :

(a) 4 (b) 8

(c) 7 (d) 9

3. If $\int_0^x f(t)\,dt = x + \int_x^1 t\,f(t)dt$, then value of $f(1)$ is :

(a) $\frac{1}{2}$ (b) 0

(c) 1 (d) $-\frac{1}{2}$

4. Let $f(x)$ be periodic function with fundamental period 'T' and $\int_0^x f(t)dt = x^2 + \int_x^{x+T} t\,f(t)dt$, then $f(T-1)$ is equal to :

(a) 2 (b) $-\frac{1}{2}$

(c) –2 (d) 1

5. The number of solutions of $x + \int_0^x ln\,t\,dt = \frac{x^2}{3}$, where $x \in R^+$, is/are :

(a) 0 (b) 1

(c) 2 (d) 3

6. If $c \neq 0$, then value of the integral

$c\int_{1+c}^{a+c} (f(cx)+1)dx - \int_c^{ac} f(x+c^2)\,dx$ is equal to :

(a) 0 (b) $c(a-1)$

(c) ac (d) $a(c+1)$

7. Let $I = \int_0^1 \frac{\sin x}{1+x}dx$, then value of integral

$\int_{4\pi-2}^{4\pi} \frac{\sin(x/2)}{4\pi+2-x}dx$ is equal to :

(a) $2I$ (b) $-I$

(c) I (d) $I/2$

8. For $x > 0$, let $f(x) = \int_1^x \frac{ln\,t}{1+t}dt$, then

$y = \sqrt{2\left(f(x) + f\left(\frac{1}{x}\right)\right)}$ is differentiable for :

(a) $x \in R$ (b) $x \in R^+$

(c) $x \in R^+/\{1\}$ (d) $x \in R^+/\{e\}$

9. Let $I_1 = \int_{-4}^{-5} \exp((x+5)^2)\,dx$ & $I_2 = \int_{1/3}^{2/3} \exp((3x-2)^2)\,dx$,

then $I_1 + 3I_2$ is equal to :

(a) e (b) $3e$

(c) $2e$ (d) 0

10. If $f(x)$ is continuous function for all $x \in R$,

$I_1 = \int_{\sin^2 t}^{1+\cos^2 t} x\,f(x(2-x))\,dx$ and

$I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f(x(2-x))\,dx$, then $\frac{I_1}{I_2}$ is equal to :

(a) 0 (b) 1 (c) 2 (d) 3

11. If $f\left(\frac{1}{x}\right)+x^2 f(x)=0\ \forall\ x>0$ and $I=\int\limits_{1/x}^{x} f(z)d\,z$ for all $\frac{1}{2}\le x\le 2$, then I is equal to :

(a) $f(2)-f\left(\frac{1}{2}\right)$ (b) $f\left(\frac{1}{2}\right)-f(2)$

(c) $f(2)+f\left(\frac{1}{2}\right)$ (d) none of these

12. $\int\limits_{-\pi/2}^{\pi/2}\frac{e^{|\sin x|}.\cos x}{(1+e^{\tan x})}dx$ is equal to :

(a) $e+1$ (b) $1-e$

(c) $e-1$ (d) none of these

13. If $I_n=\int\limits_0^{\pi/4}\tan^n x\,dx\ \forall\ n\in N$, then

(a) $I_1=I_3+2I_5$

(b) $I_n+I_{n-2}=\frac{1}{n}$

(c) $I_n+I_{n-2}=\frac{1}{n+1}$

(d) none of these

14. If $f(x)$ is periodic function with fundamental period $'T'$ and $f(x)$ is also an odd function , then value of $\left\{\int\limits_{a+T}^{b+2T} f(x)dx-\int\limits_a^b f(x)dx\right\}$ is equal to :

(a) 1 (b) 2

(c) T (d) 0

15. If $I=\int\limits_0^\infty\frac{\sin x}{x}dx=\frac{\pi}{2}$, then $\int\limits_0^\infty\frac{\sin^3 x}{x}dx$ is equal to :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) 1 (d) 0

16. If $\beta+2\int\limits_0^1 x^2e^{-x^2}dx=\int\limits_0^1 e^{-x^2}dx$, then value of β is :

(a) e (b) 1

(c) 0 (d) $1/e$

17. Let $f(x)$ be continuous for all x and not every where zero , such that $\{f(x)\}^2=\int\limits_0^x\frac{f(t)\sin t}{2+\cos t}dt$, then $f(x)$ is equal to :

(a) $\frac{1}{2}ln\left(\frac{3}{2+\sin x}\right)$ (b) $\frac{1}{2}ln\left(\frac{3}{2+\cos x}\right)$

(c) $\frac{1}{2}ln\left(\frac{2}{3+2\cos x}\right)$ (d) $\frac{1}{2}ln(3-\cos x)$

18. The least value of $F(x)=\int\limits_x^2 -log_3 t\,dt\ \forall\ x\in\left[\frac{1}{10},4\right]$ is equal to :

(a) $log_3 e-2log_3 2$ (b) $1-log_3 2$

(c) $\frac{1+2\,ln\,2}{ln\,3}$ (d) $log_2 3+1$

19. If $I_n=\int\limits_0^\infty e^{-x}x^{n-1}dx$ and $\lambda\in R^+$ then $\int\limits_0^\infty e^{-\lambda x}x^{n-1}dx$ is equal to :

(a) λI_n (b) $\frac{I_n}{\lambda}$

(c) $\frac{I_n}{\lambda^n}$ (d) $\lambda^n I_n$

20. If $\{x\}$ represents the fractional part of x , and $I=\int\limits_{-3}^{3}x^8\{x^7\}dx$, then value of I is equal to :

(a) 0 (b) 1

(c) 3^7 (d) 3^{16}

21. If $\int\limits_0^{\pi/2}ln(\sin x)dx=\frac{\pi}{2}ln\left(\frac{1}{2}\right)$, then $\int\limits_0^{\pi/2}\left(\frac{x}{\sin x}\right)^2dx$ is equal to :

(a) $\frac{\pi}{2}ln\,2$

(b) $2\pi\,ln\,2$

(c) $\pi\,ln\,2$

(d) none of these

22. If $f(2-\alpha)=f(2+\alpha)\ \forall\ \alpha\in R$, then $\displaystyle\int_{2-a}^{2+a} f(x)\,dx$ is equal to :

(a) $2\displaystyle\int_{2}^{a+2} f(x)\,dx$ (b) $2\displaystyle\int_{0}^{a} f(x)\,dx$

(c) $2\displaystyle\int_{0}^{2a} f(x)\,dx$ (d) $4\displaystyle\int_{0}^{a} f(x/2)\,dx$

23. Let $x\in\left(0,\dfrac{\pi}{4}\right)$ and $f(x)=\tan x$, $g(x)=\cot x$, where

$$I_1=\int_0^{\pi/4}(f(x))^{f(x)}dx,\ I_2=\int_0^{\pi/4}e^{-x^2}(f(x))^{g(x)}dx,$$

$$I_3=\int_0^{\pi/4}(g(x))^{f(x)}dx\ \ \&\ I_4=\int_0^{\pi/4}\sec^2 x.(g(x))^{g(x)}dx,$$

then :

(a) $I_1>I_2>I_3>I_4$ (b) $I_4>I_3>I_1>I_2$

(c) $I_3>I_1>I_2>I_4$ (d) $I_4>I_1>I_3>I_2$

24. Let $I_1=\displaystyle\int_0^a f(2a-x)dx$, $I_2=\displaystyle\int_0^a f(x)dx$, then $\displaystyle\int_0^{2a} f(x)dx$ is equal to :

(a) $2I_1-I_2$ (b) I_1-I_2

(c) I_1+I_2 (d) I_1+2I_2

25. Let $p\notin I$, $\{x\}=x-[x]$, where [.] represents greatest integer function , then value of $\displaystyle\int_0^{p^2}(x-[x])\,dx$ is equal to :

(a) $\dfrac{1}{2}[p^2]$ (b) $\dfrac{1}{2}[p^2]+\dfrac{1}{2}p^2$

(c) $\dfrac{1}{2}\left([p^2]+\{p^2\}^2\right)$ (d) $\dfrac{1}{2}[p^2]+\{p^2\}$

26. Let f be a non-negative function defined on interval $[0\ ,\ 1]$. If $\displaystyle\int_0^x\sqrt{1-(f'(t))^2}\,dt=\int_0^x f(t)dt$, $0\le x\le 1$, and $f(0)=0$, then :

(a) $f\left(\dfrac{1}{2}\right)<\dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right)>\dfrac{1}{3}$

(b) $f\left(\dfrac{1}{2}\right)>\dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right)>\dfrac{1}{3}$

(c) $f\left(\dfrac{1}{2}\right)>\dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right)<\dfrac{1}{3}$

(d) $f\left(\dfrac{1}{2}\right)<\dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right)<\dfrac{1}{3}$

27. Let $f:R\to R$ be a continuous function which satisfies $f(x)=\displaystyle\int_0^x f(t)\,dt$, then value of $f(ln\,5)$ is :

(a) 4 (b) 2

(c) 0 (d) –1

28. Let [.] represents the greatest integer function and $I=\displaystyle\int_0^{\pi}[\cot x]dx$, then value of $[I]$ is equal to :

(a) 0 (b) 1

(c) –1 (d) –2

29. Interval containing the value of definite integral $\displaystyle\int_1^5\left\{\prod_{i=1}^{5}(x-i)\right\}dx$ is given by :

(a) $\left(0,\dfrac{\pi}{2}\right)$ (b) $\left(\dfrac{\pi}{8},\dfrac{5\pi}{4}\right)$

(c) $\left(0,\dfrac{\pi}{8}\right)$ (d) $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$

30. Let $f(x)$ be continuous positive function for all $x\in[0,1]$. If $\displaystyle\int_0^1 f(x)dx=1$, $\displaystyle\int_0^1 xf(x)dx=\lambda$ and $\displaystyle\int_0^1 x^2f(x)dx=\lambda^2$, $\lambda>1$ then number of possible function(s) $f(x)$ is/are :

(a) 0 (b) 2

(c) 1 (d) infinite

Multiple choice questions with MORE than ONE correct answer : (Questions No. 31-35)

31. Let $f(x)$ be continuous function for which $f(2+x) = f(2-x)$ and $f(4-x) = f(4+x)$.

If $\int_0^2 f(x)dx = 5$, then $\int_0^{50} f(x)dx$ is equal to :

(a) $\int_1^{51} f(x)dx$ (b) 125

(c) $\int_2^{52} f(x)dx$ (d) $\int_{-4}^{46} f(x)dx.$

32. Let $f : R \to R$ be an invertible polynomial function of degree $'n'$. If the equation $f(x) - f^{-1}(x) = 0$ is having only two distinct real roots $'\alpha'$ and $'\beta'$, where $\alpha < \beta$, then :

(a) $\int_\alpha^\beta (f(x) + f^{-1}(x))dx = \beta^2 - \alpha^2.$

(b) $f''(x) = 0$ has at least one real root in (α, β).

(c) If $g(x) = f(x) + f^{-1}(x) - 2x$, then $g'(x) = 0$ has at least one real root in (α, β).

(d) Minimum degree $'n'$ of $f(x)$ is 5.

33. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ & $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$, for $n = 1, 2, 3, ...$, then

(a) $S_n < \frac{\pi}{3\sqrt{3}}$ (b) $S_n > \frac{\pi}{3\sqrt{3}}$

(c) $T_n < \frac{\pi}{3\sqrt{3}}$ (d) $T_n > \frac{\pi}{3\sqrt{3}}$

34. Let $f(x)$ be a non-constant twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1-x)$ and $f'(1/4) = 0$, then

(a) $f''(x)$ vanishes at least twice on (0 , 1)

(b) $f'\left(\frac{1}{2}\right) = 0$

(c) $\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin x dx = 0$

(d) $\int_0^{1/2} f(t)e^{\sin \pi t}dt = \int_{1/2}^{1} f(1-t)e^{\sin \pi t}dt$

35. Let $f(x)$, $f'(x)$ and $f''(x)$ be continuous positive functions for all $x \in [1, 6]$, then

(a) $f(1) + f(6) - 2f\left(\frac{7}{2}\right) > 0$.

(b) $\int_1^6 f(x)dx < \frac{5}{2}(f(1) + f(6))$.

(c) $3f^{-1}(4) - f^{-1}(2) - 2f^{-1}(5) > 0$.

(d) $\int_1^6 f(x)dx > 5f(1)$.

Assertion Reasoning questions : (Questions No. 36-40)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

36. Statement 1: Let $f(x) = ||x| - 2| - 1$ for all $|x| \le 3$ then $\int_{-2}^{2} f(x)dx = 0$

because

Statement 2 : If $f(x)$ is odd continuous function , then $\int_{-a}^{a} f(x)dx$ is always zero.

37. Statement 1 : If $f(x) = 1 + x - x^2$ for all $x \in R$ and

$g(x) = \max\{ f(t) ; 0 \le t \le x \}$, $0 \le x \le 1$

then $\int_0^1 g(x)dx = \frac{29}{24}$

because

Statement 2 : $f(x)$ is increasing in $\left(0, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, 1\right)$.

38. Statement 1 : Let $f : R \to R$ be a continuous function and $f(x) = f(2x)\ \forall\ x \in R$. If $f(1) = 3$, then value of $\int_{-1}^{1} f(f(x))dx = 6$

because

Statement 2 : $f(x)$ is constant function.

39. Statement 1 : Let $I_n = \int_{0}^{1} x^n \tan^{-1} x\, dx$, if $\alpha_n I_{n+2} + \beta_n I_n = \gamma_n\ \forall\ n \in N$, then $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots$ are in A.P.

because

Statement 2 : γ_1, γ_2, γ_3, ... are in H.P.

40. Statement 1 : Let $f(x) = \int_{0}^{x} t^3(t^2 - 4)(e^t - 1)dt$, then $f(x)$ has local maxima at location of $x = 0$

because

Statement 2 : $x = 0, \pm 2$ are the critical locations for $f(x)$.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f(x)$ be a function which satisfy the functional relationship $(x-y)f(x+y)-(x+y)f(x-y)=2(x^2y-y^3)$ for all $x,\ y \in R$ and $f(3)=12$. On the basis of definition for $f(x)$, answer the following questions.

1. If $I_1=\int_0^1 \frac{dx}{\sqrt{4-xf(x)}}$, then value of '$I_1$' lies in the interval :

(a) $\left(\frac{\pi}{4\sqrt{2}},1\right)$ (b) $\left(\frac{\pi}{12\sqrt{2}},\frac{\pi}{6}\right)$

(c) $\left(\frac{\pi}{6},\frac{\pi}{4\sqrt{2}}\right)$ (d) $\left(0,\frac{\pi}{12\sqrt{2}}\right)$

2. If $I_2=\int_{-1}^{1}\tan^{-1}\left(\frac{1}{1+f(x)}\right)dx$, then value of '$I_2$' is :

(a) greater than 2 $\tan^{-1}(2)$

(b) greater than $\tan^{-1}(2)$

(c) less than $\tan^{-1}(2)$

(d) less than $\tan^{-1}(1)$

3. If $\int_{-1}^{\alpha} f(x)dx>0$, then '$\alpha$' belongs to interval:

(a) $(-\infty,0)$ (b) $\left(\frac{1}{4},\infty\right)$

(c) $\left(\frac{1}{2},\infty\right)$ (d) $\left(-\frac{1}{2},\frac{1}{2}\right)$

Comprehension passage (2)
(Questions No. 4-6)

Let $f:R\to R$ be defined by $f(x)=\frac{1-px+x^2}{1+px+x^2}$, where $p\in(0,2)$ and $g(x)=f'(x)$ for all $x\in R$. On the basis of given information , answer the following questions :

4. Value of $\int_{-2\pi}^{2\pi} ln(f(x)\,|\sin x|)\,dx$ is equal to :

(a) 0 (b) $4\pi\, ln\,\frac{1}{4}$

(c) $2\pi\, ln\,\frac{1}{8}$ (d) $\pi\, ln\,\frac{1}{16}$

5. Let $\phi(x)=\int_0^{e^x}\frac{g(t)}{1+t^2}dt$, then

(a) $\phi(x)$ is strictly increasing function

(b) $\phi(x)$ has local maxima at location of $x=0$

(c) $\phi(x)$ has local minima at location of $x=0$

(d) $\phi(x)$ is strictly decreasing function

6. Value of $\int_{-3}^{3}\frac{(x^2+1)\,dx}{1+2^{ln(f(x))}}$ is equal to :

(a) 0 (b) 6

(c) 12 (d) 3

Comprehension passage (3)
(Questions No. 7-9)

Consider the function defined implicitly by the equation $y^3-3y+x=0$ on various intervals in the real line. If $x\in(-\infty,-2)\cup(2,\infty)$, the equation defines a unique real valued differentiable function $y=f(x)$. If $x\in(-2,2)$, the equation implicitly defines a unique real valued differentiable function $y=g(x)$ satisfying $g(0)=0$.

7. If $f\left(-10\sqrt{2}\right)=2\sqrt{2}$, then $f''\left(-10\sqrt{2}\right)$ is equal to :

(a) $\frac{4\sqrt{2}}{7^3 3^2}$ (b) $-\frac{4\sqrt{2}}{7^3 3^2}$

(c) $\frac{4\sqrt{2}}{7^3 3}$ (d) $-\frac{4\sqrt{2}}{7^3 3}$

8. The area of the region bounded by the curve $y = f(x)$, the x-axis and lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(a) $\int_a^b \frac{x}{3\left((f(x))^2-1\right)}dx + bf(b) - af(a)$

(b) $\int_a^b \frac{x}{3\left(1-(f(x))^2\right)}dx + bf(b) - af(a)$

(c) $\int_a^b \frac{x}{3\left((f(x))^2-1\right)}dx - bf(b) + af(a)$

(d) $\int_a^b \frac{x}{3\left(1-(f(x))^2\right)}dx - bf(b) + af(a)$

9. $\int_{-1}^{1} g'(x)dx$ is equal to :

(a) $2g(-1)$ (b) 0

(c) $-2g(1)$ (d) $2g(1)$

Questions with Integral Answer :
(Questions No. 10-15)

10. Let $\alpha \in R^+$ and $f(\alpha) = \int_0^\infty \frac{\ln x\, dx}{x^2+\alpha x+\alpha^2}$, where $\alpha f(\alpha) - f(1) = \frac{\pi}{\sqrt{3}}$, then value of $(\alpha)^{\ln 4}$ is ....

11. Let $f : R^+ \to R$ be a differentiable function with $f(1) = 3$ and satisfying the equation,

$\int_1^{xy} f(t)dt = y\int_1^{x} f(t)dt + x\int_1^{y} f(t)dt$ for all $x, y \in R^+$,

then value of $\frac{1}{57}f(e^{37})$ is equal to ....

12. Let $f(x)$ be continuous and twice differentiable function for all values of x and $f(\pi) = 2$, if $\int_0^{\pi}(f(x)+f''(x))\sin x\,dx = 6$, then value of $f(0)$ is equal to ....

13. Let [.] represents the greatest integer function and $I = \int_0^{\pi}\frac{5x^3\cos^4 x.\sin x}{(\pi^2-3\pi x+3x^2)}dx$, then value of $[I]$ is equal to ....

14. Let $f(x)$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t}f(x-t)dt$, then value of $\frac{1}{2}f(3)$ is equal to ....

15. Let 'α' and 'β' be two distinct real roots of the equation $\tan x - x = 0$, then $\int_0^1 \sin(\alpha x).\sin(\beta x)dx$ is equal to ....

Matrix Matching Questions :
(Questions No. 16-17)

16. Match Column (I) and (II), where [.] represent greatest integer function.

Column (I)	**Column (II)**
(a) $\int_{-2}^{2}(x-[x])dx.$	(p) 0
(b) $\int_{-3}^{3}x\lvert x\rvert dx.$	(q) 1
(c) $\int_{-1/2}^{1/2}\frac{\sin^{-1}(x)}{1+x^2}dx$	(r) 2
(d) $\int_{-1}^{1}\min\{\lvert x+1\rvert, \lvert x-1\rvert\}dx$	(s) $\frac{4\pi+1}{3}$

17. If $a \in R^+$, then match columns (I) and (II) .

Column (I)	**Column (II)**
(a) If $f(2a-x)=f(x)$, then $\int_0^{2a} f(x)dx$ is	(p) 0
(b) If $f(2a-x)=-f(x)$, then $\int_0^{a} f(x)dx$ is	(q) $2\int_0^{a} f(x)dx.$
(c) If $f(-x)=f(x)$, then $\int_{-a}^{a} f(x)dx$ is	(r) $2\int_a^{2a} f(x)dx$
(d) If $f(-x)=-f(x)$, then $\int_{-a}^{a} f(x)dx$ is	(s) $\int_{2a}^{a} f(x)dx.$

18. Macht the following columns (I) and (II).

Column (I)	**Column (II)**
(a) If $S_n=\frac{1}{2n}+\frac{1}{\sqrt{4n^2-1}}+\frac{1}{\sqrt{4n^2-4}}+....+\frac{1}{\sqrt{4n-1}}$, then $\lim\limits_{n\to\infty} S_n$ is	(p) –2
(b) If $f(x)$ is bijective in nature for all $x\in[a\,,\,b]$,	(q) 2
then $\dfrac{\int_a^b \left((f(x))^2-(f(a))^2\right)dx}{\int_{f(a)}^{f(b)} x\left(f^{-1}(x)-b\right)dx}$ is	(r) $\frac{\pi}{2}$
(c) $\lim\limits_{n\to\infty}\left\{\prod_{r=1}^{n-1}\sin\left(\frac{r\pi}{2n}\right)\right\}^{1/n}$ is equal to	(s) $\frac{\pi}{6}$
(d) $\int_0^4 \left\|\,\|\,|x-2|-1\,\|-1\right\| dx$ is	(t) $\frac{1}{2}$

ANSWERS Exercise No. (1)

1. (b)
2. (c)
3. (a)
4. (c)
5. (c)
6. (b)
7. (b)
8. (c)
9. (d)
10. (b)
11. (d)
12. (c)
13. (a)
14. (d)
15. (b)
16. (d)
17. (b)
18. (a)
19. (c)
20. (c)
21. (c)
22. (a)
23. (b)
24. (c)
25. (c)
26. (d)
27. (c)
28. (d)
29. (d)
30. (a)
31. (b , c , d)
32. (a , b , c)
33. (a , d)
34. (a , b , c , d)
35. (a , b , c , d)
36. (b)
37. (b)
38. (a)
39. (c)
40. (d)

ANSWERS Exercise No. (2)

1. (c)
2. (b)
3. (c)
4. (d)
5. (c)
6. (c)
7. (b)
8. (a)
9. (d)
10. (8)
11. (2)
12. (4)
13. (3)
14. (9)
15. (0)

16. (a) → r
(b) → p
(c) → p
(d) → q

17. (a) → q , r
(b) → s
(c) → q
(d) → p

18. (a) → r
(b) → p
(c) → t
(d) → q

Chapter
20
Area Bounded by Curves

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-20)

1. Area enclosed by curve $y = x^3$ with its normal at point $(1 , 1)$ and x-axis is :

(a) $\frac{7}{4}$ sq. units (b) $\frac{9}{4}$ sq. units

(c) $\frac{5}{4}$ sq. units (d) $\frac{11}{2}$ sq. units

2. Area (in sq. units) of region bounded by $y = 2\cos x$, $y = 3\tan x$ and y-axis is :

(a) $1+3\,ln\left(\frac{2}{\sqrt{3}}\right)$ (b) $1+\frac{3}{2}ln3-3ln2$

(c) $1+\frac{3}{2}ln3-ln2$ (d) $ln\left(\frac{3}{2}\right)$

3. Let $f(x)=|4-|10-x||$, then area (in sq. units) bounded by $f(x)$ with x-axis is :

(a) 32 (b) 16

(c) 64 (d) 8

4. Let the slope of tangent to curve $y=f(x)$ at $(x , f(x))$ is $1 - 2x$ and curve passes through point $(2 , -2)$. If area bounded by curve and line $y=\alpha x$ is $\frac{32}{3}$ square units , then value of $'\alpha'$ is :

(a) –3 (b) – 3 or 5

(c) –5 (d) 3 or 5

5. Area bounded by $|y| = \sqrt{x}$ and $x=|y|+2$ is equal to :

(a) $\frac{22}{3}$ sq. units. (b) $\frac{20}{3}$ sq. units.

(c) $\frac{16}{3}$ sq. units. (d) $\frac{14}{3}$ sq. units.

6. Area (in square units) bounded by the curves $f(x)=\max\left\{2+|x-2| , 3-|x-2|\right\}$ and $g(x)=\min\left\{2+|x-2| , 3-|x-2|\right\}$ is given by :

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) 2

7. Let $y=f(x)$ be a function such that $f(x)=\min\left\{\sqrt{x(2-x)} , (2-x)\right\}$, then area (in sq. units) bounded by $y=f(x)$ and x-axis is given by

(a) $\frac{\pi}{2}+\frac{1}{4}$ (b) $\frac{\pi}{4}+\frac{1}{4}$

(c) $\frac{\pi}{4}+\frac{1}{2}$ (d) $\frac{\pi}{4}+\frac{1}{8}$

8. Let $f(x)$ be continuous function such that the area bounded by curve $y=f(x)$, x-axis and two ordinates $x = 0$ and $x = a$ is $\left(\frac{a^2}{2}+\frac{a}{2}\sin a+\frac{\pi}{2}\cos a\right)$, where $a \in R^+$, then $f\left(\frac{\pi}{2}\right)$ is :

(a) $\frac{1}{2}$ (b) $\frac{\pi^2}{8}+\frac{\pi}{4}$

(c) $\frac{\pi+1}{2}$ (d) $\frac{2\pi+1}{4}$

9. If area of the region bounded by the curve $y=e^x$ and the lines $x(y-e)=0$ is $'A'$ square units , then incorrect value of $'A'$ is given by :

(a) $\int_1^e \ln(e+1-y)\,dy$ (b) $\int_1^e \ln y\,dy$

(c) $e-\int_0^1 e^y\,dy$ (d) $e-1$

10. The area (in square units) bounded by curves $y=x^2+2$ and $y+\cos\pi x=2|x|$ is equal to :

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{8}{3}$ (d) $\frac{4}{3}$

11. If point *'P'* moves inside the triangle formed by $A\ (0\ ,\ 0)$, $B\ (1\ ,\ \sqrt{3}\ )$ and $C\ (2\ ,\ 0)$ such that $\min\{PC,\ PB,\ PA\} = 1$, then area (in square units) bounded by the curve which is traced by moving point *'P'* is given by :

(a) $\sqrt{3} - \dfrac{\pi}{2}$ (b) $2\sqrt{3} + \dfrac{\pi}{2}$

(c) $\sqrt{3} - \pi$ (d) $\dfrac{\sqrt{3} + \pi}{2}$

12. Let area bounded by the curves $y = x^2$ and $y = 2^x$ in the I^{st} quadrant be A_1 square units , then A_1 is equal to :

(a) 0 (b) $\displaystyle\int_0^2 (2^x - x^2)\,dx$

(c) $\dfrac{56}{3} - \dfrac{12}{ln 2}$ (d) $\dfrac{64}{3} - \dfrac{2}{ln 2}$

13. Area (in square units) bounded by the curve $y - x = \sin x$ and its inverse function , satisfying the condition $x^2 - 2\pi x \le 0$, is given by :

(a) 8 (b) 16

(c) 2 (d) none of these

14. If $\displaystyle\int_1^2 e^{\alpha^2}\, d\alpha = \beta,$ then area bounded by the curve $x = \sqrt{ln\, y}$ and the lines $x = 0$, $y = e$ and $y = e^4$ is equal to :

(a) $e^4 - \beta + e$

(b) $2e^4 - \beta - e$

(c) $2e^4 + \beta - e$

(d) $e^4 + \beta + e$

15. If $\alpha \in R^+$ and the area bounded by the parabolic curves $y = x - \alpha x^2$ and $\alpha y - x^2 = 0$ is maximum , then $'\alpha\,'$ is equal to :

(a) 2 (b) $\dfrac{1}{2}$

(c) 1 (d) 4

16. The area of the region between the curves $y = \left(\dfrac{1 + \sin x}{\cos x}\right)^{\frac{1}{2}}$ and $y = \left(\dfrac{1 - \sin x}{\cos x}\right)^{\frac{1}{2}}$ bounded by the lines $x = 0$ and $x = \dfrac{\pi}{4}$ is :

(a) $\displaystyle\int_0^{\sqrt{2}+1} \frac{4t\,dt}{(1+t^2)\sqrt{1-t^2}}$ (b) $\displaystyle\int_0^{\sqrt{2}-1} \frac{4t\,dt}{(1+t^2)\sqrt{1-t^2}}$

(c) $\displaystyle\int_0^{\sqrt{2}+1} \frac{t\,dt}{(1+t^2)\sqrt{1-t^2}}$ (d) $\displaystyle\int_0^{\sqrt{2}-1} \frac{t\,dt}{(1+t^2)\sqrt{1-t^2}}$

17. Let $f(x) = \min\left\{e^x\, , 1 + e^{-x}\ , \dfrac{3}{2}\right\}$ for all real values of *x*. Area (in sq. units) bounded by *f* (*x*) with *x*-axis and the lines $x = ln\left(\dfrac{3}{2}\right)$, $x = ln\, 2$ is given by :

(a) $ln\ \dfrac{8}{3}$

(b) $ln 8 - ln\sqrt{3}$

(c) $ln\dfrac{8}{3\sqrt{3}}$

(d) $ln 3\sqrt{3} - ln 2$

18. Let point *'P'* moves in the plane of a regular hexagon such that the sum of the squares of its distances from the vertices of the hexagon is 24 square units. If the radius of circumcircle of the hexagon is 1 units , then the area (in square units) bounded by the locus of point *'P'* is equal to :

(a) π (b) 2π

(c) 3π (d) 6π

19. Area (in square units) bounded by the curves $y = |\,x - 2\,|$ and $y(x^2 - 4x + 5) - 2 = 0$ is given by :

(a) $\pi - 2$ (b) $\pi - 1$

(c) $\pi - 3$ (d) $5 - \pi$

20. Area (in square units) bounded by the curves $y = [2\sin x]$ and $y = -\left|\dfrac{12x}{\pi} - 18\right|$, where [.] represents the greatest integer function , is equal to :

(a) 0 (b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{6}$ (d) none of these

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let area (in square units) bounded by the curve $y = 2^{x^2}$ and the pair of lines $y^2 - 18y + 32 = 0$ be given by $'A'$, then which of the following statements are correct :

(a) value of $'A'$ is not greater than 56
(b) Value of $'A'$ is not less than 42

(c) value of $'A'$ is equal to $\int_{2}^{16} \sqrt{log_2 x}\, dx$

(d) value of $'A'$ is equal to $\int_{1}^{8} \left\{16\left(1 + log_2 x\right)\right\}^{\frac{1}{2}} dx$

22. Let A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0$, $y = 0$ and $4x - \pi = 0$, where $n \in N - \{1 , 2\}$, then :

(a) $A_{n+2} + A_n = \dfrac{1}{(n+1)}$ (b) $\dfrac{1}{2n+2} < A_n < \dfrac{1}{2n-2}$

(c) $A_n < A_{n+2}$ (d) $A_n \ngtr \tan^{-1}(\sqrt{2} - 1)$

23. Let the tangent to curve $f(x) = x^2 + \lambda x - \lambda$ at point $(1 , 1)$ meet the x-axis and y-axis at A and B respectively. If the area of triangle AOB is 2 square units , where $'O'$ is origin , then the values of λ can be :

(a) 3 (b) –3

(c) $1 + 2\sqrt{2}$ (d) $1 - 2\sqrt{2}$

24. Let the two branches of the curve $(y - x)^2 = \sin x$ be $y = f(x)$ and $y = g(x)$, where $f(x) \geq g(x) \quad \forall \quad x \in R$. If the area bounded by $f(x)$ and $g(x)$ in between the lines $x = 0$ and $x = \pi$ is $'A'$ square units , then :

(a) $2 < A < 4$ (b) $4 < A < 2\pi$

(c) $A > \int_{0}^{\pi/2} 4\sin^2 x dx$ (d) $A = \int_{0}^{\pi/2} 4\sqrt{\cos x}\, dx$

25. Let $f(x) = x^2 - 2|x| \quad \forall \quad x \in R$ and

$$g(x) = \begin{cases} \min\left\{f(t) : -2 \leq t \leq x\right\} ; & x \in [-2 , 0) \\ \max\left\{f(t) : 0 \leq t \leq x\right\} ; & x \in [0 , 3) \end{cases} ,$$ then

which of the following statements are correct :

(a) Area bounded by $f(x)$ with x-axis is $\dfrac{8}{3}$ square units.

(b) Area bounded by $g(x)$ with the curve $y = x^2 - 2x$ is $\dfrac{4}{3}$ square units.

(c) Area bounded by $g(x)$ with the curve $y = 1 - |x - 1|$ is 2 square units.

(d) Area bounded by $g(x)$ with the pair of lines $y + xy = 0$ is $\dfrac{2}{3}$ square units.

Assertion Reasoning questions : (Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. **Statement 1 :** Area (in square units) bounded by the curves $y = \sin^{-1}x$, $y = \cos^{-1}x$ and $y = 0$ is given by $\cot\left(\dfrac{3\pi}{8}\right)$

because

Statement 2 :

$$\int_{0}^{\pi/4} (\cos y - \sin y)dy = \left\{\int_{0}^{\frac{1}{\sqrt{2}}} \sin^{-1} x\, dx + \int_{\frac{1}{\sqrt{2}}}^{1} \cos^{-1} x\, dx\right\} = \tan\left(\frac{\pi}{8}\right).$$

27. **Statement 1 :** Let $f(x) = \lim\limits_{n\to\infty} (\sin x)^{4n}$ and $g(x) = x^2 - 6x + 8$, then area bounded by $f(x)$ and $g(x)$ is given by $\dfrac{4}{3}$ square units

because

Statement 2 : $\lim\limits_{n\to\infty} (\sin x)^{4n} = \left[\,|\sin x|\,\right] \quad \forall \quad x \in R$,

where [.] represents the greatest integer function.

28. **Statement 1 :** Area bounded by the curves $C_1 : x^2 - y - 1 = 0$ and $C_2 : y - |x| = 0$ is divided by the y-axis in two equal parts

because

Statement 2 : Curves $'C_1'$ and $'C_2'$ are symmetrical about the y-axis.

29. Let $f : [0, 1] \rightarrow [0, 1]$ be defined by the function $f(x) = 1 - \sqrt{1 - x^2}$.

Statement 1 : Area bounded by the curves $y = f(x)$ and $y = f^{-1}(x)$ is given by $\left(2 - \frac{\pi}{2}\right)$ square units

because

Statement 2 : If a function is bijective in nature , then its inverse always exist.

30. **Statement 1 :** Area bounded by the curves $y = 3x^2$ and $y = 3^x$ in between the lines $x = 3$ and $x = 4$ is given by $\left(54 log_3 e - 37\right)$ square units

because

Statement 2 : Total number of solutions for the equation $x^4 - (3^{x-1} - 1)x^2 - 3^{x-1} = 0$ are three.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f(x) = x^n \tan^{-1}(x)\ \forall\ x \in R$ and $n \in W$. If area bounded by $y = f(x)$ with x-axis and lines $x = 0$, $x = 1$ is represented by A_n, then answer the following questions.

1. Value of $(n+1)A_n + (n+3)A_{n+2}$ is equal to :

(a) $\dfrac{\pi}{2} - \dfrac{1}{n+1}$ (b) $\dfrac{\pi}{2} - \dfrac{1}{n+2}$

(b) $\dfrac{\pi}{2} + \dfrac{2}{n}$ (d) $\dfrac{\pi}{2} + \dfrac{1}{n}$

2. Value of $\sum\limits_{r=1}^{4}(r+1)A_r$ is equal to :

(a) $\pi - \dfrac{7}{12}$ (b) $\pi + \dfrac{5}{12}$

(c) $2\pi - \dfrac{1}{12}$ (d) $\dfrac{\pi}{2} - \dfrac{1}{4}$

3. Value of A_4 is equal to :

(a) $\dfrac{\pi - 1 + ln 4}{10}$ (b) $\dfrac{\pi + 1 - ln 4}{20}$

(c) $\dfrac{\pi - ln 4}{15}$ (d) $\dfrac{\pi + 2 - ln 4}{10}$

Comprehension passage (2)
(Questions No. 4-6)

In figure no. (1), the graph of two curves $C_1 : y = f(x)$ and $C_2 : y = \sin x$ are given, where 'C_1' and 'C_2' meet at $A(a, f(a))$, $B(\pi, 0)$ and $C(2\pi, 0)$. If A_1, A_2 and A_3 are the bounded area as shown in figure no. (1) and $A_1 = (a - 1)\cos a - \sin a + 1$, then answer the following questions.

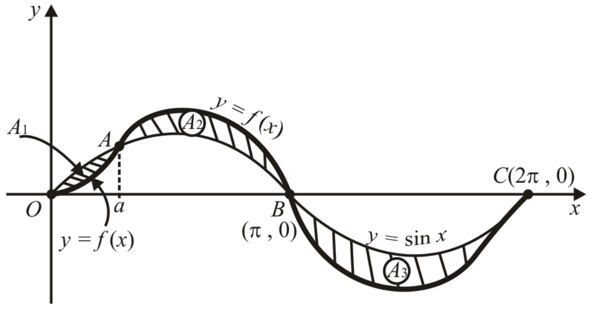


figure no. (1)

4. Area (in square units) A_2 is equal to :

(a) $\pi - 1 - \sin 1$ (b) $\pi + 1 - \sin 1$

(c) $\pi + 1 + \sin 1$ (d) $\pi - 2 + \sin 1$

5. If [.] represents the greatest integer function, then value of $[A_3]$ is equal to :

(a) 4 (b) 7

(c) 8 (d) 5

6. Let tangent to $y = f(x)$ at point 'A' meets the x-axis at $(K, 0)$, then 'K' is equal to :

(a) tan 1
(b) cot 1
(c) sin 1
(d) none of these

Comprehension passage (3)
(Questions No. 7-9)

Let $f(x) = \dfrac{px^2 + qx + 4}{x^2 + 1}$, where $f(x) = f(|x|)\ \forall\ x \in R$ and $\lim\limits_{x \to \infty} f(x) = -1$, then answer the following questions.

7. If $g(x) = [\alpha f(x)]$ for all $|x| \le 2$, where [.] represents the greatest integer function, and total number of points of discontinuity for $y = g(x)$ are 31, then value of 'α' is equal to :

(a) 3 (b) 4

(c) 5 (d) 6

8. If the vertices of rectangle 'R' lie on curve $y = f(x)$ and other two vertices lies on the line $y + 1 = 0$, then maximum area (in square units) of rectangle 'R' is equal to :

(a) 8 (b) 6

(c) 5 (d) 10

9. Let $h(x) = \begin{cases} f(x) & ; x \le 1 \\ -x + k^2 - 2k - \dfrac{1}{2} & ; x > 1 \end{cases}$ and minimum value of $h(x)$ exists at $x = 1$, then 'k' belongs to :

(a) $[-1, 3]$
(b) $R - (-1, 3)$
(c) $R - [-1, 3]$
(d) $(-1, 3)$

Questions with Integral Answer :
(Questions No. 10-14)

10. Let $d(P\,,\,L)$ represents the distance of any point $'P'$ from the line $'L'$ on $x-y$ plane. If $A(-3\,,0)\,,\,B(3\,,0)\,,\,C(3\,,\,4)$ and $D(-3\,,\,4)$ are the vertices of rectangle $ABCD$, and the moving point $'P'$ satisfy the condition $d(P\,,AB) \leq \min\{d(P\,,BC)\,,\,d(P\,,CD)\,,\,d(P\,,AD)\}$, then area (in square units) of the region in which point $'P'$ moves is equal to ....

11. Let $a \in R^{+}$ and the area of curvilinear trapezoid bounded by the curve $y = \dfrac{x}{6} + \dfrac{1}{x^2}$ and the lines whose joined equation is $y(x^2 - 3ax + 2a^2) = 0$ be $'A'$ square units. If $'A'$ is having the least value, then $'a'$ is equal to ....

12. The area enclosed by the parabolic curve $(y-2)^2 = x-1$, the tangent to parabola at $(2\,,\,3)$ and the x-axis is equal to ....

13. Let the area of region bounded by the curves $y = x^2$, $y = |\,2 - x^2\,|$ and $y - 2 = 0$, which lies to the right of the line $x - 1 = 0$, be $'A'$ square units. If [.] represents the greatest integer function, then value of $[A]$ is equal to ....

14. Let the area enclosed by the loop of the curve $2y^2 + x^2(x-2) = 0$ be $'A'$ square units, then the least integer which is just greater than $'A'$ is equal to ....

Matrix Matching Questions :
(Questions No. 15-17)

15. Match the following columns (I) and (II).

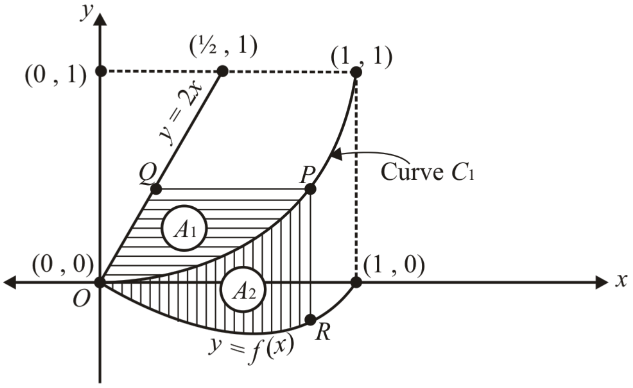
	Column (I)		Column (II)
(a)	Area of region enclosed by the curve $(y-\sin^{-1}x)^2 = x - x^2$	(p)	$\dfrac{18-2e^2}{e^2}$
(b)	Area of the finite portion of the figure bounded by $y = 2x^2e^x$ and $y + x^3e^x = 0$	(q)	π
(c)	Area of curvilinear trapezoid bounded by $y = (x^2+2x)e^{-x}$ and the x-axis	(r)	$\pi/4$
(d)	Area of figure bounded by the curves $x = \sqrt{4-y^2}$ and $\|\,y\,\| = x$	(s)	4

16. Let area (in square units) bounded by function $f(x)$ with the x-axis and the lines $x = 0$; $x = 1$ be represented by $'A'$. Match the following columns for function $f(x)$ and the interval in which area $'A'$ lies.

	Column (I)		Column (II)
(a)	$f(x) = \sqrt{x^3+2}$	(p)	$\left(ln\,2\,,\,\dfrac{\pi}{2}\right)$
(b)	$f(x) = x^{(\sin x+\cos x)^2}$	(q)	$\left(\dfrac{\pi}{6}\,,\,\dfrac{\pi}{4\sqrt{2}}\right)$
(c)	$f(x) = \dfrac{1}{\sqrt{4-x^2-x^3}}$	(r)	$\left(\dfrac{1}{3}\,,\,\dfrac{1}{2}\right)$
(d)	$f(x) = \dfrac{1}{\sqrt{x^6+1}}$	(s)	$\left(\sqrt{2}\,,\,\sqrt{3}\right)$

17. Let C_1 , C_2 and C_3 be the graph of functions $y = x^2$, $y = 2x$ and $y = f(x)$ respectively for all $x \in [0 , 1]$ and $f(0) = 0$. If point $'P'$ lies on the curve $'C_1'$ and the area of region OPQ and OPR are equal as shown in the figure , then match the following columns with reference to the function $y = f(x)\ \forall\ x \in [0 , 1]$.

{Area OPQ (A_1)}
= {Area OPR (A_2)}



Column (I)	Column (II)
(a) Value of global minima for $y = f(x)$.	(p) 1/6
(b) Area (in square units) bounded by $y = f(x)$ and $y = \lvert f(x)\rvert$	(q) $\dfrac{3\pi + 2}{24}$
(c) If $g(x) = \min\{f(t) : 0 \le t \le x\}$; $0 \le x \le 1$, then area bounded by $g(x)$ with x-axis and the line $x = 1$ is equal to :	(r) –4/27
(d) Area (in square units) bounded by $y = f(x)$ and $y = \sqrt{x - x^2}$ is :	(s) 8/81

Objective Mathematics
Er. L.K. Sharma
JEE

ANSWERS Exercise No. (1)

1. (a)
2. (b)
3. (b)
4. (b)
5. (b)
6. (b)
7. (c)
8. (a)
9. (d)
10. (c)
11. (a)
12. (c)
13. (a)
14. (b)
15. (c)
16. (b)
17. (c)
18. (c)
19. (b)
20. (b)
21. (a , b , d)
22. (a , b , d)
23. (b , c)
24. (b , c , d)
25. (a , b , d)
26. (a)
27. (b)
28. (a)
29. (d)
30. (d)

ANSWERS Exercise No. (2)

1. (b)
2. (a)
3. (b)
4. (a)
5. (b)
6. (d)
7. (b)
8. (c)
9. (b)
10. (8)
11. (1)
12. (9)
13. (1)
14. (3)

15. (a) → r
(b) → p
(c) → s
(d) → q

16. (a) → s
(b) → r
(c) → q
(d) → p

17. (a) → r
(b) → p
(c) → s
(d) → q

Chapter 21

Differential Equations

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-15)

1. If $y_1(x)$ and $y_2(x)$ are the two solutions of $\frac{dy}{dx} + f(x)y = r(x)$, then $y_1(x) + y_2(x)$ is solution of :

(a) $\frac{dy}{dx} + f(x)y = 0$ (b) $\frac{dy}{dx} + 2f(x)y = r(x)$

(c) $\frac{dy}{dx} + f(x)y = 2r(x)$ (d) $\frac{dy}{dx} + 2f(x)y = 2r(x)$

2. General solution of $\frac{dy}{dx} = y - \ln x + \frac{1}{x}$ is given by : ('c' is independent arbitrary constant)

(a) $y = x\ln x + c.$ (b) $y = e^x \ln x + c.$

(c) $y = \ln x + ce^x.$ (d) $y = x^2 \ln x + c.$

3. The equation of curve which is passing through (1 , 1) and having differential equation $y' + \frac{y}{x} = y^3$ is given by :

(a) $2x^2y^2 - xy^2 = 1$ (b) $2xy^2 + x^2y^2 = 3$

(c) $2x^2y^2 + xy^2 = 3$ (d) $2xy^2 - x^2y^2 = 1$

4. If solution of differential equation $\sin\left(\frac{dy}{dx}\right) + x\frac{dy}{dx} = y$ satisfy $y(-1) = 0$, then non-zero value of $y(1)$ is equal to :

(a) –1 (b) π (c) $-\pi$ (d) 1

5. If the length of x-intercept of tangent to the curve $y = f(x)$ is twice the length of y-intercept and $f(1) = 1$, then equation of curve is given by :

(a) $2x + y = 3$ (b) $x + 2y = 3$

(c) $2y = x + \sqrt{x}$ (d) $2y = 3\sqrt{x} - x$

6. Let $y = (a\sin x + (b+c)\cos x)e^{x+d}$, where a, b, c, d are parameters , be the general solution of a differential equation , then order of differential equation is given by :

(a) 1 (b) 2 (c) 3 (d) 4

7. If $xdy = y(dx + ydy)$, $y(1) = 1$ and $y(x) < 0$, then $y(-3)$ is equal to :

(a) 3 (b) –1

(c) –2 (d) –3

8. If a curve passes through (1 , 1) and tangent at any point 'P' on it cuts the axes at 'A' and 'B', where point 'P' bisects the segment AB, then curve is given by :

(a) $xy^2 = 1$ (b) $x^2y = 1$

(c) $x^2 + y^2 = 2$ (d) $xy = 1$

9. The degree of differential equation $y = x + 1 + \frac{dy}{dx} + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots + \infty$, is :

(a) undefined (b) 1

(c) ∞ (d) $n!$

10. A right circular cone with radius 10 m and height 20 m contains alcohol which evaporate at a rate proportional to its surface area in contact with air. If initially the cone is completely filled and the proportionality constant is 'λ', then the time in which the cone gets empty is equal to :

(a) $\frac{10}{\lambda}$ (b) $\frac{20}{\lambda}$ (c) $\frac{30}{\lambda}$ (d) $\frac{5}{\lambda}$

11. Solution of differential equation $2y\sin x\frac{dy}{dx} = \sin 2x - y^2\cos x$, satisfying $y\left(\frac{\pi}{2}\right) = 1$ is given by :

(a) $y^2 = \sin x$ (b) $y = \sin^2 x$

(c) $y^2 = \cos x + 1$ (d) $y^2 \sin x = 4\cos^2 x$

12. For differential equation $\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$, the solution can be given by :

(a) $y = 2 + x$

(b) $y = 2x$

(c) $y = 2x - 4$

(d) $y = 2x^2 - 4$

13. Let $'c'$ be independent arbitrary constant, then orthogonal trajectories of the family of curves represented by $2y^2+x^2=y+c$ is given by :

(a) $x^2=k(4y-1)$ (b) $x^2=k(4y^2+1)$

(c) $x=k(4y^2-1)$ (d) $x=k(4y+1)$

14. For differential equation

$$(1-e^x)\sec^2 y\,dy+3e^x\tan y\,dx=0,\ \text{ if }\ y(ln\,2)=\frac{\pi}{4}\ ,$$

then $y(ln\,3)$ is equal to :

(a) $\dfrac{\pi}{12}$ (b) $\dfrac{\pi}{8}$

(c) $\dfrac{\pi}{4}$ (d) none of these

15. Order of differential equation of the family of ellipse having major axis parallel to the y-axis is equal to :

(a) 2 (b) 3

(c) 4 (d) 5

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. A tangent drawn to curve $y = f(x)$ at $P(x, y)$ meet the x-axis and y-axis at A and B respectively such that $BP : AP = 3 : 1$, and $f(1)=1$, then

(a) equation of curve is $x\dfrac{dy}{dx}-3y=0$

(b) curve passes through $\left(\dfrac{1}{2}, 8\right)$

(c) normal at $(1, 1)$ is $x+3y=4$

(d) equation of curve is $x\dfrac{dy}{dx}+3y=0$

17. Let a solution $y = y(x)$ of the differential equation

$$x\sqrt{x^2-1}\,dy-y\sqrt{y^2-1}\,dx=0\ \text{ satisfy }\ y(2)=\frac{2}{\sqrt{3}}\ ,$$

then :

(a) $y(x)=\sec\left(\sec^{-1}(x)-\dfrac{\pi}{6}\right)$

(b) $\dfrac{1}{y}=\dfrac{2\sqrt{3}}{x}+\dfrac{1}{2}\sqrt{1+\dfrac{1}{x^2}}$

(c) $y(x)=\sec\left(\sin^{-1}(x)+\dfrac{\pi}{6}\right)$

(d) $\dfrac{1}{y}=\dfrac{\sqrt{3}}{2x}+\dfrac{1}{2}\sqrt{1-\dfrac{1}{x^2}}$

18. Let y_1 and y_2 be two different solutions of the differential equation $\dfrac{dy}{dx}+P(x)y=Q(x)$, where $P(x)$ and $Q(x)$ are functions of x, then :

(a) $y=y_1+k\,(y_2-y_1)$ is the gereral solution of given differential equation, (where k is parameter).

(b) If $\alpha y_1+\beta y_2$ is solution of given differential equation, then $\alpha+\beta=1$.

(c) If $\alpha y_1+\beta y_2$ is solution of given differential equation, then $\alpha+\beta=2$.

(d) If y_3 is the solution of given differential equation different from y_1 and y_2, then $\dfrac{y_2-y_1}{y_3-y_1}$ is constant.

19. Let $y=f(x)$ be a strictly increasing curve for which the length of sub-normal is twice the square of the ordinate at any point $P(x, y)$ on the curve, where $f(0)=1$, then

(a) $f''(0)=4$

(b) normal to the curve at $(0, 1)$ is $2y+x=2$

(c) $f'''(0)=4$

(d) curve passes through the point $(ln\,2, 4)$

20. A curve passing through the point $(2, 2)$ has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to distance of P from the x-axis, then

(a) curve may be represented by a line.

(b) curve may be represented by a parabola.

(c) curve may be represented by a circle.

(d) curve may be represented by an ellipse.

**Assertion Reasoning questions :
(Questions No. 21-25)**

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. Consider the differential equation

$$E_1 : \frac{d^3y}{dx^3} + 2\frac{dy}{dx} = \sin\left(\frac{d^2y}{dx^2}\right) + y$$

Statement 1 : Order of differential equation E_1 is 3

because

Statement 2 : Degree of differential equation E_1 is 1.

22. Let the family of parabolic curves of focal length 2 units and having the axis parallel to the x-axis be represented by $'C_P'$.

Statement 1 : Differential equation representing the family of curves $'C_P'$ is having order and degree as 2 and 1 repectively

because

Statement 2 : Differential equation for $'C_P'$ is given by $\frac{d^2y}{dx^2} \pm \frac{1}{4}\left(\frac{dy}{dx}\right)^3 = 0$.

23. Consider the family of curves $'C_1'$ such that any tangent to the curves intersects with the y-axis at that point which is equidistant from the point of tangency and the origin.

Statement 1 : Differential equation representing the family of curves $'C_1'$ is linear differential equation of first order and first degree

because

Statement 2 : $'C_1'$ represents the one parameteric family of circles which are passing through the origin.

24. Consider the differential equation $x^2dy + (3 - 2xy)dx = 0$, where $y(1) = 2$. Let the solution of differential equation with given condition be represented by curve $y = f(x)$.

Statement 1 : The curve of $y = f(x)$ passes through the point (–1 , 0)

because

Statement 2 : $f(x) = x^4 + \frac{1}{x}$

25. **Statement 1 :** Differential equation $(1 - x^2)\frac{dy}{dx} + xy = 2x$ can represent the family of ellipses with the centre at (0 , 2) and the axes parallel to the coordinate axes

because

Statement 2 : Each integral curve of the equation $(1 - x^2)\frac{dy}{dx} + xy - 2x = 0$ have one constant axis whose length is equal to 2 units.

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Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let any point P on a curve be joined to origin $(0\ ,\ 0)$, then OP is termed as polar radius of P. For curve C_1 passing through $(2\ ,\ 2)$, the angle of inclination of tangent with x-axis at any of its point is twice the angle of inclination with x-axis formed by polar radius of the point of tangency

1. Which one of the following differential equations satisfy curve C_1 :

(a) $(x^2+y^2)dy-2xy\ dx=0.$

(b) $d\left(\frac{x}{y^2}\right)+dy=0.$

(c) $d\left(\frac{x^2}{y}\right)+dx=0.$

(d) $d\left(\frac{x^2}{y}\right)+dy=0.$

2. Equation of curve $'C_1'$ is :

(a) $2x+2y^3-5y^2=0$ (b) $x^2+(x-4)\,y=0$

(c) $x^2+(y-2)^2=4$ (d) none of these

3. Angle of inclination with x-axis of polar radius of point having x-coordinate as 1 on curve C_1 can be given by :

(a) 30° (b) 45°

(c) 60° (d) 15°

Comprehension passage (2)
(Questions No. 4-6)

Consider a drop of water , having the initial mass M_0 g and evaporating at a rate of m g/s , falls freely in the air. The resistance force is proportional to the velocity of the drop (the proportionality factor being k). If initially the velocity of the water drop is zero and $k\neq 2m$, then answer the following questions.

4. If $'g'$ is the gravitational acceleration , then the differential equation defining the velocity-time relationship for the drop of water is given by :

(a) $\frac{dv}{dt}+\frac{(k-m)v}{(M_0-mt)}=g$. (b) $\frac{dv}{dt}+\frac{(k+m)v}{(M_0-mt)}=g$.

(c) $\frac{dv}{dt}-\frac{(k-m)v}{(M_0+mt)}=g$. (d) none of these.

5. Integrating factor for the differential equation defining the velocity-time relationship for the drop of water is equal to :

(a) $(M_0-mt)^{\frac{m+k}{m}}$

(b) $(M_0+mt)^{\frac{m-k}{m}}$

(c) $(M_0+mt)^{\frac{m+k}{m}}$

(d) $(M_0-mt)^{\frac{m-k}{m}}$

6. Let $V=f(t)$ represents the velocity of drop of water as function of time elapsed from the instant the drop started falling , then $f(t)$ is equal to :

(a) $\frac{g(M_0-mt)}{(2m-k)}\left[\left(1-\frac{mt}{M_0}\right)^{\frac{k-2m}{m}}+1\right]$

(b) $\frac{g(M_0+mt)}{(2m-k)}\left[\left(1+\frac{mt}{M_0}\right)^{\frac{k-2m}{m}}-1\right]$

(c) $\frac{g(M_0-mt)}{(2m-k)}\left[\left(1-\frac{mt}{M_0}\right)^{\frac{k-2m}{m}}-1\right]$

(d) none of these

Comprehension passage (3)
(Questions No. 7-9)

Let the curve $y=f(x)$ passes through the point $(4\ ,-2)$ and satisfy the differential equation $y(x+y^3)dx-x(y^3-x)dy=0$. If the curve $y=g(x)$ is defined for $x\in R$, where $g(x)=\left[\,|\sin x|+|\cos x|\,\right]$, $[.]$ represents the greatest integer function , then answer the following questions.

7. Total number of locations of non-differentiability for the function $y=\max\{f(x)\ ,-2x\}$ is/are :

(a) 1 (b) 2

(c) 3 (d) 4

8. Area (in square units) of the region bounded by the curves $y=f(x)\ ,\ y=g(x)$ and $x=0$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$

(c) $\frac{1}{4}$ (d) $\frac{1}{16}$

9. If [.] represents the greatest integer function , then value of $\int_{-1/2}^{1/2} [\,f(x)]\,dx$ is equal to :

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) –1

Questions with Integral Answer :
(Questions No. 10-14)

10. Let the normal at any point $'P'$ on the curve $'C_1'$ meets the x-axis and y-axis at the points $'A'$ and $'B'$ respectively such that $\frac{1}{OA}+\frac{1}{OB}=1$, where O is origin. If the curve $'C_1'$ pass through the points (5 , 4) and (4 , α) , then $'\alpha'$ is equal to ....

11. Let $y = f(x)$ be twice differentiable function such that the equation $k^2y-2k\frac{dy}{dx}+\frac{d^2y}{dx^2}=0$, provides two equal values of $'k'$ for all $x \in R$, and $f(0) = 1$, $f'(0)=2$, then value of $f(ln\,3)$ is equal to ....

12. The bottom of a vertical cylinderical vessel with the cross-sectional area $\sqrt{5}\ m^2$ is provided with a small circular hole whose area is 0.5 m^2. The hole is covered with a diaphram , and the vessel is filled with water to the height of 16 m. At time $t = 0$, the diaphram starts to open , the area of the hole being proportional to the time , and the hole opens completely in 4 seconds. If the gravitational acceleration is g = 10 m/s^2 and the velocity of flow through opening is $\sqrt{2gh}$, where h is height of water, then the height of water in the vessel in 4 seconds , after the experiment began , is equal to ....

13. Let a solution $y = y(x)$ of the differential equation $\frac{dy}{dx}=\frac{\cos x\sin y+\tan^2 x}{\sin x.\cos y}$ satisfy $y\left(\frac{\pi}{4}\right)=\frac{\pi}{4}$, then value of $y(0)$ is equal to ....

14. Let a solution $y = y(x)$ of the differential equation $\frac{dy}{dx}=\frac{2xy}{x^2-2y-1}$ satisfy $y(1)=1$, then value of $log_e\left(y\left(\sqrt{1-2e}\right)\right)$ is equal to ....

Matrix Matching Questions :
(Questions No. 15-17)

15. Match the following differential equations in column (I) with their corresponding particular solution in column (II).

Column (I)	Column (II)
(a) The solution of $(2xy)y' = x^2 + y^2$, if the curve $y = f(x)$ passes through (1 , 0).	(p) $x^2 - y^2 = x$
(b) The solution of $(2xy)y' = x^2 + y^2 + 1$, if $y = f(x)$ passes through (1 , 0).	(q) $y=\frac{2x}{2-x^2}$
(c) The solution of $y + xy^2 - xy' = 0$, if $y = f(x)$ passes through (1 , 2).	(r) $x^2y^3(3-2x)=1$
(d) The solution of $xy' + y = x^2y^4$, if $y = f(x)$ passes through (1 , 1)	(s) $x^2 - y^2 = 1$
	(t) $x^2 + y^2 = 2$

16. Match the family of curves in column (I) with the corresponding order of the differential equation in column (II).

Column (I)	Column (II)
(a) family of parabolic curves with vertex on the x-axis.	(p) 4
(b) family of circles touching the y-axis.	(q) 2
(c) family of ellipses having major axis parallel to the y-axis.	(r) 3
(d) family of rectangular hyperbolas with centre at origin.	(s) 5

17. Let $'C_1'$ represents a curve in the first quadrant for which the length of x-intercept of tangent drawn at any point $'P'$ on it is three times the x-coordinate of point $'P'$. If $y = f(x)$ represents the curve $'C_1'$ and $f(4) = 8$, then match the following columns (I) and (II).

Column (I)	Column (II)
(a) Area (in square units) bounded by $y = f(x)$ with the lines $x - 1 = 0$ and $y - 2x = 0$ is equal to :	(p) 16
(b) If [.] represents the greatest integer function , then total number of locations of discontinuity in $[1\ ,\ \infty)$ for $y = [f(x)]$	(q) 12
	(r) 15
(c) If the equation $f(x) + x - k = 0$ is having exactly two solutions , then values of $'k'$ can be	(s) 17
(d) If the equation $f(x) = \|x - \alpha\|$ is having at most two solutions , then values of α can be :	(t) 8

ANSWERS Exercise No. (1)

1. (c) **2.** (c) **3.** (d) **4.** (b) **5.** (b)

6. (b) **7.** (b) **8.** (d) **9.** (b) **10.** (b)

11. (a) **12.** (c) **13.** (a) **14.** (d) **15.** (c)

16. (b , d) **17.** (a , d) **18.** (a , b , d) **19.** (a , b , d) **20.** (a , c)

21. (c) **22.** (a) **23.** (d) **24.** (c) **25.** (b)

ANSWERS Exercise No. (2)

1. (d) **2.** (c) **3.** (d) **4.** (a) **5.** (d)

6. (c) **7.** (c) **8.** (b) **9.** (c) **10.** (5)

11. (9) **12.** (9) **13.** (0) **14.** (1)

15. (a) → p
(b) → s
(c) → q
(d) → r

16. (a) → r
(b) → q
(c) → p
(d) → q

17. (a) → s
(b) → p
(c) → p , r , s
(d) → q , t

Chapter 22

Basics of 2D-Geometry

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-20)

1. If L_1, L_2, L_3 are three non-concurrent and non-parallel lines in 2-dimesional plane , then maximum number of points which are equidistant from all the three lines is/are :

(a) 1 (b) 2
(c) 3 (d) 4

2. If circle $x^2+y^2-2x-6y+8=0$ meets the y-axis at $'A'$ and $'B'$, then circumcentre of ΔABC, where $'C'$ is the centre of circle , is given by :

(a) $\left(\frac{1}{2}, 3\right)$ (b) $(0, 3)$
(c) $\left(1, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{5}{2}\right)$

3. Total number of integral points which don't lie outside the circle $x^2+y^2-25=0$ are given by :

(a) 60 (b) 80
(c) 81 (d) 120

4. If a moving point $P(x,y)$ satisfy the condition $|x-4|+|y-2|=1$, then locus of $'P'$ is :

(a) rectangle (b) square
(c) rhombus (d) parallelogram

5. Let the vertices $'A'$ and $'D'$ of square $ABCD$ lie on positive x-axis and positive y-axis respectively , if the vertex $'C'$ is the point $(12, 17)$, then co-ordinates of vertex $'B'$ is given by :

(a) $(14, 16)$ (b) $(15, 3)$
(c) $(17, 5)$ (d) $(17, 12)$

6. In ΔABC , let the centroid and circumcentre of the triangle be $(3, 3)$ and $(6, 2)$ respectively , if point $'P'$ divides CD internally in the ratio $\dfrac{\tan A + \tan B}{\tan C}$, where D lies on side AB and CD is perpendicular to AB , then co-ordinates of point $'P'$ is given by :

(a) $(9, 5)$ (b) $(3, -1)$
(c) $(-3, 1)$ (d) $(-3, 5)$

7. If the points $(1, 1)$, $(0, \sec^2\theta)$ and $(\text{cosec}^2\theta, 0)$ are collinear , then $'\theta'$ belongs to :

(a) R (b) $R-\{n\pi\}; n\in I$
(c) $R-\left\{(2n+1)\frac{\pi}{2}\right\}; n\in I$ (d) $R-\left\{\frac{n\pi}{2}\right\}; n\in I$

8. Let $A(2, -3)$ and $B(-2, 1)$ be the vertices of ΔABC, if the centroid of ΔABC moves on the curve $y^2-4x=0$, then locus of vertex $'C'$ is

(a) circle (b) line
(c) parabola (d) ellipse

9. Let $\alpha, \beta \in R^+$ and the side lengths of triangle ABC be $3\alpha+4\beta$, $4\alpha+3\beta$ and $5\alpha+5\beta$, then triangle ABC must be :

(a) right-angled (b) obtuse-angled
(c) acute-angled (d) equilateral

10. Let a, b, c be in $A.P.$, where $a \neq c$, and p, q, r be in $G.P.$. If the real points $A(a, p)$, $B(b, q)$ and $C(c, r)$ satisfy the condition $|AB-CA| = BC$, then :

(a) $p=q=r$ (b) $p^2=q$
(c) $q^2=r$ (d) $r^2=p$

11. Let the points $'A'$ and $'B'$ be $(0, 4)$ and $(0, -4)$ respectively , then equation of the locus of moving point $P(x, y)$ such that $|PA-PB|=6$, is given by :

(a) $9x^2+7y^2=63$ (b) $7x^2+9y^2=63$
(c) $9x^2-7y^2=63$ (d) $7y^2-9x^2=63$

12. In ΔABC , let the equation of side BC be $y-4=0$ and the orthocentre and circumcentre be $(3, 5)$ and $(6, 7)$ respectively , then area of circumcircle of ΔABC is given by :

(a) 16π sq. units (b) 13π sq. units
(c) 25π sq. units (d) 20π sq. units

13. In ΔABC , let the mid points of the sides AB , BC and CA be $P(-1, 5)$, $Q(1, 3)$ and $R(4, 5)$ respectively , then area (in sq. units) of the triangle ABC is given by :

(a) 10 (b) 20
(c) 40 (d) 30

14. Let co-ordinates of a point $'P'$ be $(2\alpha, 1)$ with respect to a rectangular cartesian system, and when the system is rotated through a certain angle about origin in the clockwise sense, the co-ordinates of $'P'$ becomes $Q\ (\alpha+1, 1)$ with respect to new system, then :

(a) $\alpha = 0$ (b) $\alpha = 1$ or $\alpha = -\frac{1}{3}$

(c) $\alpha = -1$ or $\alpha = \frac{1}{3}$ (d) $\alpha = 1$ or $\alpha = -1$

15. In ΔABC, let vertex points $'A'$ and $'B'$ be $(1, 2)$ and $(2, 4)$ respectively and vertex $'C'$ lies on the line $y - 2x - 2 = 0$. If the area of ΔABC is 1 square unit, then vertex point $'C'$ can be :

(a) $(10, 25)$ (b) $(24, 100)$

(c) $(100, 200)$ (d) $(49, 100)$

16. Let α, β, γ be distinct real numbers, where $p \in R^+$, and the points $(\alpha, 2p\alpha + p\alpha^3)$, $(\beta, 2p\beta + p\beta^3)$, $(\gamma, 2p\gamma + p\gamma^3)$ are collinear, then :

(a) $\alpha\beta\gamma = 1$ (b) $\alpha + \beta + \gamma = \alpha\beta\gamma$

(c) $\alpha + \beta + \gamma = 0$ (d) $\alpha + \beta + \gamma + 1 = 0$

17. In triangle ABC, if all the vertices are rational points, then which one of the following points is not necessarily a rational point ?

(a) Centroid (b) Circumcentre

(c) Orthocentre (d) Incentre

18. Let point $P\ (x, y)$ moves in such a manner so that for all $\alpha \in R$, $x = \sqrt{3}\left(\frac{1-\alpha^2}{1+\alpha^2}\right)$ and $y = \frac{2\alpha}{1+\alpha^2}$, then locus of $'P'$ is :

(a) circle (b) ellipse

(c) parabola (d) hyperbola

19. Let $\alpha \in R$ and vertices of a variable triangle be given by $(5\cos\alpha, 5\sin\alpha)$, $(3, 4)$ and $(5\sin\alpha, -5\cos\alpha)$, then locus of the orthocentre of variable triangle is given by :

(a) $x^2 + y^2 + 6x + 8y - 25 = 0$

(b) $x^2 + y^2 - 6x + 8y - 25 = 0$

(c) $x^2 + y^2 - 6x - 8y - 25 = 0$

(d) $x^2 + y^2 + 6x + 8y + 25 = 0$

20. Let the points A, B, C be $(0, 8)$, $(0, 0)$ and $(4, 0)$ respectively, and $'P'$ is a moving point such that area of ΔPAB is four times the area of ΔPBC, then locus of point $'P'$ is given by :

(a) $x - 2y = 0$ (b) $x^2 - 4y^2 = 0$

(c) $x^2 - 16y^2 = 0$ (d) $x - 4y = 0$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let points $P(a\cos\alpha, a\sin\alpha)$, $Q(a\cos\beta, a\sin\beta)$ and $R(a\cos\gamma, a\sin\gamma)$ form an equilateral triangle, then :

(a) $\tan\alpha + \tan\beta + \tan\gamma = 0$

(b) $\sin\alpha + \sin\beta + \sin\gamma = 0$

(c) $\cos\alpha + \cos\beta + \cos\gamma = 0$

(d) $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = -3/2$

22. Let point $P(\alpha, \alpha^2)$ lies inside the triangle which is having its sides along the lines $2x + 3y - 1 = 0$, $x + 2y - 3 = 0$ and $6y - 5x + 1 = 0$. If $'S'$ is the exhaustive set for the real values of α, then $'S'$ contains :

(a) $\left(-2, -\frac{1}{\pi}\right)$ (b) $\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$

(c) $\{\sqrt{e}\}$ (d) $\left(-\sqrt{2}, -\frac{\pi}{3}\right)$

23. Let three line L_1, L_2, L_3 intersect each other at integral points A, B and C, then ΔABC may be :

(a) right-angled triangle. (b) equilateral triangle.

(c) isosceles triangle. (d) scalene triangle.

24. Let $'A'$ and $'B'$ be two fixed points on $x - y$ plane where $|AB| = a$. If $'P'$ is moving point on the plane and

(a) $|PA + PB| = b$, where $b > a$, then locus of P ellipse.

(b) $|PA - PB| = b$, where $b > a$, then locus of P is hyperbola.

(c) $|PA + PB| = b$, where $b = a$, then locus of P is line segment.

(d) $|PA - PB| = b$, where $b = a$, then locus of P is line segment.

25. Let three of the vertices of a parallelogram be $(-3, 4)$, $(0, -4)$ and $(5, 2)$, then the fourth vertex can be :

(a) $(8, -6)$ (b) $(-8, -2)$

(c) $(-10, -4)$ (d) $(2, 10)$

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. **Statement 1 :** The points $(k\,,2-2k)$, $(1-k\,,2k)$ and $(-4-k\,,6-2k)$ are collinear for all real values of $'k'$

because

Statement 2 : Area of triangle formed by three collinear points is always zero.

27. **Statement 1 :** Let $\alpha \in \left(0\,,\dfrac{\pi}{2}\right)$ be fixed angle. If $P = (\cos\theta\,,\,\sin\theta)$ and $Q = (\cos(\alpha-\theta)\,,\,\sin(\alpha-\theta))$, then Q is obtained from P by its reflection in the line through origin with slope $\tan\left(\dfrac{\alpha}{2}\right)$

because

Statement 2 : mirror image of point $(\alpha\,,\,\beta)$ about the line $y = x$ is given by the point $(\beta\,,\,\alpha)$.

28. Let $O(0\,,0)$, $P(3\,,6)$ and $Q(6\,,0)$ be the vertices of triangle OPQ and point $'R'$ lies inside the triangle OPQ.

Statement 1 : If the triangles OPR , PQR , OQR are of equal area , then co-ordinates of point $'R'$ is (3 , 2)

because

Statement 2 : In any isosceles triangle ABC , if $'G'$ is the centroid , then triangles AGB , BGC and CGA are always of equal area.

29. Let $A(2\,,\sqrt{3})$, $B(1\,,0)$, $C\,(3\,,0)$ be the vertices of triangle ABC.

Statement 1 : The ratio of circum-radius to in-radius of ΔABC is 2 : 1

because

Statement 2 : In equilateral triangle the ratio of circum-radius to in-radius is always 2 : 1

30. **Statement 1 :** Quadrilateral formed by $y = |x+2| + |x+1| + |x-1| + |x-2|$ and $y - 8 = 0$ is isosceles trapezium

because

Statement 2 : in isosceles trapezium , the non-parallel sides are always of equal length.

ANSWERS

Exercise No. (1)

1. (d)
2. (b)
3. (c)
4. (b)
5. (c)
6. (d)
7. (d)
8. (c)
9. (b)
10. (a)
11. (d)
12. (c)
13. (b)
14. (b)
15. (d)
16. (c)
17. (d)
18. (b)
19. (c)
20. (b)
21. (b , c , d)
22. (b , d)
23. (a , c , d)
24. (a , c)
25. (a , b , d)
26. (d)
27. (b)
28. (a)
29. (a)
30. (a)

Chapter
23

Straight Lines

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-20)

1. In ΔABC, the vertex point A is $(-1, 2)$ and $y^2 - x^2 = 0$ represent the combined equation of the perpendicular bisectors of AB and AC, then area of ΔABC is given by :

(a) 4 sq. units (b) 3 sq. units
(c) 12 sq. units (d) 6 sq. units

2. Let $2x + 3y = 6$ meets the x-axis and y-axis at $'A'$ and $'B'$ respectively, a variable line $\frac{x}{a} + \frac{y}{b} = 1$ meets the x-axis and y-axis at $'P'$ and $'Q'$ respectively in such a way that lines BP and AQ always meet at right angle at R, then locus of orthocentre of ΔARB is :

(a) $x^2 + y^2 - 3x - 2y = 0.$ (b) $x^2 + y^2 = 4.$
(c) $x^2 + y^2 + 3x - 2y = 0.$ (d) $x^2 + y^2 - 3x + 2y = 0.$

3. Let $'P'$ be a point on the line $y + 2x = 1$ and Q, R be two points on the line $3y + 6x = 6$ such that triangle PQR is an equilateral triangle, then length of the side of triangle is :

(a) $\sqrt{\frac{15}{4}}$ (b) $\sqrt{\frac{4}{15}}$
(c) $\frac{5}{\sqrt{15}}$ (d) $\frac{3}{\sqrt{15}}$

4. If line $(y - 7) + k(x - 4) = 0$ cuts $2x + y + 4 = 0$ and $4x + 2y - 12 = 0$ at $'P'$ and $'Q'$ respectively, where $|PQ| = 2\sqrt{5}$, then value of $'k'$ is :

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) $\frac{1}{\sqrt{3}}$ (d) 2

5. The co-ordinates of point $'P'$ on the line $2x + 3y + 1 = 0$, such that $|PA - PB|$ is maximum, where A is $(2, 0)$ and B is $(0, 2)$, is

(a) $(7, -5)$ (b) $(4, -3)$
(c) $(10, -7)$ (d) none of these

6. Let a variable line be drawn through O $(0, 0)$ to meet the lines $y - x - 10 = 0$ and $y - x - 20 = 0$ at the points A and B respectively. If a point P is taken on variable line such that $OP = \frac{2(OA)(OB)}{(OA)+(OB)}$, then the locus of P is :

(a) $3y - 3x - 40 = 0$ (b) $3x + 3y + 40 = 0$
(c) $3x + 3y - 40 = 0$ (d) $3x - 3y - 40 = 0$

7. The line $(p + 2q)x + (p - 3q)y = p - q$, for different values of p and q passes through a fixed point which is given by :

(a) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (b) $\left(\frac{2}{5}, \frac{2}{5}\right)$
(c) $\left(\frac{3}{5}, \frac{3}{5}\right)$ (d) $\left(\frac{2}{5}, \frac{3}{5}\right)$

8. If the lines $y = m_1x + c_1$ and $y = m_2x + c_2$, where $m_1, m_2 \neq 0$, meet the co-ordinate axes at four concylic points, then value of m_1m_2 is equal to :

(a) 2 (b) –1 (c) 1 (d) –2

9. If line $y = \sqrt{5}x$ meets the lines $x - r = 0$, where $r = 1, 2, 3, ...n$, at points A_r respectively, then $\sum_{r=1}^{n}(OA_r)^2$ is equal to :

(a) $3n^2 + 3n$ (b) $2n^3 + 3n^2 + n$
(c) $3n^3 + 3n^2 + n$ (d) $3n^3 + 3n^2 + 2$

10. If the point $P(a^2, a)$ lies in region corresponding to the acute angle between lines $2y = x$ and $4y = x$, then $'a'$ belongs to :

(a) $(2, 6)$ (b) $(4, 6)$
(c) $(2, 4)$ (d) $(4, 8)$

11. The locus of the orthocentre of the triangle formed by the lines $(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$, is

(a) a hyperbola (b) a parabola
(c) an ellipse (d) a straight line

12. Let triangle ABC be right angled at vertex $B(x , y)$ where vertex A and C are given by $(-4 , 2)$ and $(-1 , -2)$ respectively. If area of ΔABC is 6 square units , then number of locations for point $'B'$ is/are :

(a) 1 (b) 0

(c) 2 (d) 4

13. If the vertices of a triangle are $A(1 , 4)$, $B(5 , 2)$ and $C(3 , 6)$, then equation of the bisector of the $\angle ABC$ is given by :

(a) $x - y = 3$ (b) $y + x = 7$

(c) $x + y = 2$ (d) $y = x + 1$

14. If line $K(y - 3) + (x - 2) = 0$ forms an intercept of length 3 units in between the lines $y + 2x - 2 = 0$ and $y + 2x - 5 = 0$, then value of $'K'$ can be :

(a) $\frac{4}{3}$ or 0 (b) only $\frac{4}{3}$

(c) only 0 (d) $\frac{4}{3}$ or ∞

15. If two equal sides AB and AC of an isosceles triangle are given by $x + y - 3 = 0$ and $7x - y + 3 = 0$ respectively and its third side passes through $(1 , -10)$, then equation of line BC can be given by :

(a) $2x + y - 8 = 0$ (b) $3x + 2y - 17 = 0$

(c) $3x + y + 7 = 0$ (d) $x - y - 11 = 0$

16. If a line $L \equiv O$ is drawn through point $P(1, 2)$ so that its point of intersection with the line $x + y - 4 = 0$ is at a distance of $\frac{\sqrt{6}}{3}$ units from point P , then angle of inclination of line $L \equiv O$ may be equal to :

(a) $\frac{\pi}{8}$ (b) $\frac{5\pi}{12}$

(c) $\frac{\pi}{18}$ (d) $\frac{\pi}{6}$

17. Let the point of intersection of the lines $5x + 2y = 9$ and $Kx + y = 3$ be $P(\alpha , \beta)$. If $\alpha \in I$, then number of possible integral values of $'K'$ is/are :

(a) 0 (b) infinite

(c) 4 (d) 8

18. If the straight lines $6x + 3y - 10 = 0$, $6x + Ky - 4 = 0$ and $2x + y - 3 = 0$ are concurrent , then :

(a) $K = 3$ (b) $K \in R$

(c) $K = 1$ (d) $K \in \phi$

19. Let the rectangle $ABCD$ be formed by joining the points given by $(x^2 - 4x)^2 + (y^2 - 3y)^2 = 0$. If a straight line of slope $\frac{1}{2}$ divides the rectangle $ABCD$ into two equal parts , then its equation is given by :

(a) $2y = x + 2$ (b) $2y = x - 1$

(c) $2y = x + 1$ (d) $4y = 2x + 3$

20. Let the line segment PQ be rotated about P by an angle of 60º in the anti-clockwise direction and Q reaches to the new position Q'. If the points P and Q are $(3 , 2)$ and $(4 , 3)$ respectively , where $Q' \equiv (\alpha , \beta)$, then $2\alpha\beta$ is equal to :

(a) 25 (b) 23

(c) 17 (d) none of these

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let $'\alpha'$ and $'\beta'$ be real numbers and the lines $L_1 = 0$, $L_2 = 0$, $L_3 = 0$ form a triangle , then the equation $L_1L_2 + \alpha L_2L_3 + \beta L_3L_1 = 0$ represents

(a) a pair of straight lines if $\alpha = 0$ and $\beta \neq 0$

(b) a pair of straight lines if $\alpha \neq 0$ and $\beta = 0$

(c) a circle for all real values of α and β

(d) a circle for unique real values of α and β

22. If three straight lines $5x + 2y - 12 = 0$, $x + 3y - 5 = 0$ and $3x - \lambda y - 1 = 0$ do not form a triangle , then $'\lambda'$ can be :

(a) -9 (b) 5 (c) $\frac{5}{6}$ (d) $-\frac{6}{5}$

23. Let $\alpha , \beta \in R - \{0\}$, then the equation $(\alpha x^2 + \beta y^2 + \gamma)(x^2 - 6xy + 8y^2) = 0$ represents

(a) two straight lines and a circle if $\alpha = \beta$ and γ is of sign opposite to that of β .

(b) four straight lines if $\gamma = 0$ and α , β are of opposite sign.

(c) two straight lines and a hyperbola if α and β are of same sign and γ is of opposite sign to that of α .

(d) the 2-dimensional plane if $a = \beta = \gamma$.

24. Let the equation $y^3 - x^2y - 2y^2 + 2xy = 0$ represents three straight lines which form a triangle with vertices A , B and C , then

(a) ΔABC is right-angled triangle.

(b) area of ΔABC is 2 square units.

(c) circumcentre of ΔABC is $(1 , 0)$.

(d) ΔABC is isosceles triangle.

25. Let $p \in R$, then lines $(p-2)x + (2p-5)y = 0$, $(p-1)\,x + (p^2-7)\,y - 5 = 0$ and $x + y - 1 = 0$ are :

(a) concurrent for one value of p.

(b) concurrent for no value of p.

(c) parallel for one value of p.

(d) parallel for no value of p.

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. **Statement 1 :** The straight lines represented by $(y-mx)^2 - a^2(1+m^2) = 0$ and $(y-nx)^2 - a^2\,(1+n^2) = 0$ form a rhombus but not a square if $(mn+1)$ is non-zero

because

Statement 2 : All squares are rhombus but all rhombus are not squares.

27. **Statement 1 :** Let $k \in R^+$ and the variable line $y + kx - 4 - 9k = 0$ meets the positive axes at points *'A'* and *'B'* , then absolute minimum value of $OA + OB$, where *'O'* is origin , is 25 units

because

Statement 2 : The minimum area of triangle AOB is 72 square units.

28. Let points $A(0\ ,\ 4)$, $B(-4\ ,\ 0)$ and $C(4\ ,\ 0)$ forms a triangle , where *'D'* is mid-point of BC and *'E'* is the foot of perpendicular from *'D'* on the side AC.
Statement 1 : If *'M'* is the mid-point of ED , then circles which are described with EM and AB as the diameters touch each other externally

because

Statement 2 : AM and BE are perpendicular to each other.

29. **Statement 1 :** Straight lines $m^2x + 4y + 9 = 0$, $x + y = 1$ and $mx + 2y = 3$ are concurrent for exactly one value of *'m'*

because

Statement 2 : If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then $\Delta = 0$ is the necessary and sufficient condition for three lines to be concurrent , where the lines are given by $a_i x + b_i y + c_i = 0$, $i = 1 , 2 , 3$.

30. If ΔABC , let sides AB , BC and CA are given by $x = 0$, $y = 0$ and $x + \sqrt{3}y - 3 = 0$ respectively. The foot of perpendicular from *'B'* to side AC is *'D'*.

Statement 1 : The ratio $CD : DA$ is $3 : 1$

because

Statement 2 : The ratio $AD : DC$ is $\tan C : \tan A$.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

For any two points $A(x_1\,,\,y_1)$ and $B(x_2\,,\,y_2)$ in the x-y plane $d(AB) = |\,x_2 - x_1\,| + |\,y_2 - y_1\,|$. Let moving point $P(x\,,\,y)$, where $x \ge 0$ and $y \ge 0$, satisfy the condition $d(OP) + d(PQ) = 9$. If point $'Q'$ is $(4\,,\,3)$ and $'O'$ represents the origin , then answer the following questions.

1. Locus of moving point $'P'$ consists of the union of :

(a) two line segments.

(b) one line segment and an infinite ray parallel to y-axis.

(c) one line segment and an infinite ray parallel to x-axis.

(d) three line segments.

2. Area of region enclosed by the locus of moving point $'P'$ with the line $x + y = 5$ is equal to :

(a) $\frac{11}{2}$ square units (b) $\frac{15}{2}$ square units

(c) $\frac{7}{2}$ square units (d) $\frac{21}{2}$ square units

3. If the pair of lines $xy - 3x - 4y + 12 = 0$ form a triangle $'\Delta'$ with the locus of moving point $'P'$, then the circumcentre of $'\Delta'$ is :

(a) $\left(\frac{9}{2}\,,\,4\right)$ (b) $\left(\frac{9}{2}\,,\,\frac{7}{2}\right)$

(c) $\left(\frac{7}{2}\,,\,2\right)$ (d) $\left(\frac{7}{2}\,,\,\frac{5}{2}\right)$

Comprehension passage (2)
(Questions No. 4-6)

Let $\theta \in \left(-\frac{\pi}{2}\,,\,\frac{\pi}{2}\right)$ and $\alpha = 2\cos\theta + \sin\theta + 1$, $\beta = \cos\theta + 4\sin\theta + 1$ and $\gamma = 2\sin\theta - 3\cos\theta - 1$.
If $\alpha y - \beta x + \gamma = 0$ represents a family of straight lines $'L'$, then answer the following questions.

4. If the family of straight lines $'L'$ pass through a fixed point $'A'$, then point $'A'$ lies on the curve of :

(a) $y = log_3(x+5)$

(b) $y = \min\{2\,|\,x\,|\,,\,\sin x\}$

(c) $y = sgn\,(e^x)$

(d) $y = \dfrac{2^x + 2^{-x}}{2} + \left|\dfrac{2^x - 2^{-x}}{2}\right|$

5. If a member of the family of straight lines $'L'$ with negative slope meets the co-ordinate axes at $'P'$ and $'Q'$, then minimum area of triangle POQ , where $'O'$ is origin , is given by :

(a) 2 square units (b) 6 square units

(c) 4 square units (d) 8 square units

6. If $(1+\lambda)y + (1-\lambda)x - (7+3\lambda) = 0$ represents the family of lines $'M'$, then straight line which is common member of $'L'$ and $'M'$ is given by :

(a) $y + 2x = 9$ (b) $y - 2x = 1$

(c) $y = 3x - 1$ (d) $x - 2y + 8 = 0$

Comprehension passage (3)
(Questions No. 7-9)

Consider straight lines $L_1 : y - x = 0$, $L_2 : y + x = 0$ and a moving point $P(x\,,\,y)$. Let $d\,(\,P\,,\,L_i)$ represents the distance of $'P'$ from the line L_i , where $i \in \{1\,,\,2\}$. If point $'P'$ moves in region $'R'$ in such a way so that the inequality $2 \le d(P\,,\,L_1) + d(P\,,\,L_2) \le 4$ is satisfied , then answer the following questions.

7. If $d\,(P\,,L_1) = d\,(P\,,L_2)$, then locus of moving point $'P'$ is given by :

(a) $x^2 + y^2 = 0$

(b) $xy = 0$

(c) $x^2 - y^2 = 0$

(d) $x^2 + y^2 - xy = 0$

8. Area (in square units) of region $'R'$ is :

(a) 48 (b) 24

(c) 12 (d) 20

9. If the line $x + y = k$ divides the area of region $'R'$ in the ratio $1 : 3$, then value of $'k'$ can be :

(a) 2 (b) $\sqrt{2}$

(c) –2 (d) $-2\sqrt{2}$

Questions with Integral Answer :
(Questions No. 10-14)

10. Let $A(x_1 , y_1)$, $B(x_2 , y_2)$ and $C(x_3 , y_3)$ be the vertices of triangle ABC and the line $'L'$ is given by $ax + bx + c = 0$. If the centroid of triangle ABC is $(0 , 0)$ and the algebraic sum of the lengths of the perpendiculars from the vertices of ΔABC on the line $'L'$ is 1 , then value of $\left(\dfrac{a^2+b^2}{c^2}\right)^{1/2}$ is equal to ....

11. In triangle ABC, let $x-1=0$ and $x-y-1=0$ be the angular bisectors of the internal angles $'B'$ and $'C'$ respectively. If vertex $'A'$ is $(4 , -1)$ and the length of side BC is $P\sqrt{P}$ units , then value of $'P'$ is equal to ....

12. Let $x+y=k^2$, $k \neq 0$, meets the x-axis and y-axis at A and B respectively , and triangle APQ is inscribed in triangle OAB with right angle at Q , where $'O'$ is origin. If P and Q lie on OB and AB respectively , and area of triangle OAB is $\dfrac{8}{3}$ times the area of triangle APQ , then $\dfrac{QA}{QB}$ is equal to ....

13. If from point $P(4 , 4)$ perpendiculars to the straight lines $3x+4y+5=0$ and $y=mx+7$ meet at Q and R respectively and area of triangle PQR is maximum , then the value of $6m$ is equal to ....

14. In variable triangle PQR , let moving point $'P'$ be (h , k) and the fixed points $'Q'$ and $'R'$ are $(3 , 0)$ and $(6 , 0)$ respectively. If QP and RP meets the y-axis at $'M'$ and $'N'$ respectively and QN meets OP at $'T'$, then MT passes through a fixed point $(p , 0)$, where $'|p|'$ is equal to ....

Matrix Matching Questions :
(Questions No. 15-17)

15. Let $L_1 : (3\cos\theta)x + (4\sin\theta)y = 12$ and $L_2 : (4\sec\theta)x - (3\operatorname{cosec}\theta)y = 7$ be two variable straight lines , where $\theta \in (0 , 2\pi) - \left\{\dfrac{\pi}{2} , \dfrac{3\pi}{2}\right\}$. Match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) Minimum area (in square units) of triangle formed by line $'L_1'$ with the co-ordinate axes is :	(p) $1\frac{1}{48}$
	(q) 5
(b) Maximum area (in square units) of triangle formed by line $'L_2'$ with the co-ordinate axes is :	(r) 7
(c) If line $'L_1'$ meets the co-ordinate axes at A and B , then minimum length (in units) of AB is :	(s) 12
(d) If $'L_1'$ and $'L_2'$ meets at point (α , β) , then absolute maximum value of $(\alpha+\beta)$ is	(t) 10

16. Consider the straight lines ,

$$L_1 : px + qy + r = 0$$
$$L_2 : qx + ry + p = 0$$
$$L_3 : rx + py + q = 0.$$

If $\alpha = p+q+r$ and $\beta = p^2+q^2+r^2-pq-qr-rp$, then match the following columns for the conditions on α , β and the nature of set of lines L_1 , L_2 and L_3.

Column (I)	**Column (II)**
(a) $\alpha = 0$ and $\beta \neq 0$	(p) L_1 , L_2 and L_3 are concurrent.
(b) $\alpha \neq 0$ and $\beta \neq 0$	(q) L_1 , L_2 and L_3 are identical.
(c) $\alpha = 0$ and $\beta = 0$	(r) L_1 , L_2 and L_3 form a triangle.
(d) $\alpha \neq 0$ and $\beta = 0$	(s) L_1 , L_2 and L_3 represent the complete 2-dimensional $x-y$ plane.

17. Let there exist exactly $'n'$ lines which are at a distance of 4 units from point $'A'$ and 1 unit from point $'B'$, then match the following columns for the values of $'n'$ with the points $'A'$ and $'B'$.

Column (I)	**Column (II)**
(a) $A \equiv (2\,,-2)$ and $B \equiv (6\,,1)$	(p) $n=2$
(b) $A \equiv (-2\,,5)$ and $B \equiv (3\,,1)$	(q) $n=4$
(c) $A \equiv (-1\,,-1)$ and $B \equiv (2\,,1)$	(r) $n=1$
(d) $A \equiv (5\,,1)$ and $B \equiv (2\,,1)$	(s) $n=3$
	(t) $n=0$

ANSWERS

Exercise No. (1)

1. (b)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (a)
6. (a)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (c)
11. (d)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (c)
16. (b)	**17.** (c)	**18.** (d)	**19.** (c)	**20.** (d)
21. (a , b , d)	**22.** (a , b , d)	**23.** (a , b)	**24.** (a , c , d)	**25.** (b , c)
26. (b)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (a)

ANSWERS

Exercise No. (2)

1. (d)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)
6. (c)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (3)
11. (5)	**12.** (3)	**13.** (8)	**14.** (2)	
15. (a) → s (b) → p (c) → r (d) → q	**16.** (a) → p (b) → r (c) → s (d) → q	**17.** (a) → s (b) → q (c) → p (d) → r		

Chapter

24

Pair of Straight Lines

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-15)

1. One of the angular bisector of pair of lines $a(x-1)^2+2h(x-1)(y-2)+b(y-2)^2=0$ is $x+2y-5=0$, then other bisector is :

(a) $y-2x=0$ (b) $y+2x=0$

(c) $2x+y-4=0$ (d) $x-2y+3=0$

2. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines equidistant from origin , then

(a) $f^4+g^4=c(bf-ag)$

(b) $f^4-g^4=c(bf^2-ag^2)$

(c) $f^4+g^4=c(bf^2+ag^2)$

(d) none of these

3. If the pair of lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ intersects on the y-axis , then

(a) $2fgh=bg^2+ch^2$ (b) $bg^2\neq ch^2$

(c) $abc=2fgh$ (d) None of these

4. The lines represented by $3ax^2+5xy+(a^2-2)y^2=0$ are perpendicular to each other for

(a) two values of a.

(b) $\forall\ a\in R$.

(c) one value of a.

(d) no values of a.

5. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along the diameter of a circle and divide the circle into four sectors such that area of one of the sector is thrice the area of another sector , then

(a) $3a^2-2ab+3b^2=0$

(b) $3a^2-10ab+3b^2=0$

(c) $3a^2+2ab+3b^2=0$

(d) $3a^2+10ab+3b^2=0$

6. The area (in sq. units) of quadrilateral formed by the pair of straight lines $2x^2-3xy+y^2=0$ and $y^2-3xy+2x^2-4x+6y-16=0$ is given by :

(a) 8 (b) 16

(c) 32 (d) 20

7. If the lines $y^2-5xy+6x^2=0$ and $2y+x-4=0$ form a triangle , then its circumcentre is given by :

(a) $\left(\frac{3}{5},\frac{6}{5}\right)$ (b) $\left(\frac{2}{5},\frac{4}{5}\right)$

(c) $\left(\frac{2}{7},\frac{6}{7}\right)$ (d) $(0,0)$

8. The equation of circumcircle of the triangle formed by the pair of lines $7x^2+8xy-y^2=0$ and the line $2x+y=1$ is given by :

(a) $5x^2+5y^2-18x-12y=0$

(b) $4x^2+4y^2-17x-2y=0$

(c) $2x^2+2y^2-5x-10y=0$

(d) none of these

9. If the lines represented by $(1+K)x^2-8xy+y^2=0$ and $x^2+2Kxy+2y^2=0$ are equally inclined with each other in opposite directions , then value of 'K' is :

(a) ±1 (b) ±4

(c) ±3 (d) ±2

10. Two lines represented by the equation $x^2-y^2-2x+1=0$ are rotated about the point $(1,0)$, the line making the bigger angle with the positive direction of the x-axis being turned by 45° in the clockwise sense and the other line being turned by 15° in the anti-clockwise sense. The combined equation of the pair of lines in their new positions is

(a) $\sqrt{3}x^2-xy+2\sqrt{3}x-y+\sqrt{3}=0$

(b) $\sqrt{3}x^2-xy-2\sqrt{3}x+y+\sqrt{3}=0$

(c) $\sqrt{3}x^2-xy-2\sqrt{3}x+\sqrt{3}=0$

(d) $\sqrt{3}x^2-xy+y+\sqrt{3}=0$

11. If pair of lines $3x^2-2pxy-3y^2=0$ and $5x^2-2qxy-5y^2=0$ are such that each pair bisects the angle between the other pair , then pq is equal to :

(a) –1 (b) –5

(c) –20 (d) –15

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12. If the pair of angular bisectors of the lines $y^2 - 3xy + 2x^2 - 4x + 6y - 16 = 0$ forms a triangle with the line $3x + 4y = 12$, then the orthocentre of triangle is given by :

(a) $(5 , 8)$ (b) $(12 , 10)$

(c) $(10 , 12)$ (d) $(8 , 5)$

13. If the pair of straight line given by $2x^2 - 3xy + y^2 = 0$ is shifted to new origin (5 , 6) without any rotation , then new pair of straight lines is given by :

(a) $2x^2 + y^2 - 3xy + 2x - 3y + 4 = 0.$

(b) $y^2 - 3xy + 2x^2 - 2x - 3y - 4 = 0.$

(c) $y^2 - 3xy + 2x^2 - 2x + 3y - 4 = 0.$

(d) $x^2 + 3xy + 2y^2 - 2x + 3y - 4 = 0.$

14. If the eqution $2x^2 - 3xy + y^2 - 4x + 6y + 32 \sin\theta = 0$ represents a pair of straight lines , then possible value of $'\theta'$ is :

(a) $\dfrac{2\pi}{3}$ (b) $\dfrac{5\pi}{6}$

(c) $\dfrac{11\pi}{6}$ (d) $\dfrac{5\pi}{4}$

15. If the straight lines joining the origin to the points of intersection of $x - y = k$ and the curve $5x^2 + 12xy - 8y^2 + 8x - 4y + 12 = 0$ make equal angles with x-axes , then the value of *'k'* can be :

(a) 1 (b) –3

(c) 2 (d) 4

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. Let area of triangle formed by the intersection of a line parallel to x-axis and passing through $P(\alpha , \beta)$ with pair of lines $y^2 - x^2 - 2y + 2x = 0$ be $4\alpha^2$ square units , then locus of point *'P'* is given by :

(a) $y - 2x = 1$ (b) $y - 2x = 2$

(c) $y + 2x = 3$ (d) $y + 2x = 1$

17. Let all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$, which subtend a right angle at the origin , pass through a fixed point *'P'* , then *'P'* lie on the curves :

(a) $x^2 + y + 1 = 0$ (d) $y^2 = x + 2$

(c) $x^2 + y^2 = 5$ (d) $xy + 2 = 0$

18. Let the equations of the pair of opposite sides of parallelogram be $x^2 - 6x + 8 = 0$ and $y^2 - 4y + 3 = 0$, then equations of the diagonal of parallelogram are given by :

(a) $y - x + 1 = 0$ (b) $y = x + 2$

(c) $y + x + 4 = 0$ (d) $x + y = 5$

19. If $12x^2 + kxy + 2y^2 + 11x - 5y + 2 = 0$ represents a pair of straight lines , then angle between the lines can be given by :

(a) $\tan^{-1}\left(\dfrac{31}{25}\right)$

(b) $\tan^{-1}\left(\dfrac{1}{7}\right)$

(c) $\tan^{-1}\left(\dfrac{29}{28}\right)$

(d) $\tan^{-1}\left(\dfrac{4}{9}\right)$

20. If two of the lines represented by the equation $ax^4 + bx^3y + cx^2y^2 + dxy^3 + ay^4 = 0$ bisect the angles between the other two lines , then

(a) $6a + 5c = 0$ (b) $b + d = 0$

(c) $b + 2d = 0$ (d) $c + 6a = 0$

Assertion Reasoning questions :
(Questions No. 21-25)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. Statement 1 : Orthocentre of triangle formed by the pair of angular bisectors of $2x^2 + 3xy + y^2 - 10x - 7y + 12 = 0$ and the line $3x + 4y - 5 = 0$ is $(1 , 2)$.

because

Statement 2 : Angular bisectors are always perpendicular to each other and triangle formed by them with any line is right angled triangle.

22. Statement 1 : If line $3x + 4y - 5 = 0$ meets the curve $2x^2 + 3y^2 - 5 = 0$ at P and Q , where *'O'* is origin , then $\angle POQ = 60^\circ$

because

Statement 2 : The equation $10x^2 + 15y^2 = (3x + 4y)^2$ represents a pair of straight lines which meets at origin and passes through the points P and Q.

23. Statement 1 : Let a pair of mutually perpendicular lines $S = 0$ be drawn through the origin which forms an isosceles triangle Δ with the line $2x + 3y = 6$, then area of Δ is 3 square units

because

Statement 2 : Pair of lines $S = 0$ is given by $5x^2 - 24xy - 5y^2 = 0$

24. Statement 1 : Let the lines L_1 and L_2 be the angular bisectors of the pair of lines $ax^2 + 2h\,xy + b\,y^2 = 0$, then the angular bisectors of L_1 and L_2 is given by $(a - b)(x^2 - y^2) + 2h\,xy = 0$

because

Statement 2 : Combined equation of L_1 and L_2 is given by $h\,(x^2 - y^2) + (b - a)\,xy = 0$.

25. Statement 1 :
If $3ky^2 + 4x^2 + (2k + 6)\,xy - 4x - (9 - k)\,y - 3 = 0$ represents a pair of parallel lines , then value of $'k'$ is 3

because

Statement 2 : The distance between the given pair of lines is $\sqrt{16/13}$ units.

ANSWERS Exercise No. (1)

1. (a)
2. (b)
3. (a)
4. (a)
5. (c)
6. (b)
7. (c)
8. (d)
9. (d)
10. (b)
11. (d)
12. (c)
13. (c)
14. (c)
15. (c)
16. (a , d)
17. (a , c , d)
18. (a , d)
19. (b , c)
20. (b , d)
21. (a)
22. (d)
23. (d)
24. (d)
25. (b)

Chapter 25

Circles

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-25)

1. Let C_1 and C_2 be two concentric circles , smaller circle C_1 divides the larger circle C_2 into two regions of equal area , where radius of C_1 is 2 units , then length of tangent from any point $'P'$ on the circle C_2 to circle C_1 is :

(a) 1 unit (b) $\sqrt{2}$ units

(c) 2 units (d) 3 units

2. If tangent at any point $'P'$ on the circle $x^2 + y^2 = 9$ cuts the circle $x^2 + y^2 = 25$ at A and B , then in-radius of ΔAOB , where $'O'$ being the origin , is :

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$

(c) 2 (d) $\frac{4}{3}$

3. Let a variable circle touches a fixed straight line and cuts off an intercept of length 4 units on other fixed straight line which is perpendicular to the first line , then locus of the centre of circle is :

(a) hyperbola. (b) parabola.

(c) straight line. (d) ellipse.

4. Tangents PQ and PR are drawn to the circle $(x+4)^2 + y^2 = 1$ from point $P(4 , 4)$, then circumcentre of ΔPQR is :

(a) $(0 , 1)$ (b) $(0 , 2)$

(c) $(0 , 3)$ (d) $\left(\frac{1}{2} , 2\right)$

5. If a circle of radius 3 units is touching the pair of lines $\sqrt{3}y^2 - 4xy + \sqrt{3}x^2 = 0$ in the Ist quadrant , then length of chord of contact to the circle is :

(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{\sqrt{3}+1}{\sqrt{2}}$

(c) $3\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$ (d) $\frac{3}{2}\left(\sqrt{3}+1\right)$

6. From any point $'P'$ on the circle $x^2 + y^2 = 9$, tangents to the circle $x^2 + y^2 = 1$ are drawn which meets $x^2 + y^2 = 9$ at $'A'$ and $'B'$, locus of the point of intersection of tangents at $'A'$ and $'B'$ to the circle $x^2 + y^2 = 9$ is :

(a) $x^2 + y^2 = \left(\frac{27}{7}\right)^2$ (b) $x^2 - y^2 = \left(\frac{25}{6}\right)^2$

(c) $y^2 - x^2 = \left(\frac{27}{7}\right)^2$ (d) $x^2 + y^2 = \left(\frac{25}{6}\right)^2$

7. If common tangent is not possible for the curves $x^2 + y^2 = r^2$ and $16x^2 + 4y^2 = 64$, then :

(a) $r \in [2 , 4]$ (b) $r \in R - (2 , 4)$

(c) $r \in (4 , \infty)$ (d) $r \in (2 , \infty)$

8. If $y = mx + 2\sqrt{1+m^2}$, where $m \neq 0$, is common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 4ax + 4a^2 - 4 = 0$, $a > 2$, then value of $'m'$ is :

(a) $\frac{2}{\sqrt{a^2-4}}$ (b) $\frac{-2}{\sqrt{a^2-4}}$

(c) $\pm\frac{2}{\sqrt{a^2-4}}$ (d) $\frac{-4}{\sqrt{a^2-4}}$

9. If the line $3x - 4y = 33$ cuts the circle $x^2 + y^2 + 2x - 2y - 98 = 0$ at $'A'$ and $'B'$, where $'C'$ is the centre of the circle , then in-radius of ΔABC is :

(a) 5 units (b) 3 units

(c) 1 unit (d) 8 units

10. If $'C_2'$ is director circle of circle $'C_1'$, then angle between the pairs of tangents drawn from any point on the director circle of $'C_2'$ to $'C_1'$ is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{8}$

11. If a member of family of lines $ax + by + c = 0$, where $a + b + c = 0$, intersects the family of circles $x^2 + y^2 - 4x - 4y + \lambda = 0$ such that the length of chord generated is maximum , then equation of line is :

(a) $x + y = 0$ (b) $y - x + 1 = 0$

(c) $y - x = 0$ (d) $x - 2y = 0$

12. The centre of smallest circle which cuts the circles $x^2+y^2=1$ and $x^2+y^2+8x+8y-33=0$ orthogonally is :

(a) $(2, 2\sqrt{2})$ (b) $(2\sqrt{2}, \sqrt{3})$

(c) $(2, 2)$ (d) $(\sqrt{3}, 2)$

13. Largest circle touching the curve $xy = 1$ at $(1, 1)$ and the co-ordinate axes is given by :

(a) $x^2+y^2+(4+\sqrt{2})x-(4+\sqrt{2})y=0.$

(b) $x^2+y^2-(4+\sqrt{2})x-(4+\sqrt{2})y-2\sqrt{2}=0.$

(c) $x^2+y^2+\sqrt{2}x-(4+\sqrt{2})y+6+2\sqrt{2}=0.$

(d) none of these.

14. If a circle of diameter 6 units is inscribed in quadrilateral $ABCD$, where $CD = 3AB$, $\angle A = \frac{\pi}{2}$ and AB is parallel to CD, then area of quadrilateral $ABCD$ is :

(a) 40 sq units. (b) 48 sq units.

(c) 18 sq units. (d) 50 sq units.

15. Let C_1, C_2 and C_3 be three circles with sides of triangle ABC as their diameter. If the radical axis of the circles C_1, C_2 and C_3 in pairs meet at point $'R'$, then $'R'$ is :

(a) incentre of ΔABC.

(b) circumcentre of ΔABC.

(c) centroid of ΔABC.

(d) orthocentre of ΔABC.

16. From point P, if length of tangents to circles $x^2+y^2=9$; $x^2+y^2+4x+6y-19=0$; and $x^2+y^2-2x-2y-5=0$ are equal, then point $'P'$ is :

(a) $(2, -1)$ (b) $(2, -2)$

(c) $(1, 1)$ (d) $(1, -2)$

17. Locus of foot of perpendicular from origin to chords of circle $x^2+y^2-4x-6y-3=0$ which subtend 90° at origin is :

(a) $2x^2+2y^2-4x-6y-3=0$

(b) $x^2+y^2+4x+6y+3=0$

(c) $2x^2+2y^2+4x+6y-3=0$

(d) none of these

18. Locus of the centre of circle which externally touches the circle $x^2+y^2-6x-6y+14=0$ and also touches the y-axis is :

(a) $x^2-6x-10y+4=0$ (b) $x^2-10x-6y+5=0$

(c) $y^2-6x-10y+14=0$ (d) $y^2-10x-6y+14=0$

19. The centre of circle C_1 lies on $2x-2y+9=0$ and cuts $x^2+y^2=4$ orthogonally, then C_1 passes through two fixed points :

(a) $(1, 1)$ and $(3, 3)$

(b) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ and $(-4, 4)$

(c) $(0, 0)$ and $(5, 5)$

(d) none of these

20. The four points of intersection of lines $(2x-y+1)(x-2y+3)=0$ with co-ordinate axes lie on a circle, then centre of circle is :

(a) $\left(\frac{3}{4}, \frac{5}{4}\right)$ (b) $\left(-\frac{7}{4}, \frac{5}{4}\right)$

(c) $(2, 3)$ (d) none of these

21. The equation of smallest circle passing through intersection of $x+y=1$ and $x^2+y^2=9$ is :

(a) $x^2+y^2+x+y-8=0$

(b) $x^2+y^2-x-y-8=0$

(c) $x^2+y^2-x+y-8=0$

(d) none of these

22. Tangents are drawn to circle $x^2+y^2=12$ at the point where it is met by $x^2+y^2-5x+3y-2=0$; then point of intersection of these tangents is :

(a) $\left(6, \frac{-18}{5}\right)$ (b) $\left(6, \frac{18}{5}\right)$

(c) $\left(\frac{18}{5}, 6\right)$ (d) none of these

23. Tangents drawn from the point $P(1, 8)$ to the circle $x^2+y^2-6x-4y-11=0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is :

(a) $x^2+y^2+4x-6y+19=0$

(b) $x^2+y^2-4x-10y+19=0$

(c) $x^2+y^2-2x+6y-29=0$

(d) $x^2+y^2-6x-4y+19=0$

24. The centre of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is :

(a) 10 (b) 8

(c) 5 (d) 6

25. Two circles with radii *'a'* and *'b'* touch each other externally such that $'\theta'$ is the angle between the direct common tangents , $a > b \geq 2$, then angle $'\theta'$ is equal to :

(a) $\sin^{-1}\left(\frac{a-b}{a+b}\right)$ (b) $\sin^{-1}\left(\frac{a+b}{a-b}\right)$

(c) $2\sin^{-1}\left(\frac{a+b}{a-b}\right)$ (d) $2\sin^{-1}\left(\frac{a-b}{a+b}\right)$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. Let circles $'C_1'$ and $'C_2'$ be $x^2 + y^2 - 2x - 2y = 0$ and $x^2 + y^2 + 6x - 8y = 0$ respectively. If line $y = kx$ intersects the circle C_1 and C_2 at point *'A'* and *'B'* respectively (where *A* and *B* points are not origin) and *'S'* is the set of real values of *'k'* , then *'S'* contains :

(a) $\left(-\frac{3}{4}, \frac{3}{4}\right)$ (b) $\left(\frac{3}{4}, 1\right)$

(c) $\left(0, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, 1\right)$

27. Let a circle of unit radius lies in the first quadrant and touches the *x*-axis and *y*-axis at *'A'* and *'B'* respectively. If a variable line through origin meets the circle at points *'P'* and *'Q'* , where area of ΔPBQ is not maximum , then possible values of the slope of variable line can be :

(a) $\sqrt{2}-1$ (b) 1

(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

28. If tangent of slope $-\frac{4}{3}$ to the circle $25x^2 + 25y^2 = 144$ in first quadrant meets the co-ordinate axes at *'A'* and *'B'* , and *'O'* is the origin , then :

(a) Incentre and orthocentre of ΔAOB are integral points.

(b) Circumcentre and centroid of ΔAOB are integral points.

(c) Incentre of ΔAOB is irrational point.

(d) Circumcentre of ΔAOB is rational point.

29. Let a straight line through the vertex *'A'* of triangle *ABC* meets the side *BC* at the point *'D'* and the circumcircle of ΔABC at the point *'E'*. If point *'D'* is not the circumcentre of ΔABC , then :

(a) $\frac{1}{DA} + \frac{1}{DE} > \frac{4}{AE}$

(b) $\frac{1}{DA} + \frac{1}{DE} > \frac{2}{\sqrt{(DB)(DC)}}$

(c) $AE + BC > 4\sqrt{(AD)(DE))}$

(d) $\frac{1}{BD} + \frac{1}{CD} > \frac{4}{BC}$

30. Let T_1 and T_2 be two tangents drawn from (0 , 3) to the circle $C_1 : x^2 + (y-1)^2 = 1$. If C_2 and C_3 are two circles with centre on *y*-axis and touching C_1 externally and having T_1 and T_2 as their pair of tangents , then :

(a) (radius of C_1) × (radius of C_2) = 1.

(b) distance between the centres of C_1 and C_2 is $\frac{16}{3}$ units.

(c) sum of the area of C_1 and C_2 is 10π square units.

(d) maximum distance between the boundary of C_1 and C_2 is $\frac{26}{3}$ units.

Assertion Reasoning questions : (Questions No. 31-40)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

31. Statement 1 : Maximum number of lines which are at a distance of 3 units for point *'P'* and 2 units from point *'Q'* are four , where *'P'* and *'Q'* points are (–2 , 1) and (2 , 4) respectively

because

Statement 2 : Two mutually external circles can have at the most four common tangents.

32. **Statement 1 :** Let circles $'C_1'$ and $'C_2'$ intersect at two different points P and Q and a line passing through P meet the circles C_1 and C_2 at A and B respectively. If Y is the mid point of AB and QY meets the circle C_1 and C_2 at X and Z respectively , then Y divides XZ in the ratio 1 : 1

because

Statement 2 : if a line through point M intersects a given circle at L and N , then $(ML)(MN)$ is always constant.

33. **Statement 1 :** Let point $P(\alpha\ ,\ \beta)$ be termed as "odd point" when both α and β are odd integers. Number of "odd points" lying on the circle $x^2 + y^2 = 2012$ is zero

because

Statement 2 : if both α and β are odd , then $\alpha^2 + \beta^2$ is of form $8k + 2$, where $k \in W$.

34. Let line $L_1 = 0$ is tangential to a given circle C_1 at fixed point $'P'$. If a variable circle touches both the circle C_1 and line L_1 , then

Statement 1 : Locus of the centre of the variable circle is parabolic

because

Statement 2 : The locus of the centre of the variable circle is straight line if the points of contact with C_1 and L_1 are same.

35. Let circle $'C_1'$ be $x^2 + y^2 - 4x - 6y + 12 = 0$ and a line through point $P\ (-1\ ,\ 4)$ meets the circle $'C_1'$ at two distinct points $'A'$ and $'B'$

Statement 1 : Sum of the distances PA and PB is not less than 6

because

Statement 2 : $a + b \geq 2\sqrt{ab}$ for $a\ ,\ b \in R^+$.

36. **Statement 1 :** Let three circles with centres at A , B and C touch each other externally and $'P'$ is the point of intersection of tangents to these circles at their points of contact , then $'P'$ is the incentre of triangle ABC

because

Statement 2 : ΔABC is always an equilateral triangle in the given set of three circles.

37. **Statement 1 :** From an external point $'P'$ if tangents PA and PB are drawn to a circle with centre at C , then circumcentre of ΔPAB is the mid-point of line segment CP

because

Statement 2 : The image of orthocentre of ΔPAB about the line mirror $'AB'$ lies on the circum-circle of triangle PAB .

38. Let $'C_1'$ and $'C_2'$ be two fixed concentric circles with C_2 lying inside C_1 . A variable circle $'C'$ lying inside $'C_1'$ touches $'C_1'$ internally and $'C_2'$ externally.

Statement 1 : Locus of the centre of variable circle $'C'$ is circular in nature

because

Statement 2 : Locus of the centre of variable circle $'C'$ is elliptical in nature if $'C_1'$ and $'C_2'$ are not concentric.

39. Let A , B , C and D be four distinct points in the $x - y$ plane such that the ratio of the distance of any one of them from the point (1 , 0) to the distance from the point (–1 , 0) is equal to $\frac{1}{3}$.

Statement 1 : Quadrilateral formed by the points A , B , C and D is concyclic

because

Statement 2 : There exists a unique circle which passes through any three given points.

40. **Statement 1 :** Let a variable circle with centre $'C'$ always touches the x-axis and it touches the circle $x^2 + y^2 = 1$ externally , then locus of the centre $'C'$ is given by $x^2 - 2y - 1 = 0$, where $|\,x\,| \nless 1$

because

Statement 2 : Parabolic curve is the locus of a point which is always equidistant from a fixed point $'F'$ and a fixed line $'D'$, where $'F'$ doesn't lie on the line $'D'$.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ , QR , RP are D , E , F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. If the origin and the centre of C are on the same side of the line PC , then answer the following questions..

1. The equation of circle C is :

(a) $\left(x-2\sqrt{3}\right)^2 + (y-1)^2 = 1$

(b) $\left(x-2\sqrt{3}\right)^2 + \left(y+\frac{1}{2}\right)^2 = 1$

(c) $\left(x-\sqrt{3}\right)^2 + (y+1)^2 = 1$

(d) $\left(x-\sqrt{3}\right)^2 + (y-1)^2 = 1$

2. Points E and F are given by :

(a) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\sqrt{3}, 0\right)$ (b) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\sqrt{3}, 0\right)$

(c) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

3. Equations of the sides QR , RP are :

(a) $y = \frac{2}{\sqrt{3}}x + 1 , \ y = -\frac{2}{\sqrt{3}}x - 1$

(b) $y = \frac{1}{\sqrt{3}}x , \ y = 0$

(c) $y = \frac{\sqrt{3}}{2}x + 1 , \ y = -\frac{\sqrt{3}}{2}x - 1$

(d) $y = \sqrt{3}x , \ y = 0$

Comprehension passage (2)
(Questions No. 4-6)

Let tangents PA and PB be drawn to the circle $(x+3)^2 + (y-4)^2 = 1$ from a variable point $'P'$ on the curve $y = \sin x$. If the locus of circumcentre of triangle PAB is given by the curve $y = f(x)$, then answer the following questions :

4. If set $S = \{y : y = [f(x)] , x \in R\}$, where $[.]$ represents the greatest integer function , then total number of elements in set $'S'$ is / are :

(a) 3 (b) 1

(c) 2 (d) 4

5. Let $g(x) = \lambda^2 \, |f(x) - 2| + (6\lambda - 8) \left| f\left(\frac{\pi}{4} + x\right) - 2 \right|$,

where the fundamental period of $g(x)$ is $\frac{\pi}{4}$ then the values of λ can be :

(a) 2 or 3 (b) 2 or 6

(c) 2 or 4 (d) 3 or 6

6. Total number of integral solutions for the equation $f(x) - e^{-|x|} = 0$ is /are :

(a) 1 (b) 0

(c) 2 (d) 4

Comprehension passage (3)
(Questions No. 7-9)

Let circle $'C'$ of unit radius touches the y-axis at point A and centre Q of the circle lies in the IInd quadrant. The tangent from origin $'O'$ to the circle touches it at $'T'$ and point $'P'$ lies on it such that ΔOAP is right angled at $'A'$. If the semi-perimeter of ΔOAP is 4 units , then answer the following questions.

7. Length of QP is equal to :

(a) $\frac{3}{4}$ (b) $\frac{3}{2}$

(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

8. Equation of circle $'C'$ is :

(a) $(x+1)^2 + (y-3)^2 = 1$ (b) $(x+1)^2 + \left(y-\frac{5}{2}\right)^2 = 1$

(c) $(x+1)^2 + (y-2)^2 = 1$ (d) $(x+1)^2 + (y-4)^2 = 1$

9. If circle $x^2 + (y-2)^2 = 2$ meets the circle $'C'$ at $'M'$ and $'N'$, then length of MN is equal to :

(a) 2 (b) 1 (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

Comprehension passage (4)
(Questions No. 10-12)

Let line $'L'$ meets the circle $x^2+y^2=25$ at the points $'A'$ and $'B'$, where $PA = PB = 8$ and point $'P'$ is $(3\,,4)$. If the family of circles passing through A and B is represented by C_F , then answer the following questions :

10. If a member of C_F passes through the point $(-4\,,-4)$, then its equation is given by :

(a) $x^2+y^2-2x-4y-56=0$

(b) $3x^2+3y^2+3x+4y-68=0$

(c) $2x^2+2y^2+5x-6y-68=0$

(d) $x^2+y^2+3x-4y-12=0$

11. If a member of C_F is having minimum area , then its radius is given by :

(a) 5 (b) $\frac{28}{5}$

(c) $\frac{24}{5}$ (d) $\frac{27}{4}$

12. If tangents drawn at A and B to the member of C_F having centre at $'P'$ meets at point Q , then coordinates of $'Q'$ is given by :

(a) $(-4\,,-3)$. (b) $(-3\,,-4)$.

(c) $(-5\,,-2)$. (d) $(-3\,,3)$.

Questions with Integral Answer :
(Questions No. 13-17)

13. Let $'C_F'$ represents the family of circles passing through the points $A(6\,,5)$ and $B(3\,,7)$. If the common chords of circle $x^2+y^2-4x-6y-3=0$ and $'C_F'$ passes through a fixed point $P(\alpha\,,\beta)$, then value of $\sqrt{\alpha+3\beta}$ is equal to ....

14. Let tangents PA and PB be drawn from point $P(6\,,8)$ to the circle $x^2+y^2=r^2$. If area of triangle PAB is maximum , then radius $'r'$ is equal to ....

15. Let three circles C_1 , C_2 and C_3 with radii 3 , 4 and 5 respectively touch each other externally at point P_1 , P_2 and P_3 . If circle $'C'$ is the circumcircle of $\Delta P_1P_2P_3$, then value of $\left\{\frac{P_1P_2}{2\sin P_3}\right\}^2$ is equal to ....

16. Let circle $'C'$ passes through the point $P(1\,,-1)$ and is orthogonal to the circle which is having $(-2\,,3)$ and $(0\,,-1)$ as the diametric ends. If tangent at $'P'$ to the circle $'C'$ is $2x+3y+1=0$ and the length of x-intercept for is $'l'$ units , then value of [l] , where [.] represents the greatest integer function , is equal to ....

17. Let square $ABCD$ be inscribed in the circle $2x^2+2y^2-12x-8y+25=0$ and the variable points P , Q , R and S lie on the sides AB , BC , CD and DA respectively. If α , β , γ and δ denote the length of sides of quadrilateral $PQRS$, then minimum value of $\alpha^2+\beta^2+\gamma^2+\delta^2$ is equal to ....

Matrix Matching Questions :
(Questions No. 18-21)

18. Let curves C_1 and C_2 be the circumscribing and inscribing circles respectively for the quadrilateral $ABCD$, where the vertex points A , B , C and D in order are given by $(2\,,1)$, $(3\,,1)$, $(3\,,2)$ and $(2\,,2)$. Match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) Area (in square units) of $'C_2'$ is	(p) $\frac{\pi}{4}\left(3+2\sqrt{2}\right)$
(b) Area (in square units) of the director circle of $'C_2'$ is	(q) $\frac{\pi}{4}$
(c) Area (in square units) of $'C_1'$ *is*	(r) $\frac{\pi}{2}$
(d) Area (in square units) of incircle of ΔABC is	(s) $\frac{\pi}{2}\left(3-2\sqrt{2}\right)$

19. Match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) Family of circles touching $xy = 4$ at point $(2\,,2)$	(p) $x^2 + y^2 - 4x - 2y + 5 + \lambda(x^2 + y^2 - 5) = 0$ $\lambda \neq -1$
(b) Family of circles touching $x^2 + y^2 = 5$ at $(2\,,1)$	(q) $(x-2)^2 + (y-2)^2 + \lambda(x + y - 4) = 0.\ \lambda \in R.$
(c) Family of circles touching $2x + y - 5 = 0$ at $(2\,,1)$	(r) $(x-2)^2 + (y-1)^2 + \lambda\,(2x + y - 5) = 0.\ \lambda \in R$
(d) Family of circles touching $x^2 + y^2 + 2x + 2y - 16 = 0$ at $(2\,,2)$	(s) $(x-2)^2 + (y-2)^2 + \lambda\left((x+1)^2 + (y+1)^2 - 18\right) = 0$ $\lambda \neq -1$

20. If $'a'$ and $'b'$ satisfy the condition $12a^2 - 4b^2 + 8a + 1 = 0$ and the line $ax + by + 1 = 0$ is tangential to a fixed circle $'C'$, then match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) If $x^2 + y^2 + 2x + 4y - k = 0$ intersects circle $'C'$ orthogonally, then value of k is	(p) $\sqrt{12}$
(b) If $x^2 + y^2 = 12$ intersects the circle $'C'$ at P and Q, then length PQ is	(q) 3
(c) If OA and OB are tangents to circle $'C'$, where $'O'$ is origin, and $'r'$ is in-radius of ΔOAB, then value of $(20)^r$ is	(r) 20
(d) If line $(y+2) = m\,(x+1)$ meets the circle $'C'$ at $'M'$ and $'N'$ for some real value of m, then length MN can be :	(s) $\sqrt{10}$

ANSWERS Exercise No. (1)

1. (c) **2.** (d) **3.** (a) **4.** (b) **5.** (c)
6. (a) **7.** (c) **8.** (b) **9.** (b) **10.** (c)
11. (c) **12.** (c) **13.** (d) **14.** (b) **15.** (d)
16. (c) **17.** (a) **18.** (d) **19.** (b) **20.** (b)
21. (b) **22.** (a) **23.** (b) **24.** (b) **25.** (d)
26. (a , c) **27.** (a , b , d) **28.** (a , d) **29.** (a , b , c , d) **30.** (a , b , d)
31. (d) **32.** (a) **33.** (a) **34.** (b) **35.** (a)
36. (c) **37.** (b) **38.** (b) **39.** (c) **40.** (d)

ANSWERS Exercise No. (2)

1. (c) **2.** (d) **3.** (b) **4.** (c) **5.** (c)
6. (b) **7.** (d) **8.** (c) **9.** (a) **10.** (b)
11. (c) **12.** (b) **13.** (5) **14.** (5) **15.** (5)
16. (4) **17.** (2)

18. (a) → q
(b) → r
(c) → r
(d) → s

19. (a) → q , s
(b) → p , r
(c) → p , r
(d) → q , s

20. (a) → r
(b) → p
(c) → r
(d) → p , q , s

Chapter 26

Parabola

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-20)

1. If straight line $y = mx + c$ is tangential to parabola $y^2 = 16(x+4)$, then exhaustive set of values of 'c' is given by

(a) $R/(-4\ ,\ 4)$ (b) $R/(-8\ ,\ 8)$

(c) $R/(-12\ ,\ 12)$ (d) $R/[-4\ ,\ 4]$

2. Minimum distance between the parabolic curves $y = x^2 + 4$ and $x = y^2 + 4$ is

(a) $\frac{15}{4\sqrt{2}}$ (b) $\frac{15}{2}$

(c) $\frac{15}{\sqrt{2}}$ (d) $\frac{15}{2\sqrt{2}}$

3. Locus of the point of intersection of tangents to parabola $y^2 = 4(x + 1)$ and $y^2 = 8(x + 2)$ which are perpendicular to each other is given by :

(a) $x - 2 = 0$ (b) $x + 2 = 0$

(c) $x + 3 = 0$ (d) $x - 3 = 0$

4. If $\left(3t_i^2\ ,\ -6t_i\right)$ represents the feet of normals to the parabola $y^2 = 12x$ from $(1\ ,\ 2)$, then $\sum_{i=1}^{3}\left(\frac{1}{t_i}\right)$ is equal to :

(a) 6 (b) $-\frac{5}{2}$

(c) $\frac{3}{2}$ (d) -3

5. If chords of contact of the pair of tangents drawn from each point on the line $y = 2x + 3$ to the curve $y^2 - 8x = 0$ are concurrent, then the point of concurrency is :

(a) $(2\ ,\ 0)$ (b) $\left(2,\frac{3}{2}\right)$

(c) $\left(\frac{3}{2},2\right)$ (d) $\left(\frac{2}{3},1\right)$

6. In angle between the pair of tangents drawn from a point 'P' to the parabola $y^2 = 4ax$ is $\frac{\pi}{4}$, then locus of point 'P' is :

(a) parabola. (b) line.

(c) hyperbola. (d) ellipse.

7. From a point 'P' if common tangents are drawn to circle $x^2 + y^2 = 8$ and parabola $y^2 = 16x$, then the area (in sq. units) of quadrilateral formed by the common tangents, the chords of contact of circle and parabola is given by :

(a) 60 (b) 30

(c) 45 (d) 50

8. Let $P(h\ ,\ k)$ lies on the curve $f(x) = x - x^2$, such that $h \in (0\ ,\ 1)$, where 'O' and 'A' are $(0\ ,\ 0)$ and $(1\ ,\ 0)$ respectively, then maximum area of ΔPOA is:

(a) $\frac{1}{8}$ sq. units. (b) $\frac{1}{4}$ sq. units.

(c) $\frac{1}{2}$ sq. units. (d) $\frac{1}{16}$ sq. units.

9. If curves $C_1 : x^2 + y^2 = 5$ and $C_2 : y^2 - 4x = 0$ intersect at 'P' and 'Q' and tangents to curve 'C_1' and 'C_2' at 'P' and 'Q' intersect the x-axis at R and S respectively, then ratio of area of ΔPQR and ΔPQS is :

(a) 1 : 2 (b) 1 : 3

(c) 2 : 3 (d) 1 : 4

10. If tangent at $P(2\ ,\ 4)$ to parabola $y^2 = 8x$ meets the curve $y^2 = 8x + 5$ at Q and R, then mid-point of QR is :

(a) $(2\ ,\ 4)$ (b) $(4\ ,\ 2)$

(c) $(7\ ,\ 9)$ (d) $(2\ ,\ 5)$

11. If two parabola $y^2 = 4ax$ and $y^2 = 4c\,(x - b)$ can-not have common normal other than x-axis, then :

(a) $\frac{a-c}{b} > 2$ (b) $\frac{b}{a-c} > 2$

(c) $\frac{b}{a+c} > 2$ (d) $\frac{c}{a+b} < 2$

12. If $y-\sqrt{3}x+3=0$ cuts the parabola $2+x=y^2$ at A and B, where $P \equiv \left(\sqrt{3},0\right)$; then $PA.PB$ is :

(a) $\frac{4}{3}\left(2+\sqrt{3}\right)$ (b) $\frac{4}{3}\left(2-\sqrt{3}\right)$

(c) $\frac{4\sqrt{3}}{5}$ (d) None of these

13. If $y^2 = 4a\,(x-\alpha)$ and $x^2 = 4a\,(y-\beta)$ always touch one another, α and β being both varying, then locus of point of contact is :

(a) $xy = 4a^2$ (b) $xy = 4a$

(c) $xy = a$ (d) $xy = a/2$

14. The locus of the vertex points of the family of parabolic curve $y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$, where $'a'$ is the parameter, is given by :

(a) $xy=\frac{105}{64}$ (b) $xy=\frac{3}{8}$

(c) $xy=\frac{55}{8}$ (d) $xy=\frac{201}{10}$

15. A parabola has its vertex and focus in Ist quadrant and axis along the line $y = x$, if the distances of the vertex and focus from the origin are $\sqrt{2}$ and $2\sqrt{2}$ respectively, then equation of parabola is :

(a) $(x+y)^2 = x-y+2$

(b) $(x-y)^2 = x+y-2$

(c) $(x-y)^2 = 8(x+y-2)$

(d) $(x+y)^2 = 8\,(x-y+2)$

16. If $\theta \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, then maximum length of latus rectum of parabola whose focus is $(a\sin 2\theta\ ,\ a\cos 2\theta)$ and directrix is $y-a=0$, is :

(a) $2a$ (b) $4a$

(c) $8a$ (d) $\frac{1}{2}a$

17. Locus of all points on the curve $y^2=4a\left(x+a\sin\left(\frac{x}{a}\right)\right)$ at which the tangent is parallel to x-axis is :

(a) straight line. (b) circle.

(c) parabola. (d) hyperbola.

18. Normals PO, PA and PB are drawn to parabola $y^2 = 4x$ from $P\,(h\,,0)$, where $'O'$ is origin and $\angle AOB = 90^{\circ}$, then area of quadrilateral $OAPB$ is :

(a) 12 sq. units (b) 24 sq. units

(c) 6 sq. units (d) 18 sq. units

19. If normals at the end of a variable chord $'PQ'$ of the parabola $y^2 = 4y + 2x$ are perpendicular to each other, then locus of the point of intersection of the tangents at $'P'$ and $'Q'$ is given by :

(a) $5x+2=0$ (b) $x-y+3=0$

(c) $2x+5=0$ (d) $5y-2=0$

20. The focal chord to $y^2=16x$ is tangent to the circle $(x-6)^2+y^2=2$, then the possible values of the slope of this chord, are :

(a) $\{-1\,,1\}$ (b) $\{-2\,,2\}$

(c) $\{-2\,,1/2\}$ (d) $\{2\,,-1/2\}$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let PQ be a chord of the parabola $y^2 = 4x$ and circle on PQ as diameter passes through the vertex $'V'$ of the parabola. If the area of ΔPVQ is 20 square unit, then the possible co-ordinates for $'P'$ can be :

(a) $(2\,,-1)$ (b) $(1\,,-2)$

(c) $(16\,,8)$ (d) $(-16\,,8)$

22. Let $a \in R^+$ and the curves $x^2 = 4a\,(y-b)$ and $y^2 - x^2 = a^2$ intersect each other at four distinct points, then the values of $'b'$ may lie in the interval :

(a) $(-2a\,,-a)$ (b) $\left(a,\frac{5a}{4}\right)$

(c) $(-a\,,a)$ (d) $(0\,,a)$

23. Let any point $'P'$ lies on the parabola $y^2 = 8x$. If tangent and normal is drawn to parabola at point $'P'$ which intersects the x-axis at $'T'$ and $'N'$ respectively, then locus of the centroid of triangle PTN is parabolic curve for which :

(a) vertex is $\left(\frac{4}{3}\,,0\right)$

(b) the equation of directrix is $3x-2=0$

(c) focus is $(2\,,0)$

(d) equation of latus rectum is $2x-3=0$

24. Let a moving parabola with length of latus rectum 8 units touches a fixed equal parabola , where the axes of moving parabola and fixed parabola being parallel. If the locus of the vertex of moving parabolic curve is conic *'S'* , then :

(a) eccentricity of *'S'* is 1.

(b) length of latus rectum of *'S'* is 16 units.

(c) eccentricity of *'S'* is $\sqrt{2}$.

(d) length of latus rectum of *'S'* is 32 units.

25. Let normals drawn at points A , B (0 , 0) and C to the parabola $y^2 = 4x$ be concurrent at point P (3 , 0). If tangents drawn at *'A'* and *'C'* to the parabola intersects at point *'D'* , then :

(a) area of ΔABC is 2 square units.

(b) quadrilateral $PABC$ is cyclic.

(c) circumcentre of ΔABC lies outside the triangle.

(d) quadrilateral $ADCP$ is cyclic.

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. Statement 1 : If the curve C_1 is given parametrically by the equations $x = \sin^2 t + 2$ and $y = 1 + 2\sin t$ for all real values of *'t'* , then it represents the parabolic curve $y^2 - 2y - 4x + 9 = 0$

because

Statement 2 : The point ($2 + \sin^2 t$, $1 + 2\sin t$) lies on the curve $(y - 1)^2 = 4\,(x - 2)$ for all real values of *'t'* .

27. Statement 1 : Let tangents be drawn to $y^2 = 4\,ax$ from a variable point *'P'* moving on $x + a = 0$, then the locus of foot of perpendicular drawn from *'P'* on the chord of contact is given by $y^2 + (x - a)^2 = 0$

because

Statement 2 : The intercept made by any tangent with finile non-zero slope of the parabola between the directrix and point of tangency always subtends a right angle at focus.

28. Statement 1 : If normal drawn at any point *'P'* on the parabola $y^2 = 4ax$ meets the curve again at *'Q'* , then the least distance of Q from the axis of parabola is $4\sqrt{2}a$

because

Statement 2 : If the normal at *'t'* point meets the curve again at *'t₁'* point , then $t_1 = \left(-t - \dfrac{2}{t}\right)$ and $|t_1| \geq 2\sqrt{2}$.

29. Statement 1 : Let perpendicular tangents of the conic $y^2 + 8x - 4y - 4 = 0$ intersects each other at point $(\alpha\ ,\ \beta)$, then 'α ' must be 3 and $\beta \in R$

because

Statement 2 : Locus of the point of intersection of perpendicular tangents to a parabolic curve is the directrix of curve.

30. Statement 1 : Let a normal chord PQ be drawn for parabola $y^2 = 4x$ with point *'P'* being (4 , 4). Circle described with PQ as diameter passes through the focus F (1 , 0)

because

Statement 2 : normal chord PQ subtends an angle of $\tan^{-1}(5)$ at origin.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let the locus of the circumcentre of a variable triangle having sides $x = 0$, $y - 2 = 0$ and $lx + my - 1 = 0$, where (l , m) lies on $2y^2 - x = 0$, be curve $'C'$, then answer the following questions.

1. Curve $'C'$ is symmetric about the line :

(a) $2y + 3 = 0$ (b) $2y - 3 = 0$

(c) $2x + 3 = 0$ (d) $2x - 3 = 0$

2. Length of smallest focal chord of curve $'C'$ is :

(a) 2 units (b) $\frac{1}{2}$ unit

(c) 1 unit (d) $\frac{1}{4}$ unit

3. From point $'P'$ if perpendicular pair of tangents can be drawn to the curve $'C'$, then $'P'$ can be :

(a) $\left(-\frac{1}{4}, 4\right)$ (b) $\left(-1, \frac{3}{2}\right)$

(c) $\left(-\frac{1}{2}, 3\right)$ (d) $\left(-\frac{3}{2}, 2\right)$

Comprehension passage (2)
(Questions No. 4-6)

Let $C_1 : y = x^2 + 2ax + b$ and $C_2 : y = cx^2 + 2dx + 1$ be two parabolic curves having vertex points at $'A'$ and $'B'$ respectively. If the projection of $'A'$ and $'B'$ on the x-axis is A' and B' respectively , as shown in the figure (1) , and $AA' = BB'$, $OA' = OB'$, where $'O'$ is origin , then answer the following questions.

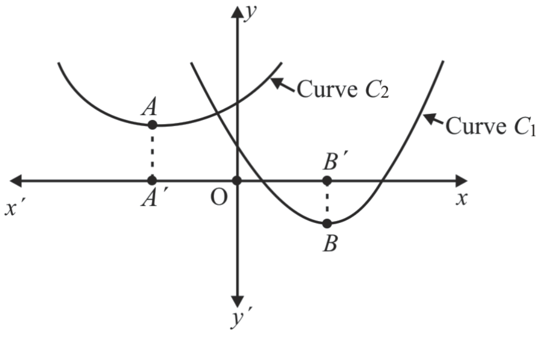

figure (1)

4. Which one of the following inequality is correct.

(a) $b > 1$ (b) $ac < 0$

(c) $cd < 0$ (d) $d \ngtr 0$

5. If b and c are non-zero real numbers , then value of a^2 is equal to :

(a) $\frac{bd}{c}$ (d) $\frac{bd^2}{c^2}$

(c) $\frac{bd^2}{c}$ (d) $\frac{cd}{b^2}$

6. In figure (1) , if $\angle A'AB' + \angle B'BA' = 180^\circ$, then which one of the following equality holds true :

(a) $(5d^2 - c)(5a^2 + b) = 1$

(b) $(5a^2 - b)(5d^2 - c) = 16ad$

(c) $(5a^2 - b)(5d^2 - c) = 16a^2d^2$

(d) $(5a^2 - b)(5d^2 + c) = 4bd$

Comprehension passage (3)
(Questions No. 7-9)

Let parabolic curves $'C_1'$ and $'C_2'$ be given by $y + x^2 + 2 = 0$ and $y^2 + x + 2 = 0$ respectively. Curve $'C'$ represents a circle with centre at $'C_0'$, where OP and OQ are tangents from origin $'O'$ to the circle $'C'$. If circle $'C'$ touches both the parabolic curves C_1 , C_2 , and have minimum area , then answer the following questions.

7. Equation of circle $'C'$ is :

(a) $4x^2 + 4y^2 + 33(x + y) + 19 = 0$

(b) $x^2 + y^2 + 11(x + y) + 10 = 0$

(c) $4(x^2 + y^2) + 11(x + 3y) + 9 = 0$

(d) $4(x^2 + y^2) + 11(x + y) + 9 = 0$

8. Area (in square units) of quadrilateral OPC_0Q is given by :

(a) $\frac{21}{2\sqrt{3}}$ (b) $\frac{21}{2\sqrt{2}}$

(c) $\frac{42}{5\sqrt{3}}$ (d) $\frac{21}{4\sqrt{2}}$

9. A common tangent to the parabolic curves $'C_1'$ and $'C_2'$ can be given by :

(a) $4x+4y+7=0$
(b) $4x+4y+5=0$
(c) $4x+8y+7=0$
(d) $8x+4y+5=0$

Comprehension passage (4)
(Questions No. 10-12)

Let variable parabolic curves be drawn through the fixed diametric ends $(0\ ,\ r)$ and $(0\ ,\ -r)$ of the circle $x^2+y^2=r^2$ such that the directrix of variable parabolic curves always touch the circle $x^2+y^2=R^2$. If the path traced by the focus of the variable parabolic curves is represented by a conic section of eccentricity $'e'$, then answer the following questions.

10. If $R^2 \in (r^2\ ,\ 2r^2)$, then eccentricity $'e'$ may be equal to :

(a) $\sqrt{\pi}$
(b) sin 4
(c) sin 1
(d) cos 2

11. If $r^2-2R^2>0$, then $'e'$ may be equal to :

(a) tan 3
(b) $\operatorname{cosec} \dfrac{\pi}{4}$
(c) $\sec \dfrac{3\pi}{8}$
(d) cos 3

12. If $r^2 \in (R^2\ ,\ 2R^2)$, then $'e'$ may be equal to :

(a) $\dfrac{1}{2}$
(b) $\sec\dfrac{3\pi}{8}$
(c) $\sqrt{2}$
(d) $\sec\dfrac{\pi}{8}$

Questions with Integral Answer :
(Questions No. 13-20)

13. Let three normals be drawn from point $'P'$ with slopes α , β and γ to the parabola $y^2=4x$. If locus of $'P'$ with the condition $\alpha\beta=k$ is a part of the parabolic curve $y^2-4x=0$, then value of $'k'$ is equal to ....

14. Let a tangent be drawn to parabola $y^2-2y-4x+5=0$ at any point $'P'$ on it. If the tangent meets the directrix at $'Q'$ and the moving point $'M'$, divides QP externally in the ratio 1 : 2 , then locus of $'M'$ passes through $(-\alpha\ ,\ 0)$. The value of $'\alpha'$ is equal to ....

15. Let the parabola $y=ax^2+2x+3$ touches the line $x+y-2=0$ at point $'P'$. If a line through $'P'$, parallel to x-axis , is drawn to meet $y+1=|x|$ at $'Q'$ and $'R'$ and the area of ΔOQR (where $'O'$ is origin) is $'A'$ square units , then value of $\dfrac{9A}{11}$ is equal to ....

16. Let the tangent at point $P(2\ ,\ 4)$ to the parabola $y^2=8x$ meets the parabola $y^2=8x+5$ at $'A'$ and $'B'$. If the midpoint of AB is point $(\alpha\ ,\ \beta)$, then $(2\alpha-\beta)$ is equal to ....

17. Let PQ be the normal chord for the parabola $y^2-4x-2y+9=0$. If PQ subtends an angle of 90º at the vertex of the parabola , then square of slope of the normal chord is equal to ....

18. Let all the sides (or the extension of sides) of on equilateral triangle ABC touch the parabola $y^2-4x=0$. If the vertices of ΔABC lie on the curve $'C'$ and curve $'C'$ passes through the point $P(1\ ,\ k)$, where $'P'$ lies above the x-axis , then value of $'k'$ is equal to ....

19. Let tangent and normal drawn to parabola at point $P(2t^2\ ,\ 4t)$, $t\neq 0$, meets the axis of parabola at points $'Q'$ and $'R'$ respectively. If rectangle $PQRS$ is completed , then locus of vertex $'S'$ of the rectangle is given by curve $'C'$. Total number of integral points inside the region of curve $'C'$ in the first quadrant is equal to ....

20. Let $'P'$ and $'Q'$ be the end points of the latus rectum of parabolic curve $y^2-4y+8x-28=0$ and point $'R'$ lies on the circle $x^2+y^2-4x-4y+7=0$. If $PR+RQ$ is minimum , then maximum number of locations for point $'R'$ is / are ....

Matrix Matching Questions :
(Questions No. 21-23)

21. Let points $P(-6, 4)$, $Q(-2, 0)$, $R(2, 4)$ and $S(-2, 8)$ form a quadrilateral $PQRS$ and a parabolic curve $'C'$ with axis of symmetry along $y-4=0$ passes through P, Q and S. With reference to curve $'C'$, match the following columns I and II.

Column (I)	Column (II)
(a) Length of latus rectum of curve $'C'$, is :	(p) 8.
(b) Length of double ordinate of curve $'C'$ which subtends an angle of 90° at the vertex of curve is :	(q) $\frac{25}{6}$.
(c) If $'F'$ is focus of curve $'C'$ and $'r'$ is the in-radius of ΔQFS, then value of $3r$ is equal to :	(r) 4.
(d) Circum-radius of ΔQFS is :	(s) $\frac{11}{4}$.

22. Match the following columns (I) and (II)

Column (I)	Column (II)
(a) Parabolic curve $y = x^2 + 5x + 4$ meets the x-axis at $'A'$ and $'B'$. Length of tangent from origin to the circle passing through $'A'$ and $'B'$ is equal to :	(p) –1
(b) Point $P(\alpha, -2)$ lies in the exterior region of both the parabolic curves $y^2 = \|x\|$. If $'P'$ is integral point, then $'\alpha'$ can be equal to :	(q) 1
(c) From point $P(9, -6)$, if two normals of slope m_1 and m_2 are drawn to parabola $y^2 = 4x$, then m_1m_2 is equal to	(r) 2
(d) If two distinct chords through the point $(a, 2a)$ of a parabola $y^2 = 4ax$ are bisected by the line $x + y = 1$, then the length of latus rectum can be equal to :	(s) 3
	(t) –2

23. Let the tangents from $P(\alpha, \beta)$ to the parabolic curve $x^2 - 2x + 8y - 15 = 0$ be PA and PB, where the chord of contact is AB. Match the possible nature of triangle PAB (in column II) with the conditions on α and β (in column I).

Column (I)	Column (II)
(a) If $\alpha = 1$; $\beta \geq 5$, then ΔPAB may be :	(p) Right-angled triangle.
(b) If $\alpha \in R$; $\beta = 4$, then ΔPAB may be :	(q) Acute-angled triangle.
(c) If $\alpha^2 - 2\alpha + 8\beta > 15$; $\beta < 4$, then ΔPAB may be :	(r) Obtuse-angled triangle.
(d) If $\alpha^2 - 2\alpha + 8\beta > 15$; $\beta > 4$, then ΔPAB may be :	(s) Scalene triangle.

ANSWERS

Exercise No. (1)

1. (b)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (c)
6. (c)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (a)
11. (b)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (c)
16. (b)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (a)
21. (b , c)	**22.** (a , b)	**23.** (a , b , c)	**24.** (a , b)	**25.** (a , c , d)
26. (d)	**27.** (a)	**28.** (a)	**29.** (a)	**30.** (b)

ANSWERS

Exercise No. (2)

1. (b)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (c)
6. (c)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (c)
11. (c)	**12.** (d)	**13.** (2)	**14.** (5)	**15.** (8)
16. (0)	**17.** (2)	**18.** (4)	**19.** (9)	**20.** (2)
21. (a) $\to$ r (b) $\to$ p (c) $\to$ r (d) $\to$ q	**22.** (a) $\to$ r (b) $\to$ p , q , r , s , t (c) $\to$ r (d) $\to$ q , r , s	**23.** (a) $\to$ q (b) $\to$ p , s (c) $\to$ r , s (d) $\to$ q , s		

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Chapter 27

Ellipse

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-20)

1. A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is intersected by the tangents at the extremities of the major axis at *'P'* and *'Q'* , then circle on PQ as diameter always passes through :

(a) one fixed point

(b) two fixed points

(c) four fixed points

(d) three fixed points

2. A variable tangent of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the co-ordinate axes at A and B , then minimum area (in sq. units) of circumcircle of ΔAOB , *'O'* being the origin , is given by :

(a) $\frac{\pi}{4}(a-b)^2$. (b) $\pi(a^2+b^2)$.

(c) $\frac{\pi}{4}(a^2+b^2)$. (d) $\frac{\pi}{4}(a+b)^2$.

3. Let $P(x,y)$ be any point on ellipse $9x^2+25y^2=225$, if $'F_1'$ and $'F_2'$ are the focal points of ellipse , then perimeter of ΔF_1PF_2 is :

(a) 10 (b) 18

(c) 25 (d) 30

4. The chords of contact of tangents to curve $x^2+8y^2=8$ from any point on its director circle intersect the director circle at *'C'* and *'D'* , then locus of the point of intersection of tangents to circle at *'C'* and *'D'* is :

(a) $16x^2+y^2=81$. (b) $64x^2+y^2=243$.

(c) $64x^2+y^2=16$. (d) None of these.

5. If normal at an end of latus rectum of an ellipse passes through one extremity of minor axis , then eccentricity *'e'* satisfy :

(a) $e^4+e^2-1=0$ (b) $e^2+e-5=0$

(c) $e^3=5/2$ (d) $e^4-e^2+1=0$

6. If tangent is drawn at $'\theta'$ point to the ellipse $x^2+27y^2=27$, where $\theta\in\left(0,\frac{\pi}{2}\right)$, then value of $'\theta'$ such that sum of intercepts on axes made by this tangent is minimum , is :

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{12}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

7. The length of latus rectum of an ellipse is one third of the major axis , then eccentricity of ellipse is equal to :

(a) $\frac{2}{3}$ (b) $\sqrt{\frac{2}{3}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

8. Minimum distance between the ellipse $x^2+2y^2=6$ and the line $x+y-7=0$ is equal to :

(a) $4\sqrt{2}$ (b) $2\sqrt{2}$

(c) $\sqrt{5}$ (d) $\sqrt{10}$

9. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2+9y^2=9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A , M and the origin O is equal to :

(a) $\frac{31}{10}$ sq. units (b) $\frac{29}{10}$ sq. unit

(c) $\frac{21}{10}$ sq. unit (d) $\frac{27}{10}$ sq. units

10. The normal at a point P on the ellipse $x^2+4y^2=16$ meets the x-axis at Q. If M is the mid point of the line segment PQ , then the locus of M intersects the latus rectums of the given ellipse at the points

(a) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{2}{7}\right)$ (b) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{\sqrt{19}}{4}\right)$

(c) $\left(\pm2\sqrt{3},\pm\frac{1}{7}\right)$ (d) $\left(\pm2\sqrt{3},\pm\frac{4\sqrt{3}}{7}\right)$

11. Maximum length of chord of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$, such that eccentric angles of the extremities of chord differ by $\frac{\pi}{2}$ is :

(a) $a\sqrt{2}$ (b) $b\sqrt{2}$

(c) $ab\sqrt{2}$ (d) $\frac{b}{a}$

12. If an ellipse with major and minor axes of length $10\sqrt{3}$ and 10 units respectively slides along the co-ordinate axes in the first quadrant , then length of the arc which is formed by the locus of centre of ellipse is given by :

(a) 10π (b) $\frac{5\pi}{4}$

(c) $\frac{5\pi}{3}$ (d) $\frac{3\pi}{2}$

13. Area of ellipse for which focal points are (3 , 0) and (–3 , 0) and point (4 , 1) lying on it , is given by :

(a) 18π sq. units (b) $9\sqrt{2}\pi$ sq. units

(c) $\sqrt{243}\,\pi$ sq. units (d) $\sqrt{18}\,\pi$ sq. units

14. Let tangents drawn from point $'P'$ to the ellipse $x^2+4y^2=36$ meets the co-ordinate axes at concylic points , then locus of point $'P'$ is given by :

(a) $x^2-y^2=27$ (b) $x^2+y^2=27$

(c) $x^2-y^2=16$ (d) $x^2+y^2=16$

15. Let the common tangent in I$^{\text{st}}$ quadrant to the circle $x^2+y^2=16$ and $4x^2+25y^2=100$ meet the axes at A and B , then area of ΔAOB , where O is origin , is :

(a) $\frac{14}{\sqrt{3}}$ (b) $\frac{28}{\sqrt{3}}$

(c) $\frac{20}{\sqrt{3}}$ (d) none of these

16. Let ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a>b$, be centered at $'O'$ and having AB and CD as its major and minor axis respectively. If one of the focus of ellipse is $'F_1'$, the in-radius of triangle DOF_1 is 1unit and OF_1 = 6 units , then director circle of ellipse is given by :

(a) $x^2+y^2=100$ (b) $x^2+y^2=97/2$

(c) $x^2+y^2=50$ (d) $x^2+y^2=105/2$

17. Let normals be drawn to the ellipse $x^2+2y^2=2$ from point (2 , 3) , then the co-normal points lie on the curve :

(a) $xy+3x-4y=0$ (b) $2xy-3x+4y=0$

(c) $3x+4y-xy=0$ (d) $4xy+4x-3y=0$

18. Let $'A'$ be the centre of ellipse $5x^2+5y^2+6xy-8=0$ and $'P'$, $'Q'$ points lie on the ellipse such that AP and AQ distances are maximum and minimum respectively , then $AP+AQ$ is equal to :

(a) 2 (b) 4

(c) 3 (d) 5

19. Let $'AB'$ be the variable chord of the ellipse $x^2+2y^2=2$ and $\angle AOB=\frac{\pi}{2}$, where $'O'$ is origin , then $\frac{OA^2+OB^2}{(OA.OB)^2}$ is equal to :

(a) $\frac{2}{3}$. (b) $\frac{3}{2}$.

(c) $\frac{3}{4}$. (d) $\frac{5}{4}$.

20. Let normal to the ellipse $4x^2+5y^2=20$ at point $P(\theta)$ touches the parabola $y^2=4x$, then $\tan\theta$ is equal to :

(a) ±2 (b) ±3

(c) ±1 (d) ±4

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let circle $'C'$ with centre (1 , 0) be inscribed in the ellipse $x^2+4y^2=16$ and the area of circle $'C'$ is maximum , then

(a) equation of director circle of $'C'$ is given by $9(x-1)^2+9y^2=121$

(b) equation of director circle of $'C'$ is given by $3(x-1)^2+3y^2=22$

(c) area of circle $'C'$ is $\frac{11\pi}{3}$ sq. units.

(d) circle $'C'$ is auxiliary circle for the ellipse $9(x-1)^2+25y^2=121$

22. Let one of the focus point of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be at $F_1(4,0)$ and its intersection point with positive y-axis be $'B'$. If the centre of ellipse is $'C'$ and circum-radius of ΔCF_1B is 2.5 units, then which of the following statements are incorrect :

(a) equation of director circle of ellipse is $x^2+y^2=34$.

(b) area of ellipse is 20π square units.

(c) director circle of auxiliary circle of the ellipse is $x^2+y^2=50$.

(d) length of latus rectum of ellipse is 4 units.

23. Let ellipse $E_1 : x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with co-ordinate axes, which in turn is inscribed in another ellipse E_2 that passes through the point $(4, 0)$. With reference to ellipse E_1 and E_2 which of the following statements are true :

(a) If point (α, β) lies in between the boundary of the director circle of E_1 and E_2, then $15<3\alpha^2+3\beta^2<52$.

(b) If point $(2\alpha, \alpha)$ lies outside the ellipse E_2, then $\alpha \in R-[-1,1]$.

(c) Total number of integral points inside the ellipse E_1 are four.

(d) If point $(2\alpha, \alpha)$ lies inside the ellipse E_1, then $\alpha \in \left(-\frac{1}{2},\frac{1}{2}\right)$.

24. Let point $'P'$ lies on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and normal to ellipse at $'P'$ meets the co-ordinate axes at A and B. If $'O'$ is the origin and M is the foot of perpendicular from origin to AB, then

(a) maximum area of ΔAOB is 2.025 square units.

(b) maximum value of OM is 2 units.

(c) maximum value of OM is 1 unit.

(d) maximum area of ΔAOB is $\frac{81}{80}$ square. units.

25. Let variable point $'P'$ lies on the curve $y=x^2$ and PA, PB are tangents to the ellipse $x^2 + 3y^2 = 9$. If $\angle APB$ is an acute angle, then x co-ordinate of point $'P'$ can be given by :

(a) $\sqrt{e+\frac{1}{e}}$ (b) $\sqrt{2}+\frac{1}{\sqrt{2}}$

(c) $\frac{3}{2}\ln 2$ (d) $\tan\left(\frac{9}{2}\right)$

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. **Statement 1 :** Total number of distinct normals which can be drawn to the ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ from point $(0, 6)$ are three.

because

Statement 2 : Maximum number of normals which can be drawn to any given ellipse from a point are four.

27. Let any point $'P'$ lies on the ellipse $\frac{x^2}{16}+\frac{y^2}{12}=1$ and PM_1, PM_2 are the distances of $'P'$ from $x-8=0$ and $x+8=0$ respectively.

Statement 1 : For point $'P'$ maximum value of $(PM_1)(PM_2)$ cannot exceed 64 square units

because

Statement 2 : Area of ΔPF_1F_2, where F_1 and F_2 are foci of ellipse, can't exceed $4\sqrt{3}$ square units.

28. Let C_1 and C_2 be two ellipse which are given by $x^2 + 4y^2 = 4$ and $x^2 + 2y^2 = 6$ respectively and any tangent to curve C_1 meets the curve C_2 at A and B.

Statement 1 : If tangents drawn to curve C_2 at points A and B meet at point P, then $\angle APB=\frac{\pi}{2}$

because

Statement 2 : Locus of point $'P'$ is the director circle of curve $'C_1'$.

29. **Statement 1 :** Let $'L'$ be variable line which is tangential to fixed ellipse with foci F_1 and F_2 , then locus of the foot of perpendicular from foci to line $'L'$ is the auxiliary circle of ellipse

because

Statement 2 : Product of the length of perpendiculars from foci F_1 and F_2 to the line $'L'$ is always the square of semi-minor axis of ellipse.

30. **Statement 1 :** If point $'P'$ lies on a given ellipse with foci at F_1 and F_2 , then perimeter of ΔPF_1F_2 is constant

because

Statement 2 : Perimeter of the ellipse is given by $\left\{\frac{\pi}{2e}(F_1F_2)(1+\sqrt{1-e^2})\right\}$ units , where $'e'$ is the eccentricity of ellipse .

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let tangent at any point on the curve $E_1 : 4x^2 + 9y^2 = 36$ meets the curve $E_2 : 10x^2 + 15y^2 = 150$ at P and Q. If tangents drawn at P and Q to curve E_2 meets at point $'R'$ and locus of $'R'$ is given by the curve $'C_1'$ then answer the following questions.

1. Locus of point from which perpendicular tangents can be drawn to curve $'C_1'$ is :

(a) $x^2 + y^2 = 50$ (b) $x^2 + y^2 = 60$

(c) $y - 8 = 0$ (d) $2y - 9 = 0$

2. Positive slope of the common tangent to curve $'C_1'$ and $2x^2 + 3y^2 = 60$ is :

(a) 1 (b) $\dfrac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $2 - \sqrt{3}$

3. If from any point $'A'$ on the line $2x + 3y = 30$ tangents AB and AC are drawn to curve $'C_1'$, then locus of the circumcentre of ΔABC is :

(a) $4x + 6y = 27$ (b) $2x + 3y = 15$

(c) $2x - 3y = 20$ (d) $2x + 3y = 20$

Comprehension passage (2)
(Questions No. 4-6)

Let variable ellipse $x^2 + 4y^2 = 4k^2$, where $k \in R^+$, and a fixed parabola $y^2 = 8x$ is having a common tangent which meets the co-ordinate axes at P and Q, then answer the following questions.

4. Let A be the point of contact of the common tangent with the ellipse and the eccentric angle of A is $\dfrac{2\pi}{3}$, then value of $'k'$ is equal to :

(a) 4 (b) 8

(c) 6 (d) 5

5. Locus of the mid-point of the intercepted length PQ is :

(a) $y^2 + 4x = 0$ (b) $y^2 + x = 0$

(c) $2y^2 + x = 0$ (d) $4y^2 + x = 0$

6. If $'O'$ is origin and the area of ΔOPQ is 2 square units , then value of $'k'$ is

(a) $\dfrac{2}{\sqrt{3}}$ (b) $\dfrac{2}{\sqrt{5}}$

(c) $\sqrt{\dfrac{5}{4}}$ (d) $\dfrac{\sqrt{5}}{4}$

Comprehension passage (3)
(Questions No. 7-9)

Let $L_1 : y - m_1 x = 0$ and $L_2 : y - m_2 x = 0$ be the variable lines for which $m_1 m_2$ is negative , and lines L_1 and L_2 are tangential to the variable ellipse $'E'$ at the points T_1 and T_2 respectively. If the ellipse $'E'$ is rotating about the point $(\alpha , 0)$ and initially its equation is given by $b^2(x - \alpha)^2 + a^2 y^2 = (ab)^2$, where $\alpha \in R^+$, then answer the following questions.

7. If $\alpha = 10$ and the angle $\angle T_1OT_2$ is constant for all the positions of variable ellipse $'E'$, where $'O'$ is origin , then the ordered pair (a , b) can be given by :

(a) (7 , 3) (b) (4 , 6)

(c) (8 , 6) (d) (12 , 6)

8. If $3a = 4b = 12$ and the angle $\angle T_1OT_2$ remains acute for all the positions of the variable ellipse $'E'$, where $'O'$ is origin , then the possible value of $'\alpha'$ can be :

(a) $\pi - e$ (b) $\pi + \dfrac{1}{\pi}$

(c) e^2 (d) 2 tan 1

9. If the $\angle T_1OT_2$ remains obtuse for all the positions of the variable ellipse $'E'$, where O is origin , then which one of the following relation must hold true :

(a) $\alpha^2 - a^2 - b^2 > 0$

(b) $\min\{2a , 2b\} < \alpha < \sqrt{a^2 + b^2}$

(c) $\max\{a , b\} < \alpha < \sqrt{a^2 + b^2}$

(d) $\dfrac{a+b}{2} < \alpha < \sqrt{a^2 + b^2}$

Questions with Integral Answer :
(Questions No. 10-15)

10. Let tangent and normal be drawn at any point $'P'$ on the ellipse $x^2 + 3y^2 = 3$, and rectangle $PAOB$ is completed , where $'O'$ is the origin. Maximum area (in square units) of the rectangle $PAOB$ is ....

11. Let common tangents of the curves $y^2 = 4x$ and $x^2 + 4y^2 = 8$ meets on the x-axis at A and intersects the positive and negative y-axis at B and C respectively. If parabola with its axis along the x-axis and vertex at A passes through B and C, then length of latus rectum of the parabola is ....

12. Let points A, B and C lie on the curve $y = -\sqrt{3 - \frac{3x^2}{4}}$, $y = \sqrt{2x - x^2}$ and $y = \sqrt{-x^2 - 2x}$ respectively, then maximum value of $(AB + AC)$ is equal to ....

13. If line $2x + 3y = \lambda$ meet the ellipse $4x^2 + 9y^2 = 36$ at points $'A'$ and $'B'$, where $\angle AOB = 90°$, $'O'$ being the origin, then positive value of λ is equal to ....

14. Let tangents drawn at A and B points on the ellipse $4x^2 + 9y^2 = 36$ meet at point $P(1, 3)$. If $'C'$ is the centre of ellipse and the area of quadrilateral $PACB$ is α square units, then value of $[\alpha]$, where [.] represents the greatest integer function, is equal to ....

15. Let $ABCD$ is a square of side length 8 units, and an ellipse of eccentricity 0.5 is drawn touching the sides of the square, where the axes of symmetry being along the diagonals of square. If the major axis and minor axis is of length $'2a'$ and $'2b'$ units respectively, then value of $\left\{\sec\left(\sin^{-1}\left(\frac{b}{a}\right)\right)\right\}^2$ is ....

Matrix Matching Questions :
(Questions No. 16-18)

16. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) Number of points on the ellipse $2x^2 + 5y^2 = 100$ from which pair of perpendicular tangents can be drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is / are :	(p) 0
(b) If the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ intersect the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at four concyclic points, then $(m_1 + m_2)$ must be :	(q) 1
(c) If all the normals of ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ intersects or touches the circle $x^2 + y^2 = r^2$, then minimum value of $'r'$ is :	(r) 2
(d) If the equation $3x^2 + 4y^2 - 18x + 16y + 43 - k = 0$ represents an ellipse, then values of $'k'$ can be :	(s) 4

17. Let $C_1 : x^2 + y^2 = a^2$ and $C_2 : x^2 + y^2 = b^2$ be two circles , where $b > a > 0$, and $'O'$ is origin. A line OPQ is drawn which meets C_1 and C_2 at points P and Q respectively. If $'R'$ is the moving point for which PR and QR is parallel to the y-axis and x-axis respectively and the locus of $'R'$ is an ellipse $'E'$, then match the following columns for eccentricity $'e'$ of the ellipse $'E'$ and the position of foci F_1 and F_2 of $'E'$.

Column (I)	**Column (II)**
(a) If F_1 and F_2 lie on the circle $'C_1'$, then eccentricity $'e'$ can be :	(p) $\left(\sec\left(\frac{1}{2}\right)\right)^{-\frac{1}{2}}$
(b) If F_1 and F_2 lie inside the circle $'C_1'$, then eccentricity $'e'$ can be :	(q) $\sin\left(\frac{1}{2}\right)$
(c) If F_1 and F_2 lie inside the circle $'C_2'$, then eccentricity $'e'$ can be :	(r) $\cos\left(\frac{\pi}{4}\right)$
(d) If F_1 and F_2 don't lie inside the circle $'C_1'$, then eccentricity $'e'$ can be :	(s) $\cos(1)$

ANSWERS Exercise No. (1)

1. (b)
2. (d)
3. (b)
4. (d)
5. (a)
6. (c)
7. (b)
8. (b)
9. (d)
10. (c)
11. (a)
12. (c)
13. (b)
14. (a)
15. (b)
16. (b)
17. (a)
18. (c)
19. (b)
20. (a)
21. (b , c)
22. (b , d)
23. (a , b)
24. (c , d)
25. (a , b , d)
26. (b)
27. (b)
28. (c)
29. (b)
30. (b)

ANSWERS Exercise No. (2)

1. (a)
2. (a)
3. (b)
4. (c)
5. (b)
6. (b)
7. (c)
8. (c)
9. (c)
10. (1)
11. (1)
12. (6)
13. (6)
14. (7)
15. (4)

16. (a) $\to$ s
(b) $\to$ p
(c) $\to$ q
(d) $\to$ q , r , s

17. (a) $\to$ r
(b) $\to$ q , s
(c) $\to$ p , q , r , s
(d) $\to$ p , r

Chapter 28

Hyperbola

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-20)

1. If the chords of contact of tangents from (– 4 , 2) and (2 , 1) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are at right angle , then eccentricity of the hyperbola is :

(a) $\sqrt{2}$ (b) $\sqrt{\frac{3}{2}}$

(c) $\sqrt{\frac{5}{2}}$ (d) $\sqrt{3}$

2. Let *'P'* be the point of intersection of $xy = c^2$ and $x^2 - y^2 = a^2$ in the first quadrant and tangents at P to both curves intersect the y-axis at *'Q'* and *'R'* respectively , then circumcentre of ΔPQR lies on :

(a) $x + y = 1$ (b) $x - y = 1$

(c) x-axis (d) y-axis

3. Slope of common tangent to the curves $y^2 = 4ax$ and $4xy = -a^2$, where $a \in R^+$, is given by :

(a) 1 (b) $\frac{a}{2}$

(c) $-\frac{a}{2}$ (d) a

4. A normal to hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$ has equal intercepts on positive x and y axes and this normal touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a > b$, then $a^2 + b^2$ is equal to :

(a) $\frac{5}{9}$ (b) $\frac{75}{9}$

(c) $\frac{5}{18}$ (d) $\frac{18}{5}$

5. Number of common tangents which are possible to curves $12y^2 - x^2 + 12 = 0$ and $4y^2 + x^2 - 16 = 0$ is / are :

(a) 1 (b) 4

(c) 2 (d) 0

6. If eccentricity of hyperbola $x^2 - y^2\sec^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of ellipse $x^2\sec^2\alpha + y^2 = 25$, then α is equal to :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

7. A common tangent to $9x^2 - 16y^2 = 144$ and $x^2 + y^2 = 9$ is :

(a) $y = \frac{3x+15}{\sqrt{7}}$ (b) $y = \frac{3\sqrt{2}x+15}{\sqrt{7}}$

(c) $y = \frac{2\sqrt{2}x+15}{\sqrt{7}}$ (d) None of these

8. If a hyperbola is passing through origin and the foci are (5 , 12) and (24 , 7) , then eccentricity of hyperbola is given by :

(a) $\frac{\sqrt{386}}{12}$ (b) $\frac{\sqrt{386}}{13}$

(c) $\frac{\sqrt{386}}{25}$ (d) $\sqrt{2}$

9. If a hyperbola passes through the focus of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse axis and conjugate axis coincides with major and minor axes of ellipse and the product of eccentricity of ellipse and hyperbola is 1 , then the incorrect statement is :

(a) eccentricity of hyperbola is 5/3.

(b) foci of hyperbola is $(\pm 5 , 0)$.

(c) equation of hyperbola is $\frac{x^2}{8}-\frac{y^2}{16}=1$.

(d) area enclosed by ellipse is 20π sq. units.

10. Let the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{p^2}-\frac{y^2}{q^2}=1$ be confocal , where $a > b$, and the length of minor axis of ellipse is equal to the length of conjugate axis of hyperbola. If e_1 and e_2 represent the eccentricity of ellipse and hyperbola respectively , then the value of $\frac{e_1^2+e_2^2}{(e_1e_2)^2}$ is equal to :

(a) 4 (b) 6

(c) 2 (d) 1

11. Let $x\cos\theta+y\sin\theta=p$ be the equation of variable chord of the hyperbola $2x^2-y^2=2a^2$ which subtends a right angle at the centre of hyperbola. If the variable chord is always tangential to a circle of radius $'R'$, then :

(a) $R^2=3a^2$. (b) $R^2=5a^2$.

(c) $R^2=2a^2$. (d) $R^2=4a^2$.

12. Let $r\in\{1,2,3,4\}$ and the normals at the points $P_r(x_r, y_r)$ on the curve $xy = 4$ be concurrent at $Q(\alpha,\beta)$, then $\frac{\left(\sum_{r=1}^{4}x_r\right)\left(\sum_{r=1}^{4}y_r\right)}{\left(\prod_{r=1}^{4}x_r\right)}$ is equal to :

(a) $\frac{\alpha\beta}{16}$ (b) $-\frac{\alpha\beta}{16}$

(c) $\frac{\alpha\beta}{4}$ (d) $-\frac{\alpha\beta}{4}$

13. Let $'F_1'$ and $'F_2'$ be the foci of the hyperbola $x^2-y^2=a^2$ and $'C'$ be its centre. If point $'P'$ lies on the hyperbola and $PF_1.PF_2=\lambda CP^2$, then value of $\tan^{-1}(\lambda)$ is equal to :

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{12}$ (d) $\frac{\pi}{3}$

14. If $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ represents a hyperbola , then area of triangle formed by the asymptotes and tangent to hyperbola at point $(a , 0)$ is equal to :

(a) $4ab$ sq. units. (b) $2ab$ sq. units.

(c) ab sq. units. (d) $\frac{ab}{2}$ sq. units.

15. If $x=9$ is the chord of contact of the hyperbola $x^2-y^2=9$, then the equation of the corresponding pair of tangents is :

(a) $9x^2-8y^2+18x-9=0$

(b) $9x^2-8y^2+18x+9=0$

(c) $9x^2-8y^2-18x-9=0$

(d) $9x^2-8y^2-18x+9=0$

16. If $xy - 1 = \cos^2\theta$, where $\theta\in[0,\pi]$, represents a family of hyperbola , then maximum area of the triangle which can be formed by any tangent to the hyperbola and the co-ordinate axes , is given by :

(a) 8 sq. units.

(b) 4 sq. units.

(c) 16 sq. units.

(d) 2 sq. units.

17. If centre of the hyperbola $xy = 4$ is $'C'$ and tangents CP and CQ are drawn to the family of circles with radius 2 units and centre lying on the hyperbola , then the locus of the circumcentre of triangles CPQ is given by :

(a) $xy = 1$. (b) $xy = 2$.

(c) $x^2+y^2=1$. (d) $x^2-y^2=1$.

18. If the product of the perpendicular distances of a moving point $'P'$ from the pair of straight lines $2x^2-3xy-2y^2+x+3y-1=0$ is equal to 10 , then locus of point $'P'$ is hyperbolic in nature whose eccentricity is equal to :

(a) $\sqrt{10}$ (b) $\sqrt{2}$

(c) $\sqrt{\frac{5}{2}}$ (d) $\frac{\sqrt{10}}{2}$

19. If tangents are drawn from any point on the hyperbola $4x^2 - 9y^2 = 36$ to the circle $x^2 + y^2 = 9$, then locus of the mid point of the chord of contact is given by :

(a) $\frac{x}{9}+\frac{y^2}{4}=\frac{(x^2-y^2)^2}{81}$.

(b) $\frac{4x^2+9y^2}{4}=\frac{(x^2+y^2)^2}{81}$.

(c) $4x^2-9y^2=\frac{4}{9}(x^2+y^2)^2$.

(d) $4x^2+9y^2=(x^2+y^2)^2$.

20. Let a tangent be drawn at any point $'P'$ on the hyperbola $\dfrac{x^2}{4}-\dfrac{y^2}{1}=1$ which meets the co-ordinate axes at $'Q'$ and $'R'$. If rectangle $QORS$ is completed, where $'O'$ is origin, then locus of vertex $'S'$ is given by :

(a) $\dfrac{4}{x^2}+\dfrac{1}{y^2}=1$

(b) $\dfrac{4}{x^2}-\dfrac{1}{y^2}=1$

(c) $\dfrac{1}{x^2}+\dfrac{4}{y^2}=1$

(d) $\dfrac{1}{x^2}-\dfrac{4}{y^2}=1$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let an ellipse $E : b^2x^2 + a^2y^2 = a^2b^2$, $a > b$, intersects the hyperbola $H : 2x^2 - 2y^2 = 1$ orthogonally. If the eccentricity of ellipse is reciprocal to that of the hyperbola, then :

(a) ellipse and hyperbola are confocal

(b) equation of ellipse is $x^2 + 2y^2 = 4$

(c) the foci of ellipse are $(\pm 1, 0)$

(d) director circle for ellipse is $x^2 + y^2 = 6$

22. Let a hyperbola having the transverse axis of length $2\sin\theta$ is confocal with the ellipse $3x^2 + 4y^2 = 12$, then :

(a) equation of hyperbola is $x^2\sec^2\theta - y^2\,\text{cosec}^2\theta = 1$.

(b) focal points of hyperbola remain constant with change in $'\theta'$.

(c) equation of hyperbola is $x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$.

(d) Directrix of hyperbola remains constant with change in $'\theta'$.

23. If the equation $4x^2 - 5y^2 - 16x - 10y + 31 = 0$ represents a hyperbolic curve $'C'$, then which of the following statements are incorrect :

(a) eccentricity of curve $'C'$ is 1.5

(b) equation of director circle for $'C'$ is $x^2 + y^2 = 1$

(c) length of latus rectum for $'C'$ is 5 units

(d) centre of curve $'C'$ is $(2, -2)$

24. If the circle $x^2+y^2=1$ meet the rectangular hyperbola $xy=1$ in four points $(x_i\,,\,y_i)$, $i=1, 2, 3, 4$, then :

(a) $x_1x_2x_3x_4=1$

(b) $y_1y_2y_3y_4=1$

(c) $x_1+x_2+x_3+x_4=0$

(d) $y_1+y_2+y_3+y_4=0$

25. A straight line touches the rectangular hyperbola $9x^2-9y^2=8$ and the parabola $y^2=32x$. The equation of the line is :

(a) $9x+3y-8=0$

(b) $9x-3y+8=0$

(c) $9x+3y+8=0$

(d) $9x-3y-8=0$

Assertion Reasoning questions : (Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. Statement 1 : Total number of points on the curve $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ from where mutually perpendicular tangents can be drawn to the circle $x^2 + y^2 = a^2$ are four

because

Statement 2 : Circle $x^2 + y^2 = 2a^2$ intersects the curve $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ at four points.

27. Statement 1 : If point $P(\theta)$ lies on the branch of hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ in the III quadrant, then eccentric angle $'\theta'$ belongs to $\left(\pi\,,\,\dfrac{3\pi}{2}\right)$

because

Statement 2 : 'θ' point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is given by $(a\sec\theta\ ,\ b\tan\theta)$, where $\theta\in[0\ ,\ 2\pi)-\left\{\frac{\pi}{2}\ ,\ \frac{3\pi}{2}\right\}$.

28. **Statement 1 :** Two branches of a given hyperbola may have a common tangent

because

Statement 2 : The asymptotes of hyperbola always meet at the centre of the hyperbola.

29. **Statement 1 :** Ellipse $E : 5x^2+9y^2=45$ and hyperbola $H : 3x^2-y^2=3$ intersect each other at an angle of 90°

because

Statement 2 : If an ellipse and hyperbola are confocal then they always meet orthogonally.

30. **Statement 1 :** If chord PQ of curve $xy=9$ is parallel to its transverse axis , then circle with PQ as diameter always passes through two fixed points

because

Statement 2 : The transverse axis of hyperbola $xy=9$ is given by $y-x=0$

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

If the curve $x^2-y^2=8$ is rotated about its centre by 45° in anti-clockwise sense , then equation of curve changes to $C : xy = 4$. Let any point $'t'$ on curve $'C'$ be $\left(2t\ ,\ \frac{2}{t}\right)$, where $t \in R-\{0\}$, then answer the following questions.

1. If tangent at $' t_1 '$ point on the curve $'C'$ touches the curve $y^2 + 2x = 0$, then value of $' t_1 '$ is equal to :

(a) 3 (b) 2

(c) 1 (d) 1/2

2. If circle $x^2+y^2=16$ meets the curve $'C'$ at t_1 , t_2 , t_3 and t_4 points , then $\sum_{i=1}^{4} t_i^2$ is equal to :

(a) 0 (b) 8

(c) 4 (d) – 4

3. If t_1 and t_2 are the roots of the equation $x^2-4x+2=0$, then point of intersection of tangents at t_1 and t_2 points on the curve $'C'$ is :

(a) (4 , 4) (b) (2 , 1)

(c) (2 , 4) (d) (6 , 3)

Comprehension passage (2)
(Questions No. 4-6)

Let point $'P'$ moves in such a way so that sum of the slopes of the normals drawn from it to the curve $xy = 16$ is equal to the sum of ordinates of the co-normal points. If the path traced by moving point $'P'$ is represented by curve $'C'$, then answer the following questions.

4. Equation of curve $'C'$ is given by :

(a) $4y-x^2=0$ (b) $x^2-12y=0$

(c) $y^2-16x=0$ (d) $x^2-16y=0$

5. If tangent to curve $'C'$ meets the co-ordinate axes at M and N , then locus of the circumcentre of ΔMON , where $'O'$ is origin , is given by :

(a) $x^2+y=0$ (b) $x^2+2y=0$

(c) $y^2-x=0$ (d) $y+2x^2=0$

6. Let normal to the curve $'C'$ at point $(8\ ,\ \beta)$, where $\beta \in R^+$, meets the co-ordinate axes at A and B , then total number of integral points inside the ΔAOB are given by :

(a) 65 (b) 60

(c) 66 (d) 55

Comprehension passage (3)
(Questions No. 7-9)

Let hyperbolic curve $'C'$ and a line $'L'$ be given by the equations $y^2-2x^2-4y+8=0$ and $y-2=0$ respectively. If tangent and normal drawn to curve $'C'$ at point $P(2\ ,\ 4)$ meets the line $'L'$ at T and N respectively , then answer the following questions.

7. Area (in square units) of ΔPTN is :

(a) 4 (b) 5

(c) 10 (d) 8

8. Area (in square units) bounded by the curve $'C'$ with its tangent at $'P'$ and the line $'L'$ in the first quadrant is equal to :

(a) $2\,ln(\sqrt{2}+1)$ (b) $\sqrt{2}\ ln(\sqrt{2}+1)+1$

(c) $\sqrt{2}\ ln(\sqrt{2}+1)-1$ (d) $\sqrt{2}\ ln(\sqrt{2}+1)+2$

9. Let from point $(1\ ,\ k)$ a perpendicular pair of tangents can be drawn to the curve $'C'$, then

(a) exactly two real values of k exist.

(b) infinite real values of k exist.

(c) no real $'k'$ exists.

(d) none of these.

Questions with Integral Answer :
(Questions No. 10-14)

10. If the locus of the mid-points of the chords of length 4 units to the rectangular hyperbola $xy = 4$ is given by the curve $(x^2+y^2)(xy-4) = \lambda xy$, then the value of $' \lambda '$ is equal to ....

11. If normal at (5 , 3) of the hyperbola $xy-y-2x-2=0$ meet the curve again at $(p\ ,\ q-29)$, then value of $\left\{\frac{q}{4p}\right\}$ is equal to ....

12. Let point $P(\alpha\ ,\ \beta)$ lies on the hyperbola $xy = 7!$, where $\alpha\ ,\ \beta \in I$. If the total number of possible locations for $'P'$ is N, then $\frac{N}{40}$ is equal to ....

13. Maximum number of different lines which are normal to parabola $y^2 = 4x$ as well as tangent to hyperbola $x^2 - y^2 = 1$ is / are ....

14. If the chords of hyperbola $x^2 - y^2 = 4$ touch the parabola $y^2 = 8x$ and the locus of middle points of these chords is given by $y^2(x-\lambda) - x^3 = 0$, then value of λ is equal to ....

Matrix Matching Questions :
(Questions No. 15-16)

15. Match the curves in column (I) with the corresponding possibility for common normal and common tangent in column (II).

Column (I)	**Column (II)**
(a) curves $x^2 + y^2 = 8$ and $y^2 - 16x = 0$ have	(p) common normal.
(b) curves $x^2 + 16y^2 = 16$ and $x^2 + y^2 = 4$ have	(q) no common tangent.
(c) curves $x^2 + 4y^2 = 16$ and $x^2 - 12y^2 = 12$ have	(r) two common tangents.
(d) curves $x^2 + y^2 = 1$ and $x^2 + y^2 - 4x - 2y - 11 = 0$ have	(s) four common tangents.

16. Match the following column (I) and column (II).

Column (I)	**Column (II)**
(a) The angle between the pair of tangents drawn to the ellipse $3x^2 + 2y^2 = 5$ from the point $(1\ ,\ 2)$ is	(p) $\tan^{-1}\left(\frac{24}{7}\right)$
(b) The inclination of the chord of the hyperbola $25x^2 - 16y^2 = 400$ which is bisected at $(6\ ,\ 2)$ with the x-axis is	(q) $\tan^{-1}\left(\frac{1}{3}\right)$
(c) The angle between the asymptotes of the hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$ is	(r) $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$
(d) The angle between the tangents at $(9\ ,\ 6)$ on $y^2 = 4x$ and the focal chord of the parabola through $(9\ ,\ 6)$ is	(s) $\tan^{-1}\left(\frac{75}{16}\right)$

ANSWERS Exercise No. (1)

1. (b) **2.** (d) **3.** (a) **4.** (b) **5.** (d)

6. (b) **7.** (b) **8.** (a) **9.** (c) **10.** (c)

11. (c) **12.** (b) **13.** (b) **14.** (c) **15.** (d)

16. (b) **17.** (a) **18.** (b) **19.** (c) **20.** (b)

21. (a , c) **22.** (b , c) **23.** (b , d) **24.** (a , b , c , d) **25.** (a , b , c , d)

26. (a) **27.** (d) **28.** (d) **29.** (a) **30.** (b)

ANSWERS Exercise No. (2)

1. (b) **2.** (b) **3.** (b) **4.** (d) **5.** (b)

6. (d) **7.** (b) **8.** (c) **9.** (c) **10.** (4)

11. (5) **12.** (3) **13.** (0) **14.** (2)

15. (a) → p , r
(b) → p , s
(c) → p , q
(d) → p , q

16. (a) → r
(b) → s
(c) → p
(d) → q

Chapter 29

Vectors

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-30)

1. If $\vec{b}$ and $\vec{c}$ are two non-collinear unit vectors and $\vec{a}$ is any vector , then $\left(\vec{a}.\vec{b}\right)\vec{b}+\left(\vec{a}.\vec{c}\right)\vec{c}+\dfrac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left|\vec{b}\times\vec{c}\right|^2}\left(\vec{b}\times\vec{c}\right)$ is equal to :

(a) $\vec{0}$ (b) $\vec{a}$

(c) $\vec{b}$ (d) $\vec{c}$

2. In a quadrilateral $PQRS$, $\overrightarrow{PQ}=\vec{a}$, $\overrightarrow{QR}=\vec{b}$ and $\overrightarrow{SP}=\vec{a}-\vec{b}$, M is mid point of QR and X is a point on SM such that $SX = kSM$, if P , X and R are collinear , then k equals to :

(a) $\dfrac{4}{7}$ (b) $\dfrac{7}{4}$

(c) $\dfrac{4}{5}$ (d) $\dfrac{5}{4}$

3. If $\vec{a}$ and $\vec{b}$ are unit vectors perpendicular to each other and $\vec{c}$ is another unit vector inclined at an angle θ to both $\vec{a}$ and $\vec{b}$, if $\vec{c}=\left\{p\left(\vec{a}+\vec{b}\right)+q\left(\vec{a}\times\vec{b}\right)\right\}$; p , $q\in R$, then

(a) $\dfrac{\pi}{4}\le\theta\le\pi$ (b) $\dfrac{\pi}{4}\le\theta\le\dfrac{3\pi}{4}$

(c) $0\le\theta\le\dfrac{\pi}{4}$ (d) $\theta\in[0,\pi]$

4. If non-zero vector $\vec{a}$ satisfy the condition $\hat{k}\times\left[\left(\vec{a}-\hat{i}\right)\times\hat{k}\right]+\hat{j}\times\left[\left(\vec{a}-\hat{k}\right)\times\hat{j}\right]+\hat{i}\times\left[\left(\hat{a}-\hat{j}\right)\times\hat{i}\right]=0$, then $\left|\vec{a}\right|$ is equal to :

(a) 1 (b) $\dfrac{1}{\sqrt{3}}$

(c) $\dfrac{3}{2\sqrt{3}}$ (d) none of these

5. If $\left[\vec{a}\ \vec{b}\ \vec{x}\right]=0$; $\vec{a}.\vec{x}=7$ and $\vec{x}.\vec{b}=0$, $\vec{a}(-1 , 1 , 1)$ and $\vec{b}(2 , 0 , 1)$, then $\vec{x}$ is :

(a) $-3\hat{i}+4\hat{j}+6\hat{k}$ (b) $-\dfrac{3}{2}\hat{i}+\dfrac{5}{2}\hat{j}+3\hat{k}$

(c) $3\hat{i}+16\hat{j}-6\hat{k}$ (d) $3\hat{i}-5\hat{j}-6\hat{k}$

6. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar non-zero vectors and $\vec{r}$ is any vector is space , then $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{r}\times\vec{c}\right)+\left(\vec{b}\times\vec{c}\right)\times\left(\vec{r}\times\vec{a}\right)+\left(\vec{c}\times\vec{a}\right)\times\left(\vec{r}\times\vec{b}\right)$ is :

(a) $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{r}$ (b) $3\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{r}$

(c) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{r}$ (d) $\vec{0}$

7. If three concurrent edges of a parallelepiped represent the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ such that $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\lambda$, $\lambda\in R^+$, then volume of parallelepiped whose three concurrent edges are the three concurrent diagonals of three faces of given parallelepiped is :

(a) λ (b) 2λ

(c) 3λ (d) $\dfrac{\lambda}{2}$

8. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors , then value of $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2$ doesn't exceed :

(a) 4 (b) 9

(c) 8 (d) 6

9. For coplanar points $A\left(\vec{a}\right)$, $B\left(\vec{b}\right)$, $C\left(\vec{c}\right)$ and $D\left(\vec{d}\right)$ if $\left(\vec{a}-\vec{d}\right).\left(\vec{b}-\vec{c}\right)=\left(\vec{b}-\vec{d}\right).\left(\vec{c}-\vec{a}\right)=0$, then point D for ΔABC is :

(a) Incentre

(b) Circumcentre

(c) Orthocentre

(d) Centroid

10. A unit vector in plane of vectors $2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\hat{j}+\hat{k}$ and orthogonal to $5\hat{i}+2\hat{j}+6\hat{k}$ is :

(a) $\dfrac{6\hat{i}-5\hat{k}}{\sqrt{61}}$ (b) $\dfrac{3\hat{j}-\hat{k}}{\sqrt{10}}$

(c) $\dfrac{2\hat{i}-5\hat{j}}{\sqrt{29}}$ (d) $\dfrac{2\hat{i}+\hat{j}-2\hat{k}}{3}$

11. Let $|\vec{b}|=|\vec{c}|=1$ and $\vec{a}$ is any vector , then value of $\left(\vec{a}\times\left(\vec{b}+\vec{c}\right)\right)\times\left(\vec{b}\times\vec{c}\right).\left(\vec{b}-\vec{c}\right)$ is always equal to :

(a) $|\vec{a}|$ (b) 1

(c) 0 (d) none of these

12. If equations $\vec{r}\times\vec{a}=\vec{b}$ and $\vec{r}\times\vec{c}=\vec{d}$ are consistent , then

(a) $\vec{a}.\vec{d}+\vec{b}.\vec{c}=0$ (b) $\vec{a}.\vec{d}=\vec{c}.\vec{d}$

(c) $\vec{b}.\vec{c}=\vec{a}.\vec{d}=0$ (d) $\vec{a}.\vec{d}+\vec{c}.\vec{d}=0$

13. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-\hat{k}$. A vector $\vec{d}$ lies in plane of $\vec{a}$ and $\vec{b}$ and its projection on $\vec{c}$ is of magnitude $\dfrac{1}{\sqrt{3}}$ units , then $\vec{b}$ is :

(a) $2\hat{i}+\hat{j}+2\hat{k}$ (b) $4\hat{i}-\hat{j}+3\hat{k}$

(c) $3\hat{i}-\hat{j}+2\hat{k}$ (d) $-\hat{i}+2\hat{j}+3\hat{k}$

14. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-coplanar vectors where

$$\vec{b_1}=\vec{b}-\frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a} \quad \text{and} \quad \vec{c_1}=\vec{c}-\frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{b_1}.\vec{c}}{|\vec{b_1}|^2}\vec{b_1},$$

then :

(a) $\vec{b_1}.\vec{b}=0$ (b) $\vec{a}\times\vec{b_1}=\vec{0}$

(c) $\vec{b_1}.\vec{c_1}=0$ (d) $\vec{c}\times\vec{c_1}=\vec{0}$

15. For non-zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ the equality $\left|\left(\vec{a}\times\vec{b}\right).\vec{c}\right|=|\vec{a}||\vec{b}||\vec{c}|$ holds if and only if :

(a) $\vec{a}.\vec{b}=0$; $\vec{b}.\vec{c}=0$.

(b) $\vec{b}.\vec{c}=0$; $\vec{c}.\vec{a}=0$.

(c) $\vec{a}.\vec{c}=\vec{a}.\vec{b}=0$.

(d) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$.

16. If a non-zero vector $\vec{a}$ is parallel to the line of intersection of the planes determined by vectors $\hat{i}$, $\hat{i}+\hat{j}$ and the plane determined by $\hat{i}-\hat{j}$, $\hat{i}+\hat{k}$, then angle between $\vec{a}$ and $\hat{i}-2\hat{j}+2\hat{k}$ is

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{3}$

(c) 0 (d) $\dfrac{\pi}{4}$

17. If $\vec{a}$ and $\vec{b}$ are non-parallel vectors and $\sqrt{3}(\hat{a}\times\vec{b})$ and $\vec{b}-(\hat{a}.\vec{b})\hat{a}$ represent two sides of a triangle , then internal angles of triangle are :

(a) 90º , 45º , 45º

(b) 90º , 60º , 30º

(c) 90º , 75º , 15º

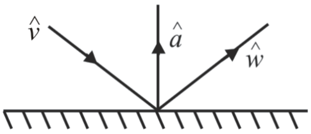
(d) none of these

18. Let $\vec{V}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{W}=\hat{i}+3\hat{k}$, if $\vec{U}$ is unit vector , then minimum value of $\left[\vec{U}\ \vec{V}\ \vec{W}\right]$ is :

(a) 0 (b) $-\sqrt{60}$

(c) $-\sqrt{59}$ (d) $-\sqrt{10}+\sqrt{6}$

19. If incident ray is along unit vector $\hat{v}$ and the reflected ray is along unit vector $\hat{w}$, the normal is along unit vector $\hat{a}$ outwards , then $\hat{w}$ is equal to :



(a) $\vec{v}+2\left(\vec{a}.\vec{v}\right)\hat{a}$ (b) $\hat{v}-2\left(\hat{a}.\hat{v}\right)\hat{a}$

(c) $\hat{v}+2\left(\hat{a}.\hat{v}\right)\hat{a}$ (d) none of these

20. If in a ΔABC , $\overrightarrow{BC}=\dfrac{\vec{e}}{|\vec{e}|}-\dfrac{\vec{f}}{|\vec{f}|}$ and $\overrightarrow{AC}=\dfrac{2\vec{e}}{|\vec{e}|}$; $|\vec{e}|\neq|\vec{f}|$, then value of $(\cos 2A+\cos 2B+\cos 2C)$ is :

(a) – 1 (b) 0

(c) 2 (d) $-\dfrac{3}{2}$

21. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=1$ and $\vec{a}.\vec{c}=\frac{1}{2}$, then

(a) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

(b) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar

(c) $\vec{b}, \vec{d}$ are non-parallel

(d) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

22. Let $\vec{a}, \vec{b}, \vec{c}$ be non-coplanar vectors and P_1, P_2, P_3, ... P_6 are six permutations of S.T.P. of $\vec{a}, \vec{b}$ and $\vec{c}$ then $\frac{P_i}{P_j}+\frac{P_k}{P_l}$, where i, j, k, l are different numbers from 1 to 6 , can not attain the value :

(a) 0 (b) 1

(c) 2 (d) –2

23. If $A(\vec{a}), B(\vec{b}), C(\vec{c})$ and $D(\vec{d})$ form a cyclic quadrilateral , then value of

$$\left\{\frac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{d}+\vec{d}\times\vec{a}\right|}{(\vec{b}-\vec{a}).(\vec{d}-\vec{a})}+\frac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{d}\times\vec{b}\right|}{(\vec{b}-\vec{c}).(\vec{d}-\vec{c})}\right\} \text{ is :}$$

(a) 1 (b) 0 (c) $\frac{1}{4}$ (d) 4

24. For non-coplanar vectors $\vec{a}, \vec{b}, \vec{c}$ if $\vec{r}=(\vec{a}.\vec{b})\vec{c}-(\vec{a}.\vec{c})\vec{b}$ then which one of the following options is incorrect ?

(a) $\vec{r}.\vec{a}=0$ (b) $\vec{r}.\vec{b}\times\vec{c}=0$

(c) $\vec{r}.\vec{a}\times\vec{c}=0$ (d) $\vec{r}=(\vec{b}\times\vec{c})\times\vec{a}$

25. If $\vec{a}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}-2\hat{k}$ are adjacent sides of a parallelogram , then angle between its diagonals is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

26. If $\vec{a}=x\hat{i}+(x-1)\hat{j}+\hat{k}$ and $\vec{b}=(x+1)\hat{i}+\hat{j}+a\hat{k}$ always form an acute angle with each other $\forall\ x\in R$, then

(a) $a\in(-\infty, 2)$ (b) $a\in(2, \infty)$

(c) $a\in(-\infty, 1)$ (d) $a\in(1, \infty)$

27. Let $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be any four vectors , then $\left[\vec{a}\times\vec{b}\ \ \vec{a}\times\vec{c}\ \ \vec{d}\right]$ is always equal to :

(a) $(\vec{a}.\vec{d})\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ (b) $(\vec{a}.\vec{c})\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

(c) $(\vec{a}.\vec{b})\left[\vec{a}\ \vec{b}\ \vec{d}\right]$ (d) 0

28. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors , if $\vec{u_1}=\vec{a}-(\vec{a}.\vec{b})\vec{b}$ and $\vec{u_2}=\vec{a}\times\vec{b}$, then $\left|\vec{u_2}\right|$ is equal to :

(a) $\left|\vec{u_1}\right|+\left|\vec{u_1}.\vec{a}\right|$ (b) $\left|\vec{u_1}.(\vec{a}+\vec{b})\right|$

(c) $\left|\vec{u_1}\right|+\left|\vec{u_1}.\vec{b}\right|$ (d) $\left|\vec{u_1}.(\vec{a}-\vec{b})\right|$

29. Let $\vec{r}, \vec{a}, \vec{b}$ and $\vec{c}$ be four non-zero vectors such that $\vec{r}.\vec{a}=0$, $\left|\vec{r}\times\vec{b}\right|=\left|\vec{r}\right|\left|\vec{b}\right|$ and $\left|\vec{r}\times\vec{c}\right|=\left|\vec{r}\right|\left|\vec{c}\right|$, then $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is equal to :

(a) 0 (b) $\left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|$

(c) $\left|\vec{a}\right|+\left|\vec{b}\right|+\left|\vec{c}\right|$ (d) $-\left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|$

30. Let $ABCD$ be parallelogram , where A_1 and B_1 are the midpoints of side BC and CD respectively , if $\overrightarrow{AA_1}+\overrightarrow{AB_1}=\lambda\overrightarrow{AC}$, then '$\lambda$' is equal to :

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$

(c) $\frac{4}{5}$ (d) $\frac{5}{4}$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 31-35)

31. In triangle ABC, let $\overrightarrow{CB}=\vec{a}$, $\overrightarrow{CA}=\vec{b}$ and the altitude from vertex B on the opposite side meets the side CA at D. If $\overrightarrow{CD}=\vec{\lambda}$ and $\overrightarrow{DB}=\vec{\mu}$, then :

(a) $\vec{\lambda}=\frac{(\vec{a}.\vec{b})\vec{a}}{\left|\vec{a}\right|^2}$ (b) $\vec{\lambda}=\frac{(\vec{a}.\vec{b})\vec{b}}{\left|\vec{b}\right|^2}$

(c) $\vec{\mu}=\frac{\left|\vec{b}\right|^2\vec{a}-(\vec{a}.\vec{b})\vec{b}}{\left|\vec{b}\right|^2}$ (d) $\vec{\mu}=\frac{\vec{b}\times(\vec{a}\times\vec{b})}{\left|\vec{b}\right|^2}$

32. Let $\vec{b}=\left(\frac{e}{e^{\cos^2 x}}\right)\hat{i}+(\cos x)\hat{j}+[\,|\sin x|+|\cos x|\,]\hat{k}$ and $\vec{a}=\left(e^{\sin^2 x}\right)\hat{i}+\left(xe^{\sin x}\right)\hat{j}+\hat{k}$, where [.] represents the greatest integer function. If $\vec{a}\times\vec{b}=\vec{0}$, then :

(a) unique value of x exists in $\left(0\,,\frac{\pi}{2}\right)$.

(b) exactly two values of x exist in $\left(0\,,\frac{\pi}{2}\right)$.

(c) no value of x exist in $\left(-\frac{3\pi}{2}\,,-\pi\right)$.

(d) unique value of x exists in $\left(-\frac{3\pi}{2}\,,-\pi\right)$.

33. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $\hat{a}.\hat{b}>0$. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ is given by $(\cos t)\hat{a}+(\sin t)\hat{b}$. When $'P'$ is farthest from origin $'O'$, let $'L'$ be the length of $\overrightarrow{OP}$ and $\hat{n}$ be the unit vector along $\overrightarrow{OP}$, then :

(a) $\hat{n}=\frac{\hat{a}+\hat{b}}{\left|\hat{a}+\hat{b}\right|}$ (b) $\hat{n}=\frac{\hat{a}-\hat{b}}{\left|\hat{a}-\hat{b}\right|}$

(c) $L=\sqrt{1+\hat{a}\,.\,\hat{b}}$ (d) $L=\sqrt{1+2\hat{a}\,.\,\hat{b}}$

34. If $\lambda\in R$, $\vec{a}=(-\lambda^2)\,\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-(\lambda^2)\,\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-(\lambda^2)\hat{k}$, then which of the following statements are correct ?

(a) $\left(\vec{a}\times\vec{b}\right).\vec{c}$ is zero for exactly one positive value of λ.

(b) $\left(\vec{a}\times\vec{b}\right).\vec{c}$ is zero for exactly four real values of λ ,

(c) $\left(\vec{a}\times\vec{b}\right).\vec{c}$ is zero for exactly one negative value of λ.

(d) $\left(\vec{a}\times\vec{b}\right).\vec{c}$ is zero for at least four real values of λ.

35. Let a , b , c be the sides of a scalene triangle and $\lambda\in R$. If angle between the vectors $\vec{\alpha}$ and $\vec{\beta}$ is not more that $\frac{\pi}{2}$, where $\vec{\alpha}=(a+b+c)\hat{i}-3\lambda\,\hat{j}+ac\,\hat{k}$ and $\vec{\beta}=(a+b+c)\hat{i}+(ab+bc)\hat{j}-3\lambda\,\hat{k}$, then exhaustive set of values of $'\lambda'$ contains :

(a) $[-1\,,0]$ (b) $\left[0\,,\frac{4}{3}\right]$

(c) $\left(\tan\frac{\pi}{8}\,,\tan\frac{3\pi}{8}\right)$ (d) $\left[1\,,\frac{5}{4}\right]$

Assertion Reasoning questions :
(Questions No. 36-40)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

36. Statement 1 : Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-zero vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ is perpendicular to $\left(\vec{a}\times\vec{b}\right)\times\vec{c}$, then value of $\vec{a}.\vec{c}$ must be zero

because

Statement 2 : $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ reprsents a vector which lie in the plane of vectors $\vec{b}$ and $\vec{c}$, and is perpendicular to $\vec{a}$ where the magnitude of $\vec{a}$, $\vec{b}$, $\vec{c}$ is non-zero.

37. Statement 1 : Let $\vec{a}=\hat{i}+2\hat{j}+4\hat{k}$, $\vec{b}=\hat{i}-\hat{j}-6\hat{k}$ be two vectors such that $\vec{r}\times\vec{a}=\vec{a}\times\vec{b}$ and $\vec{r}\times\vec{b}=\vec{b}\times\vec{a}$, then unit vector along the direction of $\vec{r}$ is given by $\pm\frac{1}{9}\left(2\hat{i}+\hat{j}-2\hat{k}\right)$

because

Statement 2 : $\vec{r}$ is parallel to $\vec{a}+\vec{b}$.

38. Statement 1 : If $\vec{u}$, $\vec{v}$, $\vec{w}$ are non-coplanar vectors and $p\,,q\in R$, then the equality $\left[3\vec{u}\;\;p\vec{v}\;\;p\vec{w}\right]-\left[p\vec{v}\;\;\vec{w}\;\;q\vec{u}\right]-\left[2\vec{w}\;\;q\vec{v}\;\;q\vec{u}\right]=0$ holds for exactly one ordered pair $(p\,,q)$

because

Statement 2 : if $ax^2+bxy+cy^2=0$ where $a\,,b\,,c\in R$ and $a\neq 0$, $b^2-4ac<0$, then $x=y=0$, provided $x\,,y\in R$.

39. **Statement 1 :** Let $\vec{a}$ and $\vec{b}$ be two perpendicular unit vectors such that $\vec{r} = \vec{b} + \left(\vec{r} \times \vec{a}\right)$, then $\left|\vec{r}\right|$ is equal to $\frac{\sqrt{2}}{2}$

because

Statement 2: $2\vec{r} = \vec{b} + \vec{a} \times \vec{b}$

40. **Statement 1 :** Let $\vec{a}, \vec{b}, \vec{c}$ be non-coplanar and non-zero vectors such that $\vec{r} = \left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} \times \vec{c}\right)$, then $\vec{r}$ and $\vec{a}$ are linearly dependent vectors

because

Statement 2 : $\vec{r}$ is perpendicular to the vectors $\vec{b}$ and $\vec{c}$.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

For triangle ABC , let the position vector of the vertices A , B , C be $\hat{i}-2\hat{j}+2\hat{k}$, $\hat{i}+4\hat{j}$ and $-4\hat{i}+\hat{j}+\hat{k}$ respectively. If point D lies on the side AC, where $\overrightarrow{AD}.\overrightarrow{BD}=0$, then answer the following questions.

1. If $'O'$ represents the origin , then value of $\left|\overrightarrow{OD}\right|$ is equal to :

(a) $3\sqrt{\frac{5}{7}}$ (b) $2\sqrt{\frac{15}{7}}$

(c) $\sqrt{\frac{50}{7}}$ (d) $\sqrt{\frac{39}{7}}$

2. Area (in square units) of the triangle CDB is equal to :

(a) $\frac{150\sqrt{6}}{49}$ (b) $\frac{75\sqrt{6}}{49}$

(c) $\frac{10\sqrt{3}}{7}$ (d) $\frac{60\sqrt{5}}{7}$

3. The angle DBC is equal to :

(a) $\frac{\pi}{12}$ (b) $\cos^{-1}\left(\frac{2\sqrt{10}}{7}\right)$

(c) $\cos^{-1}\left(\frac{3\sqrt{5}}{7}\right)$ (d) $\cos^{-1}\left(\frac{\sqrt{13}}{7}\right)$

Comprehension passage (2)
(Questions No. 4-6)

Let $P\left(\vec{p}\right)$, $Q\left(\vec{p}+\vec{r}\right)$, $R\left(\vec{r}\right)$, $S\left(\lambda\vec{p}\right)$ and $T\left(\lambda\vec{r}\right)$ represents the vertices of a regular polygon $PQRST$, where the area (in square units) enclosed by the polygon is given by $\mu\left|\vec{p}\times\vec{r}\right|$. If the centre of polygon $PQRST$ is C_0 , then answer the following questions.

4. The value of $\frac{\left|\overrightarrow{PS}\right|}{\left|\overrightarrow{QR}\right|}$ is equal to :

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{2}$

(c) $\frac{\sqrt{5}+1}{4}$ (d) $\frac{\sqrt{5}-1}{2}$

5. The value of $'\mu'$ is equal to :

(a) $\frac{5-\sqrt{5}}{2}$ (b) $\frac{\sqrt{5}-1}{2}$

(c) $\frac{\sqrt{5}+1}{4}$ (d) $\frac{5+\sqrt{5}}{4}$

6. The position vector of centre $'C_0'$ is :

(a) $\frac{5+\sqrt{5}}{10}\left(\vec{p}+\vec{q}\right)$ (b) $\frac{5+\sqrt{5}}{2}\left(\vec{p}+\vec{q}\right)$

(c) $\frac{5-\sqrt{5}}{5}\left(\vec{p}+\vec{q}\right)$ (d) $\frac{5-\sqrt{5}}{10}\left(\vec{p}+\vec{q}\right)$

Comprehension passage (3)
(Questions No. 7-9)

Let $\vec{e}_1,\vec{e}_2,\vec{e}_3$ and $\vec{f}_1$, $\vec{f}_2$, $\vec{f}_3$ be two sets of non-coplanar vectors such that $\vec{e}_m.\vec{f}_n=\begin{cases}1 & ; \quad m=n\\ 0 & ; \quad m\neq n\end{cases}$, where m , $n\in\{1 , 2 , 3\}$. If values of $\left[\vec{e}_1\ \vec{e}_2\ \vec{e}_3\right]$ and $\left[\vec{f}_1\ \vec{f}_2\ \vec{f}_3\right]$ are positive , then answer the following questions.

7. The least value of $16\left[\vec{e}_1\ \vec{e}_2\ \vec{e}_3\right]+9\left[\vec{f}_1\ \vec{f}_2\ \vec{f}_3\right]$ is equal to :

(a) 10 (b) 24

(c) 12 (d) 20

8. Let $\alpha=\left[\vec{e}_1+\vec{e}_2\quad \vec{e}_2+\vec{e}_3\quad \vec{e}_3+\vec{e}_1\right]$ and $\beta=\left[\vec{f}_1+\vec{f}_2\quad \vec{f}_2+\vec{f}_3\quad \vec{f}_3+\vec{f}_1\right]$, then roots of the equation $\left[2\vec{e}_1\ 4\vec{e}_2\ 3\vec{e}_3\right]x^2+(\alpha\beta)x+\left[2\vec{f}_1\ \vec{f}_2\ 3\vec{f}_3\right]=0$ are :

(a) real and distinct (b) real and equal

(c) imaginary (d) real

9. Let $\alpha = \left[\vec{e}_1\times\vec{e}_2 \ \ \vec{e}_2\times\vec{e}_3 \ \ \vec{e}_3\times\vec{e}_1\right]$ and $\beta = \left[\vec{f}_1\times\vec{f}_2 \ \ \vec{f}_2\times\vec{f}_3 \ \ \vec{f}_3\times\vec{f}_1\right]$, then the incorrect statement is :

(a) there exists some x such that $\sin x + \cos x = \alpha\beta$

(b) equation $x^2 + (\alpha\beta)x + 1$ is having two different roots

(c) least value of $(9\alpha + 4\beta)$ is 12

(d) there exists some x such that $|\sin x| + |\cos x| = \alpha + \beta$

Questions with Integral Answer :
(Questions No. 10-15)

10. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors such that $\left|\dfrac{\vec{a}+\vec{b}}{2} + \vec{a}\times\vec{b}\right| = 1$, then value of $\dfrac{\left|\vec{a}-\vec{b}\right|}{\left|\vec{a}\times\vec{b}\right|}$ is equal to ....

11. Let $\sum\limits_{r=1}^{3}(a_r + b_r + c_r) = 6$, where a_r , b_r , c_r are non-negative real numbers and $r \in \{1 , 2 , 3\}$. If $'V'$ be the volume of the parallelepiped formed by three coterminous edges representing the vectors $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then the maximum value of $'V'$ is equal to ....

12. If $\vec{b} = \vec{a}\times\left(\hat{i}\times\vec{a}\right) + \vec{a}\times\left(\hat{j}\times\vec{a}\right) + \vec{a}\times\left(\hat{k}\times\vec{a}\right)$ and $\vec{a}.\left(\hat{i}+\hat{j}+\hat{k}\right) = 0$, then value of $\left\{\dfrac{\left|\vec{b}\right|^2}{\left|\vec{a}\right|^4}\right\}$ is equal to ....

13. Let $\vec{a}$ be unit vector and $\vec{b} = 2\hat{i} - 2\hat{j} - \hat{k}$, $\vec{c} = 2\hat{i} - \hat{j}$, where $\vec{a}$ is non-collinear with $\vec{b}$ and $\vec{c}$. If $P = \left\{(\vec{a}-\vec{b})\times(\vec{a}-\vec{b}-\vec{c})\right\}.(\vec{a}+2\vec{b}-\vec{c})$, then maximum value of $'P'$ is equal to ....

14. Let $\vec{u}$, $\vec{v}$, $\vec{w}$ be three non-coplanar unit vectors , where $\vec{u}.\vec{v} = \cos\alpha$, $\vec{v}.\vec{w} = \cos\beta$ and $\vec{w}.\vec{u} = \cos\gamma$. If $\vec{x}$, $\vec{y}$ and $\vec{z}$ are the unit vectors along the bisector of the angles α , β and γ respectively , then value of

$$\left\{\frac{\left[\vec{u} \ \ \vec{v} \ \ \vec{w}\right]^2 \sec^2\dfrac{\alpha}{2}\sec^2\dfrac{\beta}{2}\sec^2\dfrac{\gamma}{2}}{\left[\vec{x}\times\vec{y} \ \ \vec{y}\times\vec{z} \ \ \vec{z}\times\vec{x}\right]}\right\}^{\frac{1}{2}}$$

is equal to ....

Matrix Matching Questions :
(Questions No. 15-16)

15. Match the following columns (I) and (II).

Column (I)	Column (II)
(a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ form sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ of ΔABC , then	(p) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$
(b) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are forming three adjacent sides of regular tetrahedron , then	(q) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$
(c) If $\vec{a}\times\vec{b} = \vec{c}$, $\vec{b}\times\vec{c} = \vec{a}$, where $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero vectors , then	(r) $\left|\vec{a}\times\vec{b}\right| = \left|\vec{b}\times\vec{c}\right| = \left|\vec{c}\times\vec{a}\right|$
(d) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors , and $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, then	(s) $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\dfrac{3}{2}$

16. Match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three collinear vectors , then	(p) the vectors are position vectors of three collinear points
(b) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three coplanar vectors , then	(q) the volume of parallelopiped formed by the vectors is non-zero
(c) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar vectors , then	(r) the volume of parallelopiped formed by the vectors is zero
(d) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero vectors such that exactly two of them are collinear , then	(s) there exists a plane which contain all the three vectors

ANSWERS Exercise No. (1)

1. (b)
2. (c)
3. (b)
4. (c)
5. (b)
6. (a)
7. (b)
8. (b)
9. (c)
10. (b)
11. (c)
12. (a)
13. (a)
14. (c)
15. (d)
16. (d)
17. (b)
18. (c)
19. (b)
20. (a)
21. (c)
22. (b)
23. (b)
24. (c)
25. (c)
26. (b)
27. (a)
28. (c)
29. (a)
30. (b)
31. (b , c , d)
32. (a , c)
33. (a , c)
34. (a , c)
35. (a , d)
36. (d)
37. (a)
38. (a)
39. (c)
40. (c)

ANSWERS Exercise No. (2)

1. (d)
2. (a)
3. (b)
4. (b)
5. (d)
6. (d)
7. (b)
8. (c)
9. (d)
10. (2)
11. (8)
12. (3)
13. (9)
14. (4)

15. (a) → r
(b) → p , r
(c) → q , p , r
(d) → p , r , s

16. (a) → p , r , s
(b) → r , s
(c) → q
(d) → r ,s

Chapter
30

3-Dimensional Geometry

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-25)

1. If the line of intersection of planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=3$ and $\vec{r}.\left(2\hat{i}+3\hat{j}+\hat{k}\right)=9$ is normal to the plane $\vec{r}.\left(a\hat{i}+b\hat{j}+4\hat{k}\right)=5$, then value of $(a+b)$ is :

(a) 4 (b) – 4
(c) 8 (d) –8

2. If the line $\frac{x+4}{3}=\frac{y+6}{5}=\frac{z-1}{-2}$ and the line of intersections of plane $3x-2y+z+5=0$ and $2x+3y+4z-K=0$ are coplanar , then value of 'K' equals to :

(a) 4 (b) 2
(c) –1 (d) 3

3. If line $\frac{x-1}{2k}=\frac{y+k}{1}=\frac{z-1}{-4}$ is contained by the plane $3x+4y+(k+2)z+1=0$, then :

(a) $k=1$ (b) $k=-2$
(c) $k=2$ (d) no real 'k' exists

4. Minimum distance between the lines given by $\frac{x+2}{1}=\frac{y+1}{2}=\frac{z-2}{1}$ and $\frac{x-1}{-1}=\frac{y+3}{2}=\frac{z-1}{1}$ is equal to :

(a) $\sqrt{3}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{4}{\sqrt{5}}$ (d) none of these

5. Let $P(3,2,6)$ be a point in space and Q be a point on the line $\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k})$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x-4y+3z=1$ is :

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

6. A line with positive direction cosines passes through the point $P(2,-1,2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x+y+z=9$ at point Q. The length of the line segment PQ equals to :

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

7. A plane $P_1=0$ passes through $(1,-2,1)$ and is normal to two planes : $2x-2y+z=0$ and $x-y+2z+4=0$, then distance of the plane $P_1=0$ from $(1,2,2)$ is :

(a) $\sqrt{2}$ (b) $2\sqrt{2}$

(c) $3\sqrt{2}$ (d) 4

8. The lines whose vector equation are $\vec{r}=\vec{a}+\lambda\vec{b}$ and $\vec{r}=\vec{c}+\mu\vec{d}$ are coplanar , where $\lambda , \mu\in R$, then :

(a) $\left(\vec{a}-\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=0$ (b) $\left(\vec{a}-\vec{c}\right).\left(\vec{b}\times\vec{d}\right)=0$

(c) $\left(\vec{b}-\vec{c}\right).\left(\vec{a}\times\vec{d}\right)=0$ (d) $\left(\vec{b}-\vec{d}\right).\left(\vec{a}\times\vec{c}\right)=0$

9. If the equations , $ax+by+cz=0$, $bx+cy+az=0$ and $cx+ay+bz=0$ represents the line $x=y=z$, then

(a) $ab+bc+ac=a^2+b^2+c^2$; $a+b+c=0$

(b) $ab+bc+ac\neq a^2+b^2+c^2$; $a+b+c=0$

(c) $ab+bc+ac=a^2+b^2+c^2$; $a+b+c\neq 0$

(d) $ab+bc+ac\neq a^2+b^2+c^2$; $a+b+c\neq 0$

10. Let plane $P=0$ passes through the intersection of planes $2x-y+z-3=0$ and $3x+y+z-5=0$. If distance of plane $P=0$ from $(2,1,-1)$ is $\frac{1}{\sqrt{6}}$ then its equation can be :

(a) $2x-y+z+3=0$ (b) $62x+29y+19z-105=0$
(c) $2x+y-z-3=0$ (d) $62x-29y+19z+105=0$

11. Let plane $P_1=0$ passes through the points $(1,-1,1)$, $(1,1,1)$ and $(-1,-3,-5)$. If point $(3,\alpha,7)$ lies on the plane $P_1=0$, then number of possible values of 'α' is / are :

(a) 1 (b) 2 (c) 0 (d) infinite

12. The angle between the lines whose direction cosines are given by the relations , $l^2 + m^2 - n^2 = 0$ and $l + m + n = 0$, is given by :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$

(c) 0 (d) $\frac{\pi}{4}$

13. If a plane passing through the point $(4\,,-5\,,6)$ meets the co-ordinate axes at $A\,,B$ and C such that centroid of triangle ABC is the point $(1\,,K\,,K^2)$, then value of $'K'$ can be :

(a) 1 (b) –4

(c) 3 (d) –1

14. Let a system of three planes be given by :

$$\lambda x + y + z - 1 = 0$$
$$x + \lambda y + z - \lambda = 0$$
$$x + y + \lambda z - \lambda^2 = 0$$

If no common point exists which may satisfy all the three planes simultaneously , then :

(a) $\lambda \in R - \{1\}$ (b) $\lambda \neq -2$

(c) $\lambda = -2$ (d) $\lambda \neq 1$ and -2

15. The distance of the point (1 , –2 , 3) from the plane $x - y + z - 5 = 0$, measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{-6}$, is equal to :

(a) 1 unit (b) 2 units

(c) 3 units (d) 5 units

16. If a variable plane passes through the point (1 , 1 , 1) and meets the co-ordinate axes at A , B and C , then locus of the common point of intersection of the planes through A , B and C and parallel to the coordinate planes is given by :

(a) $x + y + z = xyz$ (b) $xy + yz + zx = xyz$

(c) $x^2 + y^2 + z^2 = xyz$ (b) $xy + yz + zx = x + y + z$

17. Let $P_1 : \vec{r}.\vec{n_1} - d_1 = 0$, $P_2 : \vec{r}.\vec{n_2} - d_2 = 0$ and $P_3 : \vec{r}.\vec{n_3} - d_2 = 0$ be three planes , where $\vec{n_1}$, $\vec{n_2}$ and $\vec{n_3}$ are three non-coplanar vectors. If three lines are defined in unsymmetrical form by , $P_1 = P_2 = 0$, $P_2 = P_3 = 0$ and $P_1 = P_3 = 0$, then the lines are :

(a) concurrent at a point.

(b) coincident.

(c) coplanar.

(d) parallel to each other.

18. Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$ be three unit vectors which are equally inclined to each other at an angle of $\frac{2\pi}{5}$. The angle between line $\vec{r} = \lambda\vec{a}$ and the plane $(\vec{r} - \vec{b}).(\vec{b} \times \vec{c}) = 0$, where $'\lambda'$ is parameter and $'O'$ is origin , is given by :

(a) $\cos^{-1}\left(\frac{\sqrt{5}+1}{\sqrt{5}-1}\right)$ (b) $\cos^{-1}\left(\frac{\sqrt{5}-2}{\sqrt{5}+1}\right)$

(c) $\cos^{-1}\left(\frac{3-\sqrt{5}}{2}\right)$ (d) $\cos^{-1}\left(\frac{1}{3+\sqrt{5}}\right)$

19. Let plane $P_1 = 0$ passes through $(1\,,1\,,1)$ and parallel to the lines L_1 and L_2 having direction ratios $\langle 1\,,0\,,-1\rangle$ and $\langle 1\,,-1\,,0\rangle$ respectively. If plane $P_1 = 0$ intersects the co-ordinate axes at A , B and C , then volume of tetrahedron $OABC$, where $'O'$ is origin , is given by :

(a) $\frac{18}{5}$ cubic units. (b) $\frac{9}{4}$ cubic units.

(c) $\frac{9}{6}$ cubic units. (d) $\frac{18}{4}$ cubic units.

20. If a line with direction ratios $\langle 0, 2\,,-1\rangle$ meet the lines $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ and $\frac{x-1}{1} = \frac{y+2}{3} = \frac{z-2}{-2}$ at $'A'$ and $'B'$ respectively , then the length of line segment AB is given by :

(a) $2\sqrt{5}$ (b) $4\sqrt{2}$

(c) $\sqrt{5}$ (d) $3\sqrt{5}$

21. If the plane $4x + 3y + 2 = 0$ is rotated about its line of intersection with the plane $z = 0$ by an angle of $\frac{\pi}{4}$, then the length of perpendicular from origin to the plane in new position is given by :

(a) $\frac{2}{\sqrt{5}}$ (b) $\frac{\sqrt{3}}{5}$ (c) $\sqrt{5}$ (d) $\frac{\sqrt{2}}{5}$

22. A variable plane is at a constant distance of 2 units from the origin $'O'$ and meets the co-ordinate axes at A , B and C. Locus of the centroid of the tetrahedron $OABC$ is given by :

(a) $x^2 + y^2 + z^2 = 1$ (b) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 16$

(c) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 4$ (d) $x^2 + y^2 + z^2 = 4$

23. If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a unique straight line , then value of $a^2 + b^2 + c^2 + 2abc$ is equal to :

(a) 0 (b) 2 (c) 1 (d) 4

24. Let plane $P_1 = 0$ passes through the point $P(\alpha\, ,\, \beta\, ,\, \gamma)$ and meets the co-ordinate axes at A , B and C. If $'O'$ is origin and OP is normal to plane $P_1 = 0$, then area of ΔABC, where $OP = \delta$, is given by :

(a) $\dfrac{\delta^3}{|2\alpha\beta\gamma|}$ (b) $\dfrac{\delta^5}{|\alpha\beta\gamma|}$ (c) $\dfrac{2\delta^5}{|\alpha\beta\gamma|}$ (d) $\dfrac{\delta^5}{|2\alpha\beta\gamma|}$

25. To form a rectanglar parallelopiped if planes are drawn through the points $(5\, ,\, 0\, ,\, 2)$ and $(3\, ,-2\, ,\, 5)$ parallel to the coordinate planes , then volume of the parallelopiped , in cubic units , is given by :

(a) 20 (b) 8

(c) 12 (d) 15

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. Let $P_1 : \vec{r}.\hat{a}_1 - d_1 = 0$, $P_2 : \vec{r}.\hat{a}_2 - d_2 = 0$

and $P_3 : \vec{r}.\hat{a}_3 - d_3 = 0$ be the vector equations of three distinct non-parallel planes such that $\vec{a}_1.\left(\vec{a}_2 \times \vec{a}_3\right) = 0$, where $d_1^2 + d_2^2 + d_3^2 \neq 0$, then which of the following statements are incorrect :

(a) for point $P\left(\vec{r}_1\right)$, if $\vec{r}_1.\hat{a}_1 - d_1 = 0$, $\vec{r}_1.\hat{a}_2 - d_2 = 0$ and $\vec{r}_1.\hat{a}_3 - d_3 \neq 0$, then there exists infinitely many points which are equidistant from the given three planes.

(b) for point $P\left(\vec{r}_1\right)$, if $\vec{r}_1.\hat{a}_1 - d_1 = 0$, $\vec{r}_1 . \hat{a}_2 - d_2 = 0$ and $\vec{r}_1.\hat{a}_3 - d_3 = 0$, then $P_2 = \lambda P_1 + \mu P_3$ for some scalar quantities λ and μ .

(c) number of common solutions of the plane $\vec{r}.\hat{n} - d_4 = 0$ with given three planes P_1 , P_2 and P_3 is either zero or one.

(d) for point $P\left(\vec{r}_1\right)$, if $\vec{r}_1 . \hat{a}_1 - d_1 = 0$, $\vec{r}_1.\hat{a}_2 - d_2 = 0$ and $\vec{r}_1 . \hat{a}_3 - d_3 = 0$, then point $'P'$ can be origin (i.e. $(0\, ,0\, ,0)$).

27. If the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersects in a straight line , then possible values of $'k'$ are

(a) 2 (b) 6

(c) 1 (d) 4

28. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$.

If $\vec{r}.\left(\vec{a} \times \vec{b}\right) = 0$ and projection of $\vec{r}$ on $\vec{c}$ is $\dfrac{1}{\sqrt{3}}$,

then $\vec{r}$ can be given by :

(a) $-2\hat{i} + 5\hat{j} - 2\hat{k}$ (b) $\hat{i} + \hat{j} + \hat{k}$

(c) $2\hat{i} + \hat{j} + 2\hat{k}$ (d) $-\hat{i} + \hat{j} - \hat{k}$

29. Let a variable plane be passing through the point $(1\, ,\, 1\, ,\, 1)$ and meeting the positive direction of coordinate axes at A , B and C , then volume of tetrahedron $OABC$, where $'O'$ represents the origin , can be :

(a) 4 cubic units (b) 5 cubic units

(c) 8 cubic units (d) 3 cubic units

30. Let A , B , C , D be four non-coplanar points and at the maximum N different planes are possible which are equidistant from A , B , C and D , then

(a) N is prime number (b) N is even integer

(c) N is more than 4 (d) N is less than 6

Assertion Reasoning questions : (Questions No. 31-35)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

31. Consider the following planes ,

$P_1 : ax + by + cz = 0$
$P_2 : bx + cy + az = 0$
$P_3 : cx + ay + bz = 0$

Statement 1 : If a , b , c are three distinct rcal numbers , then the planes P_1 , P_2 , P_3 have a common line of intersection when $a + b + c = 0$.

because

Statement 2 : $\dfrac{a^2 + b^2 + c^2}{ab + bc + ca} > 1$, if a , b , c are three distinct real numbers.

32. Let the vector equation of the lines L_1 and L_2 be given by $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \mu\left(4\hat{i} + 6\hat{j} + 8\hat{k}\right)$ respectively.

Statement 1 : Shortest distance between L_1 and L_2 is equal to $\frac{5}{\sqrt{29}}$ units

because

Statement 2 : for L_1 and L_2 there exists infinite lines of shortest distance.

33. In tetrahedron $OABC$, let the position vectors of A, B, C be $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively, where $\vec{c} + \left(\vec{c} \times \vec{a}\right) = \vec{b}$

Statement 1 : If $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = 1$, then maximum volume of the tetrahedron $OABC$ is $\frac{1}{12}$ cubic units

because

Statement 2 : the volume of tetrahedron $OABC$ is maximized if the faces OAB and OAC form right angled trianges.

34. Let A, B, C be the internal angles of triangle ABC, and the plane $\frac{x}{\sin A} + \frac{y}{\sin B} + \frac{z}{\sin C} = 1$ meet the co-ordinate axes at P, Q and R. If $'O'$ represents the origin , then

Statement 1 : volume of tetrahedron $OPQR$ cannot exceed $\frac{\sqrt{3}}{16}$ cubic units

because

Statement 2 : maximum value of $\sin A \sin B \sin C$ is $\frac{3\sqrt{3}}{8}$, where $A + B + C = \pi$.

35. **Statement 1 :** Let the direction cosines of a variable line in two adjacent positions be l, m, n and $l + \delta l$, $m + \delta m$, $n + \delta n$, where $\delta\theta$ is the small angle in radians between the two positions of the line , then $(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$

because

Statement 2: $\sin^2\left(\frac{\delta\theta}{2}\right) = \frac{1}{2}\left((\delta l)^2 + (\delta m)^2 + (\delta n)^2\right)$

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Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

If the planes , $\pi_1 = 0$, $\pi_2 = 0$ and $\pi_3 = 0$ have common line of intersection , where

$\pi_1 : x + y + 3z - 4 = 0$; $\pi_2 : x + 2y + z + 1 = 0$ and

$\pi_3 : \lambda x + 3y + \mu z - 3 = 0$, then answer the following questions.

1. Value of $(\lambda + 3\mu)$ is :

(a) 10 (b) 12

(c) 14 (d) 20

2. Common line of intersection of the planes $\pi_1 = 0$, $\pi_2 = 0$, $\pi_3 = 0$ can be given by :

(a) $\frac{x-1}{5} = \frac{y+1}{-2} = \frac{z-1}{-1}$ (b) $\frac{x+1}{5} = \frac{y-1}{-2} = \frac{z+1}{-1}$

(c) $\frac{x+1}{-5} = \frac{y+1}{2} = \frac{z-2}{1}$ (d) none of these

3. If plane $3x + \beta y + 7z + \alpha = 0$ contains the common line of intersection of planes $\pi_1 = 0$, $\pi_2 = 0$ and $\pi_3 = 0$, then value of $(\alpha + 2\beta)$ is :

(a) 0 (b) 1

(c) –1 (d) 2

Comprehension passage (2)
(Questions No. 4-6)

Let the line of intersection of the planes $3x + y - 2z + 3 = 0$ and $x + y + z - 7 = 0$ be $'L_1'$ and the incident ray along L_1 meet the plane mirror $2x + 2y - z - 2 = 0$ at point $'A'$. If the reflected ray is along the line $' L_2 '$, then answer the following questions.

4. Minimum distance of point $'A'$ from the surface of sphere $(x-3)^2 + (y-1)^2 + (z-2)^2 = 4$ is equal to :

(a) 1 (b) 4

(c) 5 (d) $\sqrt{3}$

5. Equation of line $'L_2'$ can be given by :

(a) $\frac{x+1}{2} = \frac{y+2}{4} = \frac{z+8}{12}$ (b) $\frac{x-18}{17} = \frac{y+5}{-7} = \frac{z-6}{2}$

(c) $\frac{x-8}{-7} = \frac{y+3}{5} = \frac{z-2}{2}$ (d) $\frac{x-1}{5} = \frac{y-2}{19} = \frac{z-4}{21}$

6. If the plane $'P'$ contains the point $'A'$ then the maximum distance of plane $'P'$ from the origin is equal to :

(a) $\frac{27}{\sqrt{35}}$ (b) $\frac{49}{\sqrt{18}}$

(c) $\frac{23}{\sqrt{27}}$ (d) none of these

Comprehension passage (3)
(Questions No. 7-9)

Consider four spherical balls S_1 , S_2 , S_3 and S_4 which are touching each other externally , where the radius of all the four balls is $\sqrt{12}$ units. Let the centre of the spherical balls S_1 , S_2 , S_3 and S_4 be $C_1\left(-\sqrt{12}\ , -2\ , 0\right)$, $C_2\left(\sqrt{12}\ , -2\ , 0\right)$, $C_3\left(x_3\ , y_3\ , 0\right)$, $C_4\left(x_4\ , y_4\ , z_4\right)$ respectively , where y_3 and z_4 is positive in nature. If the spherical ball $'S'$ of minimum volume enclose all the spherical balls S_1 , S_2 , S_3 and S_4 , where the points of contact are respectively P_1 , P_2 , P_3 and P_4 , then answer the following questions.

7. The radius of spherical ball $'S'$ is equal to :

(a) $4\sqrt{3} - 2\sqrt{2}$ (b) $3\sqrt{2} + 2\sqrt{3}$

(c) $4\sqrt{2} - \sqrt{3}$ (d) $\sqrt{3} + \sqrt{2}$

8. If the centre of $'S'$ is $(\alpha\ , \beta\ , \gamma)$, then value of $\log_2 \gamma$ is equal to :

(a) 1 (b) 1/2

(c) 1/3 (d) 1/4

9. If the point $'P_3'$ is $(a\ , b\ , c)$, then value of b is equal to :

(a) $2 - \sqrt{\frac{3}{2}}$ (b) $6 - 2\sqrt{\frac{3}{2}}$

(c) $4 + \sqrt{\frac{2}{3}}$ (d) $4 + 4\sqrt{\frac{2}{3}}$

Questions with Integral Answer :
(Questions No. 10-14)

10. Let the faces of tetrahedron $ABCD$ be represented by the planes $x + y = 0$, $y + z = 0$, $z + x = 0$ and $x + y + z = 2\sqrt{6}$. The shortest distance between any two opposite edges of the tetrahedron $ABCD$ is equal to ....

11. Let the lines L_1 and L_2 for which the direction cosines are given by the relation $l+m+n=0$ and $6lm-5mn+2nl=0$, include an angle α, then value of $\left\{\dfrac{3\tan\alpha}{11}\right\}^2$ is equal to ....

12. Let the image of line $\dfrac{x-1}{2}=\dfrac{y-2}{1}=\dfrac{z-3}{4}$ with respect to the plane mirror $2x+y+z-6=0$ passes through the point $(-1,\alpha,\beta)$, then the value of $(2\beta-\alpha)$ is equal to ....

13. Let plane *'P'* contain the lines $\dfrac{x-3}{2}=\dfrac{y+1}{-3}=\dfrac{z+2}{1}$ and $\dfrac{x-7}{3}=\dfrac{y}{-1}=\dfrac{z+7}{-2}$, then the minimum distance of plane *'P'* from the surface of the sphere $x^2+y^2+z^2-2\sqrt{3}(x+y+z)+8=0$ is equal to ....

14. If the line of shortest distance between the lines $\dfrac{x-1}{1}=\dfrac{y+1}{-1}=\dfrac{z+1}{1}$ and $\dfrac{x+2}{-1}=\dfrac{y-1}{-1}=\dfrac{z-2}{1}$ passes through the point $(\alpha,3,\beta)$, then value of $8(\alpha+\beta)$ is equal to ....

Matrix Matching Questions :
(Questions No. 15-17)

15. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) If the straight lines $\vec r=\vec r_1+\lambda\vec a$ and $\vec r=\vec r_2+\mu\vec b$ are coplanar, where λ, μ are scalars, and $\vec c.(\vec a\times\vec b)=0$, then $\vec c$ is equal to	(p) $(\vec r_1-\vec r_2)$
	(q) $\vec a\times\vec b$
(b) If the straight lines $\vec r=\vec r_1+\lambda\vec a$ and $\vec r=\vec r_2+\mu\vec b$ are intersecting at a point, where λ, μ are scalars, then	(r) $(\vec r_1-\vec r_2).(\vec a\times\vec b)=0$
(c) If $\vec r=\vec r_1+\lambda\vec a$ and $\vec r=\vec r_2+\mu\vec b$ are two skew lines, then vector along the line of shortest distance is parallel to	(s) $(\vec r_1+\vec r_2).(\vec a\times\vec b)=0$
(d) If line joining $P(\vec r_1)$ and $Q(\vec r_2)$ is L_1 and point with position vector $\vec a\times\vec b$ lies on the line L_1, then	(t) $(\vec r_1\times\vec r_2).(\vec a\times\vec b)=0$

16. Consider the following linear equations

$$ax+by+cz=0$$
$$bx+cy+az=0$$
$$cx+ay+bz=0$$

Match the conditions in Column I with statements in Column II.

Column (I)	**Column (II)**
(a) $a+b+c\neq 0$ and $a^2+b^2+c^2=ab+bc+ca$	(p) the equations represent planes meeting only at a single point.
(b) $a+b+c=0$ and $a^2+b^2+c^2\neq ab+bc+ca$	(q) the equations represent the line $x=y=z$.
(c) $a+b+c\neq 0$ and $a^2+b^2+c^2\neq ab+bc+ca$	(r) the equations represent identical planes.
(d) $a+b+c=0$ and $a^2+b^2+c^2=ab+bc+ca$	(s) the equations represent the whole of the three dimensional space.

17. Let the points A , B , C and D form a regular tetrahedron $ABCD$ in 3-dimensional space , where the edge length of the tetrahedron is $\sqrt{2}$ units , then match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) The angle between any two faces of the tetrahedron $ABCD$ is	(p) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(b) The angle between any edge and a face not containing that edge is	(q) $\tan^{-1}\left(2-\sqrt{3}\right)$
	(r) $\cos^{-1}(1/2)$
(c) The angle between two opposite edges of the tetrahedron is	(s) $\sin^{-1}\left(\frac{\sqrt{5}-1}{4}\right)$
(d) The volume (in cubic units) of the tetrahedron is more than	(t) $\sin^{-1}(1)$

ANSWERS Exercise No. (1)

1. (b)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (a)
6. (c)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (b)
11. (d)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (a)
16. (b)	**17.** (a)	**18.** (c)	**19.** (d)	**20.** (c)
21. (d)	**22.** (c)	**23.** (c)	**24.** (d)	**25.** (c)
26. (c , d)	**27.** (a , d)	**28.** (a , c , d)	**29.** (b , c)	**30.** (a , c)
31. (b)	**32.** (d)	**33.** (a)	**34.** (a)	**35.** (c)

ANSWERS Exercise No. (2)

1. (c)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (b)
6. (d)	**7.** (b)	**8.** (b)	**9.** (d)	**10.** (4)
11. (3)	**12.** (5)	**13.** (2)	**14.** (2)	
15. (a) → p (b) → r (c) → q (d) → t	**16.** (a) → r (b) → q (c) → p (d) → s	**17.** (a) → r (b) → p (c) → t (d) → q , s		

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Chapter 31

Trigonometric Ratios and Identities

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-20)

1. $\{\cos 43^o + \cos 29^o - \sin 11^o - \cos 65^o\}$ is equal to :

(a) sin 7° (b) cos 36°

(c) sin 83° (d) none of these

2. If $x \in R$, then maximum value of the expression

$$\left\{a\sin^2 x + b\sin x.\cos x + c\cos^2 x - \frac{1}{2}(a+c)\right\} \text{ is :}$$

(a) $\frac{1}{2}\sqrt{a^2+b^2+c^2}$

(b) $\frac{1}{2}\sqrt{a^2+b^2+c^2-2ac}$

(c) $\frac{1}{2}\sqrt{a^2+b^2+c^2-2bc}$

(d) $\frac{1}{2}\sqrt{a^2+b^2+c^2-2ab}$

3. If $(2-\cos\beta)\cos\alpha = 2\cos\beta - 1$; $0 < \alpha < \beta < \pi$,

then value of $\dfrac{\tan\beta/2}{\tan\alpha/2}$ is equal to :

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) 1 (d) $\frac{1}{\sqrt{2}}$

4. The value of $\left\{32.\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}\right\}$

is equal to :

(a) –2 (b) 1

(c) –1 (d) 2

5. If $a\cos\alpha + b\sin\alpha = c$ and $a\cos\beta + b\sin\beta = c$, then

value of $\tan\left(\frac{\alpha+\beta}{2}\right)$ is equal to :

(a) $\frac{a}{b}$ (b) $\frac{b}{c}$

(c) $\frac{b}{a}$ (d) $\frac{b+c}{a}$

6. If $\tan\alpha$, $\tan\beta$ are the roots of quadratic equation $x^2 + px + q = 0$, then value of expression $\{\sin^2(\alpha+\beta) + q\cos^2(\alpha+\beta) + p\sin(\alpha+\beta).\cos(\alpha+\beta)\}$ is equal to :

(a) $\frac{p+q}{2q}$ (b) $\frac{p}{q}$

(c) $p-q$ (d) q

7. If $\tan\theta = \dfrac{1+\sqrt{1-p}}{1+\sqrt{1+p}}$, then $\cos(8\theta)$ is equal to :

(a) $2p^2-1$ (b) $-2p\sqrt{1-p^2}$

(c) $2p^2+p$ (d) none of these

8. The value of {sin 144°. sin 108°. sin72°. sin36°} is equal to :

(a) $\frac{3}{16}$ (b) $\frac{5}{16}$ (c) $\frac{7}{16}$ (d) $\frac{1}{16}$

9. The value of $\tan^6 20° - 33\tan^4 20° + 27\tan^2 20°$ is :

(a) 2 (b) 4

(c) 3 (d) none of these

10. The value of

$$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)$$

is equal to :

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$

(c) $\frac{1}{32}$ (d) none of these

11. If $\dfrac{\cos A}{\cos B} = n$, $\dfrac{\sin A}{\sin B} = m$, then $\sin^2 B$ is equal to :

(a) $\frac{1+n^2}{m^2-n^2}$ (b) $\frac{1-n^2}{m^2-n^2}$

(c) $\frac{1-n}{m+n}$ (d) $\frac{1+n}{m^2+n^2}$

12. If

$f(\theta)=(a^2\cos^2\theta+b^2\sin^2\theta)^{1/2}+(a^2\sin^2\theta+b^2\cos^2\theta)^{1/2}$

then maximum value of $f(\theta)$ is :

(a) $\sqrt{a^2+b^2}$ (b) $\sqrt{2(a^2+b^2)}$

(c) $2\sqrt{a^2+b^2}$ (d) none of these

13. Let $f(x)=\dfrac{\tan x}{\tan 3x}$ and $x\neq n\pi$ or $\dfrac{n\pi}{3}$; $n\in I$, then inteval in which $f(x)$ lies is :

(a) $R-\left(\dfrac{1}{2}\,,\,2\right)$ (b) $R-\left[\dfrac{1}{3}\,,\,3\right]$

(c) $R-\left(\dfrac{1}{3}\,,\,3\right)$ (d) $R-\left[\dfrac{1}{2}\,,\,2\right]$

14. If $\cos^6\alpha+\sin^6\alpha+K\sin^2(2\alpha)=1$; $0<\alpha<\dfrac{\pi}{2}$, then value of K is equal to :

(a) $\dfrac{3}{4}$ (b) $\dfrac{1}{4}$

(c) $\dfrac{1}{3}$ (d) $\dfrac{1}{8}$

15. The value of $\cos^2 10^\circ-\cos 10^\circ.\cos 50^\circ+\cos^2 50^\circ$ is :

(a) $\dfrac{4}{3}$ (b) $\dfrac{1}{3}$

(c) $\dfrac{3}{4}$ (d) 3

16. If $A+B+C=0$, then value of the expression {sin²A+cos C(cos A cos B–cos C)+ cos B (cos$A$ cos$C$ –cos$B$)} is equal to :

(a) 1 (b) 2

(c) 0 (d) –1

17. Value of $(\tan 40^\circ+2\tan^\circ 10)$ is :

(a) cot 50° (b) cot 40°

(c) cot 10° (d) cot 20°

18. $\displaystyle\sum_{r=1}^{18}\sin^2(5r)^\circ$ is equal to :

(a) 9 (b) $\dfrac{19}{2}$

(c) $\dfrac{21}{2}$ (d) $\dfrac{17}{2}$

19. If $f(\theta)=\sin^2\theta+\sin^2\left(\dfrac{2\pi}{3}+\theta\right)+\sin^2\left(\dfrac{4\pi}{3}+\theta\right)$;

then value of $f\left(\dfrac{\pi}{15}\right)$ is equal to :

(a) $\dfrac{2}{3}$ (b) $\dfrac{3}{2}$ (c) $\dfrac{1}{3}$ (d) 1

20. If $\cot A$, $\cot B$, $\cot C$ are in A.P. for ΔABC, then 2 sin A cos B sin C is :

(a) tan²B (b) sin²B

(c) sec²B (d) cot² B

21. Let θ_i , $\phi_i\in R$ for all $i\in\{1\,,2\,,3\}$. if

$$\sin^2\alpha=\frac{\left(\sum_{i=1}^{3}\sin^2\theta_i\right)\left(\sum_{i=1}^{3}\cos^2\phi_i\right)}{\left(\sum_{i=1}^{3}\sin\theta_i.\cos\phi_i\right)^2}$$ and

$$\cos^2\beta=\frac{\left(\sum_{i=1}^{3}\sin^2\phi_i\right)\left(\sum_{i=1}^{3}\cos^2\phi_i\right)}{\left(\sum_{i=1}^{3}\sin\phi_i.\cos\phi_i\right)^2}$$, then

(a) $\sin^2\alpha+\cos^2\beta=1.$ (b) $\sin^4\alpha+\cos^4\beta=1.$

(c) $\sin^4\alpha+\cos^8\beta=2.$ (d) $\sin^8\alpha+\cos^8\beta=1.$

22. Let $\sqrt{2}\cos A=\cos B+\cos^3 B$ and $\sqrt{2}\sin A=\sin B-\sin^3 B$, then $\sin^2(2B)$ is :

(a) $\dfrac{1}{25}$ (b) $\dfrac{8}{9}$

(c) $\dfrac{1}{4}$ (d) $\dfrac{1}{36}$

23. Let for all $x\in R$, $\tan\theta=\left\{\dfrac{x^2-x+1}{x^2+x+1}\right\}^{1/2}$, where

$\theta\in(0\,,\,2\pi)-\left\{\dfrac{\pi}{2}\,,\dfrac{3\pi}{2}\right\}$, then value of $'\theta'$ can be :

(a) $\dfrac{3\pi}{8}$ (b) $\dfrac{5\pi}{12}$

(c) $\dfrac{6\pi}{5}$ (d) $\dfrac{13\pi}{12}$

24. The minimum value of $\left\{(81)^{\sin x+1/2}+(27)^{\cos x+2/3}\right\}$ is equal to :

(a) $\sec\left(\frac{\pi}{3}\right)$ (b) $\tan\left(\frac{\pi}{8}\right)$

(c) $\sin\left(\frac{\pi}{12}\right)$ (d) $\operatorname{cosec}\left(\frac{2\pi}{3}\right)$

25. Let α , $\beta \in R^+$ and $\alpha+\beta=\frac{\pi}{2}$, then maximum value of $\{\sin\alpha+\sin\beta\}$ is equal to :

(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{2}$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. Let $f_n(\theta)=\tan\theta.\left\{\prod_{r=1}^{n}(1+\sec(2^r\theta))\right\}$, then

(a) $f_2\left(\frac{\pi}{16}\right)=1$

(b) $f_3\left(\frac{\pi}{64}\right)=\sqrt{2}-1$

(c) $f_2\left(\frac{\pi}{48}\right)=2-\sqrt{3}$

(d) $f_5\left(\frac{\pi}{128}\right)=\sqrt{3}-1$

27. Which of the following are rational numbers ?

(a) $\sin\frac{\pi}{12}.\cos\frac{\pi}{12}$ (b) $\sqrt{3}.\operatorname{cosec}\frac{\pi}{9}-\sec\frac{\pi}{9}$

(c) $\sin\frac{\pi}{10}.\cos\frac{\pi}{5}$ (d) sin 12° . sin 48° . sin 54°

28. Solution set $\{x\ ,\ y\}$ for the system of equations $x-y=\frac{1}{3}$ and $\cos^2(\pi x)-\sin^2(\pi y)=\frac{1}{2}$ can be given by :

(a) $\left\{\frac{7}{6},\frac{5}{6}\right\}$ (b) $\left\{\frac{2}{3},\frac{1}{3}\right\}$

(c) $\left\{-\frac{5}{6},-\frac{7}{6}\right\}$ (d) $\left\{\frac{13}{6},\frac{11}{6}\right\}$

29. If $\sin^3 x.\sin 3x=\sum_{m=0}^{6}a_m\cos^m x$, where $a_0, a_1, a_2, \ldots a_6$ are constants , then

(a) $a_1=a_3=a_5=0$ (b) $a_0+a_2+a_4+a_6=0$

(c) $a_2-a_6+2a_0=0$ (d) $\sum_{r=1}^{6}a_r=0$

30. Value of $\left\{\prod_{r=1}^{10}(1+\tan(r^o))\right\}.\left\{\prod_{r=46}^{55}(1+\cot(r^o))\right\}$ is equal to :

(a) 1024 (b) $\sum_{r=0}^{10}{}^{10}C_r$

(c) 2^{20} (d) $\sum_{r=0}^{20}{}^{20}C_r$

Assertion Reasoning questions : (Questions No. 31-35)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

31. In a triangle ABC with fixed base BC , the vertex A moves such that $\cos B+\cos C=4\sin^2\left(\frac{A}{2}\right)$.

If a , b and c denote the side lengths of triangle opposite to the angles A , B and C respectively , then

Statement 1 : locus of vertex point A is an ellipse

because

Statement 2 : In the given ΔABC , b , a and c form an arithmetic progression.

32. Let $\frac{\sin^4\theta}{3}+\frac{\cos^4\theta}{7}=\frac{1}{10}$, where $\theta \in R$, then

Statement 1 : Value of $\left\{\frac{\sin^8\theta}{27}+\frac{\cos^8\theta}{343}\right\}$ is equal to

$sgn\left(ln\frac{1}{2}\right).\ log_{\sqrt[3]{10}}10$

because

Statement 2 : Value of $\tan^2\theta=\frac{3}{7}$.

33. Let $\theta_1, \theta_2, \theta_3 \in R$, and $\cos\theta_1 = \dfrac{a}{b+c}$, $\cos\theta_2 = \dfrac{b}{a+b}$ and $\cos\theta_3 = \dfrac{c}{a+b}$, where the sides a, b, c of triangle ABC are in $A.P.$

Statement 1 : Value of $\tan^2\left(\dfrac{\theta_1}{2}\right) + \tan^2\left(\dfrac{\theta_3}{2}\right)$ is equal to $\dfrac{2}{3}$

because

Statement 2 : $\displaystyle\sum_{p=1}^{3} \tan^2\left(\frac{\theta_p}{2}\right) = 1$ and $\tan^2\left(\dfrac{\theta_2}{2}\right) = \dfrac{1}{3}$

34. Statement 1 : For triangle ABC, if $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then triangle must be right angled

because

Statement 2 : In any triangle PQR,
$\sin^2 P + \sin^2 Q + \sin^2 R = (2 + 4\cos P.\cos Q.\cos R)$

35. Consider any triangle ABC having internal angles α, β and γ, where $\alpha, \beta, \gamma \neq \dfrac{\pi}{2}$.

Statement 1 : If $\tan\alpha + \tan\beta + \tan\gamma = 6 - 4x + x^2$ for all $x \in R$, then triangle ABC is essentially an acute angled triangle

because

Statement 2 : In any triangle except the right-angled, sum of the tangent of internal angles is always equal to the product of tangent of internal angles.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $\alpha \neq \frac{n\pi}{2} + 3\theta$; where $n \in I$, and

$$\frac{\cos^3\theta}{\cos(\alpha - 3\theta)} = \frac{\sin^3\theta}{\sin(\alpha - 3\theta)} = \lambda \text{....(1)}$$

On the basis of given relation , answer the following questions.

1. Using the identity $\cos^4\theta - \sin^4\theta = \cos 2\theta$, the value of $\tan 2\theta$ which is obtained from the given relation ..... (1) of passage is equal to :

(a) $\frac{1+\lambda\cos\alpha}{\sin\alpha}$ (b) $\frac{1-\lambda\cos\alpha}{\lambda\sin\alpha}$

(c) $\frac{1+\lambda\cos\alpha}{\lambda\sin\alpha}$ (d) $\frac{1+\lambda\sin\alpha}{\lambda\cos\alpha}$

2. Using the identity $\sin\theta.\cos^3\theta + \cos\theta\sin^3\theta = \sin\theta\cos\theta$, the value of $\tan 2\theta$ which is obtained from the given relation ...(1) of passage is equal to :

(a) $\frac{2\lambda\cos\alpha}{1-2\lambda\sin x}$ (b) $\frac{2\lambda\sin\alpha}{1+\cos x}$

(c) $\frac{2\lambda\sin\alpha}{1+2\lambda\cos\alpha}$ (d) $\frac{\lambda\sin\alpha}{1+\cos\alpha}$

3. If 'θ' is eliminated from relation ...(1) of passage , then quadratic in λ which is obtained , is equal to :

(a) $2\lambda^2 + \lambda\cos\alpha + 1 = 0$

(b) $2\lambda^2 - \lambda\sin\alpha + 1 = 0$

(c) $2\lambda^2 - \lambda\cos\alpha - 1 = 0$

(d) $2\lambda^2 - \lambda\sin\alpha - 1 = 0$

Comprehension passage (2)
(Questions No. 4-6)

Let value of $\tan\left(\frac{19\pi}{24}\right) = a + \sqrt{a} - \sqrt{b} - \sqrt{ab}$, where $b > a > 0$, then answer the following questions.

4. The value of $\left\{\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{14\pi}{15}\right\}$ is equal to :

(a) $\frac{b}{a^2}$ (b) $\frac{1}{a^4}$ (c) $\frac{1}{b^4}$ (d) $\frac{a+b}{b^3}$

5. The value of $\left\{\prod_{r=0}^{3}\left(1+\cos(2r+1)\frac{\pi}{8}\right)\right\}$ is equal to :

(a) $\frac{1}{b^3}$ (b) $\frac{1}{2a+b}$

(c) $\frac{1}{2b+a}$ (d) $\frac{b}{a+b}$

6. The value of $\left\{\tan 6^o.\tan 42^o.\tan 66^o.\tan 78^o\right\}$ is equal to :

(a) $\frac{b+1}{a^2}$ (b) $\frac{b+2}{a}$

(c) $\frac{2b+1}{3a}$ (d) $\frac{a}{b+1}$

Questions with Integral Answer :
(Questions No. 7-10)

7. If $T_n = \left\{\frac{\sin^n x + \cos^n x}{n}\right\}$, then value of $\frac{1}{2}\{T_4 - T_6\}^{-1}$ is equal to ....

8. If $\sin\left(\frac{\pi}{14}\right)$ is a root of the cubic equation $8x^3 - 4x^2 - 4x + \alpha = 0$ and [.] represents the greatest integer function , then value of $\left[\frac{\alpha}{2}\right]$ is equal to ....

9. If $\prod_{r=1}^{7}\left\{\sin\left(\frac{(2r-1)\pi}{14}\right)\right\} = \left(\frac{1}{\sqrt{2}}\right)^n$, then value of $\left(\frac{n}{4}\right)^2$ is equal to ....

10. Let $\alpha^2 + 3\alpha + 8$, $\alpha^2 + 2\alpha$ and $2\alpha + 3$ be three sides of a triangle , then least possible integral value of 'α' is equal to ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Let $\sin\theta+\sin\phi=a$ and $\cos\theta+\cos\phi=b$, where $a\neq b$, then match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) $\tan\theta+\tan\phi$	(p) $\dfrac{(a^2+b^2)^2-4b^2}{4(a^2+b^2)}$
(b) $\cos\theta.\cos\phi$	(q) $\dfrac{2ab}{(a^2+b^2)}$
(c) $\cos(\theta+\phi)$	(r) $\dfrac{8ab}{(a^2+b^2)^2-4b^2}$
(d) $\sin(\theta+\phi)$	(s) $\dfrac{4ab}{(a^2+b^2)^2+2b^2}$
	(t) $\dfrac{b^2-a^2}{b^2+a^2}$

12. Match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) If $x\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$, then the output set of $f(x)=4^{\sin x}-2^{1+\sin x}+4$ contain the interval(s)	(p) $(1,2]$
	(q) $[4,5)$
(b) If $x\in\left[-\dfrac{\pi}{2},0\right]$, then the output set of $f(x)=\sin^6 x+3\sin^4 x+5\sin^2 x+2\cos^2 x$ contain the interval(s)	(r) $(5,9]$
(c) If $x\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$, then the output set of $f(x)=\tan^6 x+4\tan^3 x+5$ contain the interval(s)	(s) $[3,4)$
(d) If $x\in\left(\dfrac{\pi}{2},\pi\right]$, then output set of $f(x)=9^{\sec x}-4(3)^{\sec x}+5$ contain the interval(s)	(t) $[1,4)$

ANSWERS Exercise No. (1)

1. (c) **2.** (b) **3.** (a) **4.** (d) **5.** (c)

6. (d) **7.** (a) **8.** (b) **9.** (c) **10.** (b)

11. (b) **12.** (b) **13.** (b) **14.** (a) **15.** (c)

16. (c) **17.** (b) **18.** (b) **19.** (b) **20.** (b)

21. (c) **22.** (b) **23.** (c) **24.** (d) **25.** (d)

26. (a , b , c) **27.** (a , b , c , d) **28.** (a , c d) **29.** (a , b , c) **30.** (a , b)

31. (a) **32.** (b) **33.** (a) **34.** (c) **35.** (a)

ANSWERS Exercise No. (2)

1. (b) **2.** (c) **3.** (c) **4.** (b) **5.** (c)

6. (a) **7.** (6) **8.** (0) **9.** (9) **10.** (6)

11. (a) $\to$ r
(b) $\to$ p
(c) $\to$ t
(d) $\to$ q

12. (a) $\to$ s
(b) $\to$ q , r , s
(c) $\to$ p , q , r , s , t
(d) $\to$ q

Chapter
32

Trigonometric Equations and Inequations

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-15)

1. Total number of integral values of *'n'* such that the equation (cos x + sin$x$) sin$x$ = n is having atleast one real solution is/are :

(a) 3 (b) 1

(c) 2 (d) 0

2. The equation $\cos x - x + 2 = 0$ is having one real root in the interval :

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2}, \pi\right)$

(c) $\left(\pi, \frac{3\pi}{2}\right)$ (d) $\left(\frac{3\pi}{2}, 2\pi\right)$

3. The equation $\tan^4 x - 2\sec^2 x + a^2 = 0$ will have at least one solution , if :

(a) $|a| \le 2$ (b) $|a| \le 4$

(c) $|a| \le \sqrt{3}$ (d) $|a| \le 1$

4. The number of solutions of the equation $\max\{\sec x, \operatorname{cosec} x\} = 3$ in interval $[0, 2\pi]$ are given by :

(a) 4 (b) 8

(c) 6 (d) 10

5. If $4\sin^2 x + \tan^2 x + \operatorname{cosec}^2 x + \cot^2 x - 6 = 0$, then for all $n \in I$, x belongs to :

(a) $n\pi \pm \frac{\pi}{4}$ (b) $2n\pi \pm \frac{\pi}{4}$

(c) $n\pi + \frac{\pi}{4}$ (d) $n\pi - \frac{\pi}{4}$

6. If $x \in [0, 2\pi]$, then total number of solutions of equation $\sin^4 x + \cos^4 x = \sin x.\cos x$ is equal to :

(a) 0 (b) 1

(c) 2 (d) 4

7. General solution of the trinometric equation , $(\sqrt{3}-1)\sin\theta + (\sqrt{3}+1)\cos\theta = 2$ is :

(a) $n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{12}$; $n \in I$

(b) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{12}$; $n \in I$

(c) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$; $n \in I$

(d) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{12}$; $n \in I$

8. If $4\sin^2 x - 8\sin x + 3 \le 0$ and $x \in [0, 2\pi]$, then the solution set for x is :

(a) $\left[0, \frac{\pi}{6}\right]$ (b) $\left[\frac{5\pi}{6}, \frac{11\pi}{6}\right]$

(c) $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$ (d) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

9. Let $x \in \left(-\frac{\pi}{2}, \frac{7\pi}{2}\right)$ and $y \in R$, then number of ordered pairs (x , y) which satisfy the inequation $2^{\sec^2 x}\left\{\sqrt{\frac{1}{2} - y^2 + y^4}\right\} \le 1$ are given by :

(a) 4 (b) 8

(c) 12 (d) 16

10. If $\cos^6 x + \sin^6 x + \lambda \sin^2 2x = 1$, where $x \in \left(0, \frac{\pi}{2}\right)$, then '$\lambda$' is equal to :

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

11. Number of solutions of the pair of equations , $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta - 3\sin\theta = 0$, in the interval $[0 , 2\pi]$ is/are :

(a) 0 (b) 2

(c) 4 (d) 3

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12. If $x \in \left[0\,,\frac{\pi}{2}\right]$, then number of solutions of the equation $2\sin^2 x.\cos^2\left(\frac{x}{2}\right) = 2^x + 2^{-x}$ is/are :

(a) 0 (b) 1

(c) 2 (d) 3

13. The number of ordered pairs $(p\,,\,q)$, where $p\,,q \in (-\pi,\pi)$, satisfying the conditions $\cos(p+q) = \lim\limits_{\alpha\to 1}(1+\sin\pi\alpha)^{\cot\pi\alpha}$ and $\cos(p-q) = 1$ is/are :

(a) 0 (b) 1 (c) 2 (d) 4

14. Let $'\alpha'$ be the smallest positive number for which the equation $\cos(\alpha\sin x) - \sin(\alpha\cos x) = 0$ is having a solution for $x \in [0\,,\,2\pi]$, then $\tan\left(\frac{\alpha}{2\sqrt{2}}\right)$ is :

(a) 1 (b) $\sqrt{2}-1$

(c) $\sqrt{3}-1$ (d) $2-\sqrt{3}$

15. The smallest positive root of the equation $\sqrt{\sec^2 x - 1} - x = 0$ lies in :

(a) $\left(0\,,\frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2},\pi\right)$

(c) $\left(\pi\,,\frac{3\pi}{2}\right)$ (d) $\left(\frac{3\pi}{2},2\pi\right)$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. Let $\theta \in \left(0\,,\frac{\pi}{2}\right)$, then the solutions of the equation

$$\sum_{p=1}^{6} \operatorname{cosec}\left(\theta+(p-1)\frac{\pi}{4}\right).\operatorname{cosec}\left(\theta+p\frac{\pi}{4}\right) = 4\sqrt{2}$$

is / are :

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{12}$ (c) $\frac{3\pi}{8}$ (d) $\frac{5\pi}{12}$

17. If the equation $4\,|\sin x\cos x| - 2\,|x| - \lambda = 0$ is having atleast two real solutions , then possible values of the parameter $'\lambda'$ can be :

(a) $\tan\left(\frac{\pi}{8}\right)$ (b) $\tan\left(\frac{13\pi}{12}\right)$

(c) $\sin\left(\frac{\pi}{10}\right)$ (d) $\cos\left(\frac{\pi}{5}\right)$

18. Let $f(x) = 2\sin x + 3\cos(\lambda x)$, where $\lambda \in R$. If the equation $f(x) - \sec\left(\sin^{-1}\left(\frac{12}{13}\right)\right) - \tan\left(\cos^{-1}\left(\frac{5}{13}\right)\right) = 0$ is having atleast one real solution , then values(*s*) of $'\lambda'$ can be equal to :

(a) $\frac{8}{5}$ (b) $-\frac{4}{3}$

(c) $\frac{2}{3}$ (d) $\frac{12}{17}$

19. If *'S'* represents the exhaustive set of values of x in $(-\pi\,,\,\pi]$ which satisfy the inequality $2\sin^2 x + |\sin x| - 1 \le 0$, then set *'S'* contains :

(a) $\left[\frac{\pi}{12}\,,\frac{\pi}{4}\right]$

(b) $\left[-\frac{\pi}{6}\,,\frac{-\pi}{8}\right]$

(c) $\left[\frac{5\pi}{6}\,,\frac{7\pi}{8}\right]$

(d) $\left(-\frac{5\pi}{6},-\pi\right)$

20. If the inequality $x + \sin x \ge |\,p\,|\,x^2$ is satisfied for all $x \in \left[0\,,\frac{\pi}{2}\right]$, then the possible value(s) of *'p'* can be :

(a) $\frac{\pi+4}{\pi^2}$ (b) $\tan\left(\frac{7\pi}{8}\right)$

(c) $\frac{\pi+4}{\pi}$ (d) $\frac{2\pi+4}{\pi^2}$

Assertion Reasoning questions : (Questions No. 21-25)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. Statement 1 : The equation $2\cos^2 x+\sqrt{3}\sin x+1=0$ is having four solutions in $[-3\pi\ ,\ \pi]$

because

Statement 2 : $\sin x=\frac{-\sqrt{3}}{2}\Rightarrow x=n\pi-(-1)^{n+3}.\frac{\pi}{3}$, where $n\in I$.

22. Statement 1 : If $x\in(0\ ,\ 2\pi)$, then the equation $\tan x+\sec x=2\cos x$ is having 3 distinct solutions

because

Statement 2 : The graphs of $y=1+\sin x$ and $y=2+\cos^2 x$ intersect each other at three distinct locations if $x\in(0\ ,\ 2\pi)$.

23. Statement 1 : If $\sin^4 x-\cos^6 3x=1$, then no solution exists for the equation in $\left(-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right)$

because

Statement 2 : $\cos x+\sec x=2\ \Rightarrow\ \sin^4 x+\sin^6 x=0$.

24. Statement 1 : If [.] denotes the greatest integer function , then the equation $2+[\sin x]+[\cos x]=0$ is having infinitely many solutions is $\left(-\pi\ ,\ -\frac{\pi}{2}\right)$

because

Statement 2 : The values of both $\sin x$ and $\cos x$ lies in between -1 and 0 for all $x\in\left(-\pi\ ,\ -\frac{\pi}{2}\right)$.

25. Statement 1 : If [.] denotes the greatest integer function , then number of solutions of the system of equations $2y=\left[\cos x+[\cos x]\right]$ and $\left[\,y+[y+[y]]\,\right]=6\sin x$, where $x\in[-2\pi\ ,\ 2\pi]$, are two

because

Statement 2 : The graphs of $y=2\cos x$ and $y=[\sin x]$ intersect each other at two location for $x\in\left[-2\pi\ ,\ 2\pi\right]$.

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Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Consider the system of equations :

$4|\sin x|\sin y + 1 = 0$, and

$\cos(x+y)+\cos(x-y)=3/2$

If $x \in [0\,,\,2\pi]$ and $y \in [\pi\,,\,2\pi]$, then answer the following questions

1. Let the ordered pair $(x\,,\,y)$ satisfy the given system of equations , then number of ordered pair(s) for which $x \in (0\,,\,\pi)$, is/are :

(a) 2 (b) 1

(c) 0 (d) 4

2. Number of ordered pairs $(x\,,\,y)$ which satisfy the given system of equations and hold the conditions $y - x = 0$, is/are :

(a) 4 (b) 1

(c) 2 (d) 0

3. Number of ordered pairs $(x\,,y)$ which satisfy the given system of equations and hold the condition

$y - x \geq \dfrac{\pi}{4}$, is/are :

(a) 2 (b) 1

(c) 0 (d) 4

Comprehension passage (2)
(Questions No. 4-6)

Let 'α' be a real parameter for which the equation $\sin^4 x + \cos^4 x + (\sin x + \cos x)^2 + \alpha - 1 = 0$ is having atleast one real solution. If 'β' is another real parameter for which the equation $\sin^4 x + \cos^4 x = \beta$ is having real solution , then answer the following questions.

4. Exhaustive set of values of 'α' belong to :

(a) $\left[-\dfrac{3}{2}\,,\,\dfrac{3}{2}\right]$ (b) $\left[-\dfrac{1}{2}\,,\,\dfrac{1}{2}\right]$

(c) $\left[-\dfrac{3}{2}\,,\,-\dfrac{1}{2}\right]$ (d) $\left[-\dfrac{3}{2}\,,\,\dfrac{1}{2}\right]$

5. If the exhaustive set of permissible values of α and β are represented by A and B respectively , then number of integral element(s) which lies in $A \cap B$ is/are :

(a) 2 (b) 0

(c) 1 (d) 4

6. Let for some permissible values of 'α' and 'β' the given system of equations in the passage is having common solution , then the common solution can be :

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{5\pi}{4}$

(c) $\dfrac{3\pi}{4}$ (d) $\dfrac{\pi}{2}$

Questions with Integral Answer :
(Questions No. 7-10)

7. Let $\dfrac{k\pi}{32}$ be the smallest angle in $[0\,,\,2\pi]$ for which the equation $16\sin^{10}x + 16\cos^{10}x = 29\cos^4 2x$ is satisfied , then value of 'k' is equal to ....

8. Total number of values of x in $(-\pi\,,\,\pi)$ for which the equation $\left(\sqrt{3}\sin x + \cos x\right)^{\sqrt{\sqrt{3}\sin 2x - \cos 2x + 2}} = 4$ is satisfied is/are ....

9. Total number of solution(s) of the equation $|4\sin \pi x| - x^2 + 2x = 1$ is /are ....

10. If the equation $K\cos x - 3\sin x = K + 1$ is solvable for x , then maximum possible integral value of 'K' is equal to ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Match the equations in column (I) with their number of solutions in column (II).

Column (I)	**Column (II)**
(a) $3x+2\tan x=\frac{5\pi}{2}$, $x\in[0\ ,\ 2\pi]$	(p) 4
(b) $\sin\{x\}=\cos\{x\}$, $x\in[0\ ,\ 2\pi]$, $\{.\}$ denotes the fractional part of x.	(q) 3
	(r) 0
(c) $\cos 2x=\lvert\sin x\rvert$, $x\in\left(-\frac{\pi}{2}\ ,\ \pi\right)$	(s) 6
(d) $\sin(\cos x)-\cos(\sin x)=0$, $x\in[0\ ,\ 2\pi]$	(t) 1

12. Match columns (I) and (II).

Column (I)	**Column (II)**
(a) If the equation $2\cot^2 x-5\operatorname{cosec} x-1=0$ is having at least seven distinct solutions in $[0\ ,\ n\pi]$, then natural number $'n'$ can be	(p) 8
(b) Number of solution(s) of the equation $\frac{\tan x+\cot x}{2}+\left\lvert\frac{\tan x-\cot x}{2}\right\rvert=x$ for $x\in\left[0\ ,\ \frac{3\pi}{2}\right)$ is/are	(q) 0
	(r) 2
(c) Number of ordered pairs $(x\ ,\ y)$ satisfying the equation $\lvert x\rvert+\lvert y\rvert=1$ and $\sin(x+y)-\sin x-\sin y=0$ is/are	(s) 7
(d) If the equation $4\operatorname{cosec}^2(\pi(\lambda+x))+\lambda^2-4\lambda=0$ is having real solution , then $'\lambda'$ can be	(t) 6

ANSWERS Exercise No. (1)

1. (c) **2.** (b) **3.** (c) **4.** (a) **5.** (a)

6. (c) **7.** (c) **8.** (d) **9.** (b) **10.** (b)

11. (b) **12.** (a) **13.** (d) **14.** (b) **15.** (b)

16. (b , d) **17.** (a , b , c) **18.** (a , b , d) **19.** (b , c) **20.** (a , b , d)

21. (c) **22.** (d) **23.** (b) **24.** (a) **25.** (b)

ANSWERS Exercise No. (2)

1. (a) **2.** (c) **3.** (a) **4.** (d) **5.** (b)

6. (c) **7.** (4) **8.** (2) **9.** (7) **10.** (4)

11. (a) → q
(b) → s
(c) → q
(d) → r

12. (a) → p , s
(b) → r
(c) → t
(d) → r

Chapter 33

Solution of Triangle

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-20)

1. In ΔABC, if angles A, B, C are in geometric sequence with common ratio 2 , then $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)$ is :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 0 (d) 2

2. Let ABC and ABC' be two non-congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and angle $B = 30°$. The absolute value of the difference between the area of these triangles is :

(a) 8 (b) 4 (c) 6 (d) 2

3. In an isosceles triangle if one angle is 120° and radius of its incircle is $\sqrt{3}$, then area of the triangle in square units is :

(a) $7+12\sqrt{3}$ (b) $12-7\sqrt{3}$

(c) $12+7\sqrt{3}$ (d) 4π

4. If a, b and c denote the length of the sides opposite to angles A, B and C of a triangle ABC, then the correct relation is given by :

(a) $(b+c)\sin\left(\frac{B+C}{2}\right) = a\cos\left(\frac{A}{2}\right)$

(b) $(b-c)\cos\left(\frac{A}{2}\right) = a\sin\left(\frac{B-C}{2}\right)$

(c) $(b-c)\cos\left(\frac{A}{2}\right) = 2a\sin\left(\frac{B-C}{2}\right)$

(d) $(b-c)\sin\left(\frac{B-C}{2}\right) = a\cos\left(\frac{A}{2}\right)$

5. Three circular coins each of radii 1 cm are kept in an equilateral triangle so that all the three coins touch each other and also the sides of the triangle. Area of the triangle is

(a) $(4+2\sqrt{3})\,\text{cm}^2$ (b) $\left(\frac{1}{4}\right)(12+7\sqrt{3})\,\text{cm}^2$

(c) $\left(\frac{1}{4}\right)(48+7\sqrt{3})\,\text{cm}^2$ (d) $(6+4\sqrt{3})\,\text{cm}^2$

6. If the angles of a triangle are in the ratio $4:1:1$, then the ratio of the longest side to the perimeter is :

(a) $\sqrt{3}:(2+\sqrt{3})$

(b) $1:\sqrt{3}$

(c) $1:(2+\sqrt{3})$

(d) $2:3$

7. In a triangle ABC, let $\angle C = \pi/2$. If r is the in-radius and R is the circum-radius of the triangle then $2(r+R)$ is equal to :

(a) $a + b$ (b) $b + c$

(c) $c + a$ (d) $a + b + c$

8. In a triangle ABC, $\angle B = \pi/3$ and $\angle C = \pi/4$. Let D divides BC internally in the ratio $1:3$ then $\frac{\sin\angle BAD}{\sin\angle CAD}$ equal to :

(a) $1/\sqrt{6}$ (b) $1/3$

(c) $1/\sqrt{3}$ (d) $\sqrt{2/3}$

9. If $\angle B = \frac{\pi}{4}$, $\angle C = \frac{\pi}{3}$ and $a = \left(2\sqrt{3}+2\right)$ units , then area (in sq. units) of traingle ABC is :

(a) $6+2\sqrt{3}$ (b) 4

(c) $\sqrt{3}+1$ (d) $2\sqrt{3}+4$

10. Let r, R be respectively the radii of the inscribed and circumscribed circles of a regular polygon of n sides such that $\frac{R}{r} = \sqrt{5}-1$, then n is equal to :

(a) 5 (b) 6
(c) 10 (d) 8

11. In a triangle ABC, $\frac{r_1}{bc}+\frac{r_2}{ca}+\frac{r_3}{ab}$ is equal to :

(a) $\frac{1}{2R}-\frac{1}{r}$ (b) $2R-r$

(c) $r-2R$ (d) $\frac{1}{r}-\frac{1}{2R}$

12. If for a triangle ABC , $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then

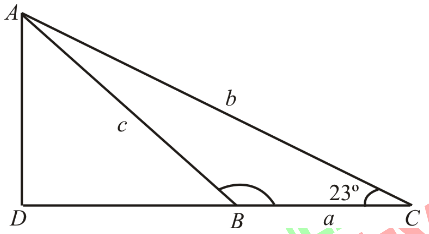
$\sin^3 A + \sin^3 B + \sin^3 C$ is equal to :

(a) $\sin A + \sin B + \sin C$
(b) $3 \sin A \sin B \sin C$
(c) $\sin 3A + \sin 3B + \sin 3C$
(d) $\sin^3 A \sin^3 B \sin^3 C$

13. In a triangle ABC if $\frac{a}{4} = \frac{b}{5} = \frac{c}{6}$, then ratio of the radius of the circumcircle to that of the incircle is

(a) 15/4 (b) 11/5
(c) 16/7 (d) 16/3

14. In a triangle ABC let AD be the altitude form A. If $b > c$, $\angle C = 23^o$ and $AD = \frac{abc}{b^2 - c^2}$ then $\angle B$ is equal to



(a) 113^o (b) 123^o
(c) 147^o (d) 157^o

15. In triangle ABC, if

$$\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$$, then

(a) $A = 90^o$ (b) $B = 90^o$
(c) $C = 90^o$ (d) $C = 75^o$

16. In a triangle ABC , if $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ then $\angle C$ is equal to :

(a) 30^o (b) 60^o
(c) 75^o (d) 90^o

17. In a triangle with one angle $\frac{2\pi}{3}$, the lengths of the sides form an A.P. If the length of the greatest side is 7 cm , the radius of the circumcircle of the triangle is

(a) $\frac{7\sqrt{3}}{3}$ cm (b) $\frac{5\sqrt{3}}{3}$ cm

(c) $\frac{2\sqrt{3}}{3}$ cm (d) $\sqrt{3}$ cm

18. If D is the mid-point of side BC of a triangle ABC and AD is perpendicular to AC , then

(a) $3b^2 = a^2 - c^2$ (b) $3a^2 = b^2 - 3c^2$
(c) $b^2 = a^2 - c^2$ (d) $a^2 + b^2 = 5c^2$

19. If two sides of a triangle are the roots of the equation $4x^2 - (2\sqrt{6})x + 1 = 0$ and the included angle is 60^o , then the third side is

(a) $\sqrt{3}$ (b) $\sqrt{3}/2$
(c) $1/\sqrt{3}$ (d) $2/\sqrt{3}$

20. In a triangle ABC , if $(a + b + c)(b + c - a) = \lambda bc$, then :

(a) $\lambda < 0$ (b) $\lambda > 6$
(c) $0 < \lambda < 4$ (d) $\lambda > 4$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Internal bisector of angle A of triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F. If a , b , c represent sides of ΔABC , then

(a) AE is $H.M.$ of b and c
(b) $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$
(c) $EF = \frac{4bc}{b+c}\sin\frac{A}{2}$
(d) the triangle AEF is isosceles

22. If a triangle ABC with side $a = 12$ units is inscribed in a circle of radius 10 units , then in-radius of triangle ABC can be :

(a) 4 units (b) 8 units
(c) 5 units (d) 2 units

23. Let the two adjacent sides of a cyclic quadrilateral be 2 , 5 and the angle between them is $\frac{\pi}{3}$. If the area of quadrilateral is $4\sqrt{3}$ square units , then the remaining sides can be :

(a) 2 (b) 4 (c) 3 (d) 6

24. Which of the following expressions on solving reduce to the area of triangle ABC ? (all the notations are having their usual meaning).

(a) $\sqrt{r(r_1 r_2 r_3)}$ (b) $r_1 r_2 \sqrt{\frac{4R - (r_1 + r_2)}{r_1 + r_2}}$

(c) $r^2 \cot\frac{A}{2} + 2Rr(\sin A)$ (d) $r_1 r\left(\frac{r_3 - r_2}{c - b}\right)$

25. For triangle ABC, which of the following statements are true ?

(a) Product of all the side lengths of $\Delta ABC = 2(r\,s\,R)$.

(b) $\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$

(c) If $2R = r_1 - r$, then ΔABC is right-angled.

(d) If $R = 2r$, then ΔABC is equilateral.

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. Let A_1 be the area of n-sided regular polygon inscribed in a circle $'C'$ of unit radius and A_2 be the area of n-sided regular polygon circumscribing the circle $'C'$.

Statement 1 : If $\frac{A_2}{A_1} = 4(2-\sqrt{3})$, then the number of sides $'n'$ of the regular polygon are 12

because

Statement 2 : $\frac{A_2}{A_1} = 4\tan\left(\frac{\pi}{n}\right)$.

27. In triangle ABC, let the side lengths be $a = 6$, $b = 8$ and $c = 10$.

Statement 1 : Distance between the circum-centre and in-centre of ΔABC is equal to $\sqrt{5}$ units

because

Statement 2 : For any triangle , distance between the circum-centre and in-centre is equal to $\sqrt{R^2 - 2rR}$, where R , r represents the circum-radius and in-radius of the triangle.

28. Consider an acute-angled triangle ABC in which the altitudes are AP , BQ and CR.

Statement 1 : Incentre of triangle PQR is the orthocentre of triangle ABC

because

Statement 2 : orthocentre of triangle $I_1I_2I_3$ is the in-centre of triangle ABC, where I_1, I_2 , I_3 denote the centre of escribed circles for triangle ABC.

29. Consider a triangle ABC , having side lengths a , b , c and circum-radius (R). If r_1 , r_2 , r_3 denote the ex-radii of triangle ABC , then

Statement 1 : $\left\{\frac{ab}{r_3}+\frac{ac}{r_2}+\frac{bc}{r_1}\right\} \geq 6R$

because

Statement 2 : $\left\{\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)\right\} \geq 6$

30. Statement 1 : In triangle ABC, if the sides b , c and the angle $\angle ABC$ is known , then a unique triangle can only be formed if $\sin B = \frac{b}{c}$ and $\angle B$ is acute

because

Statement 2 : If $\sin B = \frac{b}{c}$ and $\angle B$ is obtuse , then ΔABC doesn't exist.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let circum-radius of ΔABC be $'R'$ and the line joining the circum-centre $'O'$ and in-centre $'I'$ is parallel to side BC. If R_1, R_2, R_3 are the radii of circumcircles of triangles OBC, OCA and OAB respectively, then answer the following questions.

1. Value of $\left\{\dfrac{a}{R_1}+\dfrac{b}{R_2}+\dfrac{c}{R_3}\right\}$ is equal to :

(a) $\dfrac{a+b+c}{R}$ (b) $\dfrac{abc}{R^3}$

(c) $\left(\dfrac{abc}{R}\right)^2$ (d) $\dfrac{a^2+b^2+c^2}{R^2}$

2. Value of ($\cos B+\cos C$) is :

(a) 1 (b) 3/2

(c) 1/2 (d) 1/3

3. For given ΔABC the in-radius is given by :

(a) $R\cos B$ (b) $R\cos A$

(c) $R\cos C$ (d) none of these

Comprehension passage (2)
(Questions No. 4-6)

In triangle ABC, let the altitude, internal angular bisector and the median from vertex A meet the opposite side BC at D, E and F respectively. If $\angle BAD=\alpha$, and $\angle DAE=\angle EAF=\angle CAF=\alpha$, then answer the following questions.

4. If $\{p\}$ denotes the fractional part of p, where $p=[p]+\{p\}$, then :

(a) $\{\tan B\}=0$ (b) $\{\sin A\}=1/2$

(c) $\{\cos B\}=\dfrac{1}{2}$ (d) $\{\tan B\}=\{\tan C\}$

5. Value of $\tan\left(\dfrac{1}{2}\cos^{-1}(\cos(2C))\right)$ is equal to :

(a) $\tan\left(\dfrac{B}{2}\right)$ (b) $\tan\left(\dfrac{3A}{4}\right)$

(c) $\sin(2B)$ (d) $\tan B+\tan C$

6. If $BC=4$ units and the area of ΔABC is $'\delta'$ square units, then :

(a) $\tan\left(\sin^{-1}\left(\dfrac{4}{\delta}\right)\right)=1$

(b) $\tan\left(2\tan^{-1}\left(\dfrac{\delta-2}{2}\right)\right)=1$

(c) $\cot\left(2\tan^{-1}\left(\dfrac{\delta+2}{2}\right)\right)=1$

(d) $\tan\left(3\tan^{-1}(\delta+1)\right)=1$

Comprehension passage (3)
(Questions No. 7-9)

Let triangle ABC of area Δ square units be inscribed in a circle of radius 4 units, where $\Delta\in\left(0, 12\sqrt{3}\right]$. If p_1, p_2 and p_3 denote the length of altitudes of triangle ABC from the vertices A, B and C respectively, then answer the following questions.

7. The value of $4\left\{\dfrac{\cos A}{p_1}+\dfrac{\cos B}{p_2}+\dfrac{\cos C}{p_3}\right\}$, is equal to :

(a) 2 (b) 1

(c) 3 (d) 4

8. If sides a, b, c are in $A.P.$, then maximum value of $\left\{\dfrac{1}{p_1}+\dfrac{1}{p_2}+\dfrac{1}{p_3}\right\}$ is equal to :

(a) $\dfrac{18}{\Delta}$ (b) $\dfrac{24}{\Delta}$

(c) $\dfrac{6}{\Delta}$ (d) $\dfrac{12}{\Delta}$

9. Minimum value of the expression $\left\{\dfrac{a^2p_3}{b}+\dfrac{b^2p_1}{c}+\dfrac{c^2p_2}{a}\right\}$ is equal to :

(a) 4Δ (b) 6Δ

(c) 8Δ (d) 2Δ

Questions with Integral Answer :
(Questions No. 10-14)

10. Let a, b, c represent the sides of triangle ABC, where $(b-a)=(c-b)=1$ and $a, b, c\in N$. If $\angle C=2\angle A$, then value of $(3c-b-a)$ is equal to ....

11. If sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one , then the largest side of triangle is ....

12. Let a , b and c represent the sides of triangle ABC opposite to the vertices A , B and C respectively. If $a^4+b^4+c^4+b^2c^2-2a^2(b^2+c^2)=0$, then value of $\sec^2(A)$ is equal to ....

13. Let three circles touch one-another externally and the tangents at their points of contact meet at a point whose distance from any point of contact is 2 units. If ratio of the product of radii to the sum of radii of cricles is $k:1$, then k is equal to ....

14. If Δ_0 is the area of Δ formed by joining the points of contact of incircle with the sides of the given triangle whose area is Δ , similarly Δ_1 , Δ_2 and Δ_3 are the corresponding area of the Δ formed by joining the points of contact of excircles with the sides , then value of $\frac{\Delta_1}{\Delta}+\frac{\Delta_2}{\Delta}+\frac{\Delta_3}{\Delta}-\frac{\Delta_0}{\Delta}$ is equal to ....

Matrix Matching Questions :
(Questions No. 15-17)

15. In triangle ABC , let the orthocentre (H) and circum-centre (C_0) be (3 , 3) and (4 , 3) respectively. If side BC of the triangle lies on line $y - 2 = 0$ and internal angles are $\angle A=\alpha$, $\angle B=\beta$, $\angle C=\gamma$, then match the following columns (I) and (II).

Column (I)	Column (II)
(a) $(AC_0)\cos\alpha$	(p) $\sec\gamma$
(b) HB	(q) 2
(c) HA	(r) 4
(d) HC	(s) $\sec\beta$
	(t) 1

16. In triangle ABC , let CH and CM be the lengths of the altitude and median to base AB. If side lengths $a=5$, $b=\sqrt{97}$ and $c=12$, then match the following columns I and II.

Column (I)	Column (II)
(a) Value of $\cos(\tan^{-1}(\sqrt{MH}))$ is	(p) 2
(b) Length of in-radius of triangle MHC is	(q) 1
(c) If BC is extended to P such that triangle APB is right angled at P , and area of ΔAPC is 'δ' square units ,	(r) 5
then integer(s) less than $\left(\frac{\delta}{MH}\right)$ can be	(s) 3
(d) If $\angle APH=\theta$, then value of $\tan\theta$ is more than	(t) 1/2

ANSWERS Exercise No. (1)

1. (c)
2. (b)
3. (c)
4. (b)
5. (d)
6. (c)
7. (a)
8. (a)
9. (a)
10. (a)
11. (d)
12. (b)
13. (c)
14. (a)
15. (a)
16. (b)
17. (a)
18. (a)
19. (b)
20. (c)
21. (a , b , c , d)
22. (a , d)
23. (a , c)
24. (a , b , c , d)
25. (b , c , d)
26. (c)
27. (a)
28. (b)
29. (a)
30. (d)

ANSWERS Exercise No. (2)

1. (b)
2. (a)
3. (b)
4. (d)
5. (b)
6. (b)
7. (b)
8. (d)
9. (b)
10. (9)
11. (6)
12. (4)
13. (4)
14. (2)

15. (a) → t
(b) → p
(c) → q
(d) → s

16. (a) → t
(b) → q
(c) → p , q , s
(d) → p , q , t

Chapter

34

Inverse Trigonometric Functions

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-25)

1. If $1 < x < \sqrt{2}$, then number of solutions of equation $\tan^{-1}(x-1)+\tan^{-1}(x)+\tan^{-1}(x+1)=\tan^{-1}(3x)$ is :

(a) 0 (b) 1

(c) 2 (d) 3

2. If $\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right)=\tan^{-1}x$, then x is equal to :

(a) $\tan 3\theta$ (b) $3\tan\theta$

(c) $\frac{1}{3}\tan\theta$ (d) $3\cot\ \theta$

3. If $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+........n$ terms is equal to $\tan^{-1}(\theta)$, then θ is equal to :

(a) $\frac{n}{n+1}$ (b) $\frac{n+1}{n+2}$

(c) $\frac{n}{n+2}$ (d) $\frac{n-1}{n+2}$

4. A root of the quadratic equation $17x^2+17x\,\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)-10=0$ is :

(a) $\frac{10}{17}$ (b) -1

(c) $-\frac{7}{17}$ (d) 1

5. The value of $\left\{\sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)+\cos\left(\tan^{-1}\left(2\sqrt{2}\right)\right)\right\}$ is :

(a) $\frac{14}{13}$ (b) $\frac{14}{15}$

(c) $\frac{15}{7}$ (d) $\frac{1}{2}$

6. If $4\sin^{-1}(x)+\cos^{-1}(x)=\pi$, then x is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

7. Sum of infinite series :

$\cot^{-1}(2)+\cot^{-1}(8)+\cot^{-1}(18)+\cot^{-1}(32)+........$

is equal to :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

8. Which one of the following is equivalent to $2\tan^{-1}(-3)$?

(a) $\pi+\cos^{-1}\left(\frac{4}{5}\right)$ (b) $-\frac{\pi}{2}+\tan^{-1}\left(-\frac{4}{3}\right)$

(c) $\frac{\pi}{2}+\sin^{-1}\left(\frac{3}{5}\right)$ (d) $-\frac{\pi}{2}+\tan^{-1}\left(\frac{4}{3}\right)$

9. The principal value of $\sin^{-1}(\sin 10)-\cos^{-1}(\cos 5)$ is :

(a) $\pi+5$ (b) $25+\pi$

(c) $\pi-5$ (d) $2\pi-10$

10. Complete solution set of $\sin^{-1}x\le\cos^{-1}x$ is :

(a) $x\in\left[\frac{1}{\sqrt{2}},1\right]$ (b) $x\in\left[-\frac{1}{\sqrt{2}},1\right]$

(c) $x\in\left[-1,\frac{1}{\sqrt{2}}\right]$ (d) $x\in\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$

11. If $3\sin^{-1}x=-\pi-\sin^{-1}(3x-4x^3)$, then

(a) $x\in\left[-1,-\frac{1}{2}\right]$ (b) $x\in\left[\frac{1}{2},1\right]$

(c) $|x|\le 1$ (d) none of these

12. If $\cot^{-1}x+\cot^{-1}y+\cot^{-1}z=\dfrac{\pi}{2}$, then $(x+y+z)$ is :

(a) $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}$ (b) xyz

(c) $xy+yz+zx$ (d) $\dfrac{xyz}{x+y+z}$

13. The value of $\cos^{-1}\left(\sqrt{\dfrac{2}{3}}\right)-\cos^{-1}\left(\dfrac{\sqrt{6}+1}{2\sqrt{3}}\right)$ is :

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{6}$

14. If $x\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$, then value of the summation

$\tan^{-1}\left(\dfrac{\tan x}{4}\right)+\tan^{-1}\left(\dfrac{3\sin 2x}{5+3\cos 2x}\right)$ is :

(a) $\dfrac{x}{2}$ (b) $2x$

(c) $3x$ (d) x

15. If $x_1=2\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$; $x_2=\sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$, where $x\in(0\,,1)$, then (x_1+x_2) is equal to :

(a) 0 (b) 2π

(c) π (d) $-\pi$

16. $\cos^{-1}\left(\cos(2\cot^{-1}(\sqrt{2}-1))\right)$ is equal to :

(a) $\sqrt{2}-1$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{3\pi}{4}$ (d) $\dfrac{\pi}{8}$

17. The maximum value of $(\sec^{-1}x)^2+(\operatorname{cosec}^{-1}x)^2$ is :

(a) $\dfrac{\pi^2}{2}$ (b) $\dfrac{\pi^2}{4}$

(c) π^2 (d) none of these

18. Range of $f(x)=\sin^{-1}x+\tan^{-1}x+\sec^{-1}x$ is :

(a) $\left\{\dfrac{\pi}{4},\dfrac{3\pi}{4}\right\}$ (b) $\left[\dfrac{\pi}{4},\dfrac{3\pi}{4}\right]$

(c) $\left(-\dfrac{\pi}{4},0\right)$ (d) none of these

19. The value of $\sin^{-1}(\sin 12)+\cos^{-1}(\cos 12)$ is equal to :

(a) 0 (b) $24-2\pi$

(c) $4\pi-24$ (d) none of these

20. If $\sin^{-1}\left(x-\dfrac{x^2}{2}+\dfrac{x^3}{4}-\ldots\right)+\cos^{-1}\left(x^2-\dfrac{x^4}{2}+\dfrac{x^6}{4}-\ldots\right)=\dfrac{\pi}{2}$,

for $0<|x|<\sqrt{2}$, then x equals to :

(a) 1/2 (b) 1

(b) –1/2 (d) –1

21. If $x\in\left(\dfrac{\pi}{2},\pi\right)$, then value of the expression

$\sin^{-1}\left(\cos(\cos^{-1}(\cos x)+\sin^{-1}(\sin x))\right)$ is equal to :

(a) $\dfrac{\pi}{2}$ (b) $-\pi$

(c) π (d) $-\dfrac{\pi}{2}$

22. Complete solution set of $\tan^2(\sin^{-1}x)>1$ is :

(a) $\left(-1\,,-\dfrac{1}{\sqrt{2}}\right)\cup\left(\dfrac{1}{\sqrt{2}}\,,1\right)$

(b) $\left(-\dfrac{1}{\sqrt{2}}\,,\dfrac{1}{\sqrt{2}}\right)-\{0\}$

(c) $(-1\,,1)-\{0\}$

(d) none of these

23. The value of $\sin\left(\dfrac{1}{4}\sin^{-1}\left(\dfrac{\sqrt{63}}{8}\right)\right)$ is :

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{1}{\sqrt{10}}$

(c) $\dfrac{1}{\sqrt{8}}$ (d) $\dfrac{1}{3\sqrt{3}}$

24. If $x\in\left(\pi,\dfrac{3\pi}{2}\right)$, then $\tan^{-1}\left\{\dfrac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right\}$

is :

(a) $\dfrac{\pi}{4}+\dfrac{x}{2}$ (b) $-\dfrac{x}{2}$

(c) $\dfrac{\pi}{4}-\dfrac{x}{2}$ (d) $\dfrac{\pi}{2}-x$

25. If $x \in \left(0\,,\,\frac{\pi}{4}\right)$, then the value of summation

$$\tan^{-1}\left(\frac{1}{2}\tan 2x\right)+\tan^{-1}(\cot x)+\tan^{-1}(\cot^3 x)$$ is :

(a) 0 (b) π

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. Let $\sin(2\cos^{-1}\{\cot(2\tan^{-1}\alpha)\})=0$, then possible values of $'\alpha'$ can be :

(a) $\sqrt{2}+1$ (b) $2+\sqrt{3}$

(c) $sgn(\pi)$ (d) $1-\sqrt{2}$

27. Let the equation $\sin^{-1}(x)-|x-\alpha|=0$ is having at least one real solution , then possible values of $'\alpha'$ can be :

(a) $\tan^{-1}(\tan 3)$ (b) $\cos^{-1}(\cos 2)$

(c) $\sin^{-1}(\sin 4)$ (d) $\mathrm{cosec}^{-1}(\mathrm{cosec}\,7)$

28. Let the system of equations $\cos^{-1}x+(\sin^{-1}y)^2=\frac{n\pi^2}{4}$ and $(\sin^{-1}y)^2-\cos^{-1}x=\frac{\pi^2}{16}$ be consistent , where $n \in R$, then :

(a) Least positive integral value of n is 2.

(b) Greatest positive integral value of k , where $k=4n$, is 7.

(c) Possible number of integral values of $2n$ are 3.

(d) Least positive integral value of n is 1.

29. If $[\alpha]$ represents the greatest integer just less than or equal to α , then solution set of the equation $\left[\cot^{-1}x\right]+2\left[\tan^{-1}x\right]=0$ contains :

(a) $\left[\frac{3}{4}\,,\,\frac{5}{4}\right]$ (b) $(\cot 1\,,1)$

(c) $(1\,,\,\tan 1]$ (d) $[\sin 1\,,\,\sin 2]$

30. Let $P(x\,,y)$ satisfy the equation

$$\cos^{-1}(axy)+\cos^{-1}(y)-\cos^{-1}(bx)=0.$$

If $a=0$ and $b=1$ then P lies on curve C_1. For curve C_1 which of the following statements are correct :

(a) C_1 passes through origin and have constant slope of $sgn(e)$.

(b) all points on C_1 are equidistant from origin.

(c) C_1 bounds a region of π square unit area.

(d) C_1 bounds a region of $\frac{\pi}{4}$ square units with coordinate axes.

Assertion Reasoning questions : (Questions No. 31-35)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

31. **Statement 1 :** Sum of the infinite series :

$$S=\left\{\cot^{-1}(3)+\cot^{-1}\left(\frac{9}{2}\right)+\cot^{-1}\left(\frac{33}{4}\right)+\cot^{-1}\left(\frac{129}{8}\right)+\ldots\ldots..\right\}$$

is equal to $\frac{\pi}{4}$

because

Statement 2 : If $S_n=\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$, then $S_n=\left(\tan^{-1}\left(2^{n-1}\right)-\frac{\pi}{4}\right)$, and hence $\lim_{n\to\infty}S_n=\frac{\pi}{4}$.

32. Let $p=\cot\left[\frac{1}{2}\sin^{-1}\left\{\cos(3\tan^{-1}(\sqrt{3}+2))\right\}\right]$ and $q=\tan\left[\frac{1}{2}\cos^{-1}\left\{\cos(2\cot^{-1}(\sqrt{2}-1))\right\}\right]$, then

Statement 1 : $p+q=0$

because

Statement 2 : $p=(\sqrt{2}+1)$ and $q=-(\sqrt{2}+1)$.

33. **Statement 1 :** If $\left[\cos^{-1}x\right]+\left[\cot^{-1}x\right]\le\left[\sin^{2}x\right]$, where [.] represents the greatest integer function , then exhaustive set of values of $'x'$ is (cot 1 , 1]

because

Statement 2 : $\left[\sin^{2}x\right]=0\ \forall\ |x|\le 1.$

34. Consider the ordered pairs $(x\ ,\ y)$ satisfying the conditions $|y| - \cos x = 0$ and $y = \sin^{-1}(\sin x)$.

Statement 1 : If $x\in[-\pi\ ,\ 3\pi]$, then four ordered pairs of $(x\ ,\ y)$ exist

because

Statement 2 : $|y| = \cos x$ and $y^2 - x^2 = 0$ intersects at four distinct points.

35. Consider a triangle ABC , where $\angle B = 90^{\circ}$, and

$$M = \tan^{-1}\left(\frac{a}{b+c}\right)+\tan^{-1}\left(\frac{c}{a+b}\right).$$

Statement 1 : Value of $\cot\left(\frac{M}{3}\right)=\sqrt{3}+2$

because

Statement 2 : Value of M is 45°.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Consider the functions $f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $g(x) = (x-1)^2 + k$ for all $x \in R$, where $'k'$ is a parameter. On the basis of definitions of $f(x)$ and $g(x)$ answer the following questions.

1. If [.] represents the greatest integer function , and α , β are the maximum and minimum values respectively of $y=[f(x)]$, then $(\alpha-\beta)$ is equal to :

(a) 7 (b) 4

(c) 3 (d) 2

2. If the equation $f(x)-g(x)=0$ is having at least one real solution then complete set of values of k is :

(a) $\left(-\infty , \frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) $\left[-\frac{\pi}{2}, \infty\right)$ (d) $(-\infty , \infty)$

3. Number of values of x satisfying the equation $(\tan^{-1}x)^2 + f^2(x) = 2f(x)(\tan^{-1}x)$ is/are :

(a) 0 (b) 1

(c) 3 (d) 4

Comprehension passage (2)
(Questions No. 4-6)

Let P and Q be the positive integral ordered pairs of (x , y) , where $x < y$, which satisfy the equation $\tan^{-1}(x)+\cos^{-1}\left(\frac{y}{\sqrt{1+y^2}}\right)=\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$.

On $x-y$ plane if $OP < OQ$, where $'O'$ is origin , then answer the following questions.

4. Let points $'R'$ and $'S'$ be the reflection of $'P'$ and $'Q'$ respectively about the line mirror $y-x=0$, then area (in square units) of the quadrilateral $PRSQ$ is equal to :

(a) 8 (b) 10

(c) 18 (d) 4

5. Let points $'P'$ and $'Q'$ be (a , b) and (c , d) respectively , where $f:[a , c]\rightarrow[b , d]$ is linear function which is surjective in nature , then $f(x)$ can be :

(a) $2x-3$ (b) $5x-2$

(c) $12-5x$ (d) $6-2x$

6. Diametric length of circle passing through $'P'$ and $'Q'$ and orthogonal to $x^2+y^2=10$, is :

(a) $\sqrt{130}$ (b) $\sqrt{150}$

(c) $\sqrt{105}$ (d) 10

Questions with Integral Answer :
(Questions No. 7-10)

7. Let $\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{2r^2}\right)=\alpha$, then the least integer just greater than the value of $\cot\left(\frac{\alpha}{3}\right)$ is equal to ....

8. Let the equation $(\sin^{-1}x)^3+(\cos^{-1}x)^3=\frac{p\pi^3}{8}$ is having real solution of x , where $p \in I$, then total number of possible values of p are ....

9. If $\frac{1}{\pi}\cos^{-1}(\cos x)=\left|\log_{12}|x|\right|$, then number of solutions of $'x'$ is/are ....

10. Let $u=\tan\left(2\tan^{-1}(\sqrt{2}-1)+\frac{1}{2}\cos^{-1}\left(\frac{1}{4}\right)\right)$ and $v=\tan\left(3\tan^{-1}(2-\sqrt{3})-\frac{1}{2}\cos^{-1}\left(\frac{1}{4}\right)\right)$, then value of $(u + v)$ is equal to ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) If $n\pi - \tan^{-1}(3)$ is *a* solution of the equation $12\tan 2x + \sqrt{10}\sec x + 1 = 0$, then value of n can be	(p) 1
	(q) 2
(b) If $\cot^{-1}\left(\dfrac{n^2 - 10n + 7\pi}{\pi}\right) > \dfrac{\pi}{4}$, then value of n can be	(r) 3
(c) Value of $\tan^{-1}(\tan 3) + \sin^{-1}(\sin 2)$ is	(s) 4
(d) Maximum value of $\dfrac{3}{\pi}.\sec^{-1}\left(\dfrac{7-5(3+x^2)}{2(2+x^2)}\right)$ is less than	(t) 5

12. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) If $x \in (-\infty, 0)$, then value of $2\tan^{-1}x + \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ is :	(p) 0
(b) If $x \in \left(\dfrac{\sqrt{3}}{2}, 1\right)$, then value of $2\sin^{-1}x + \sin^{-1}(2x\sqrt{1-x^2})$ is :	(q) π
(c) If $x \in (-\pi, -e)$, then value of $\tan^{-1}\left(\dfrac{1}{x}\right) - \cot^{-1}(x)$ is :	(r) $-\pi$
(d) If $x \in (1, \infty)$, then value of $3\tan^{-1}x - \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$ is :	(s) 2π
	(t) -2π

ANSWERS Exercise No. (1)

1. (a) **2.** (c) **3.** (c) **4.** (d) **5.** (b)

6. (a) **7.** (c) **8.** (b) **9.** (c) **10.** (c)

11. (a) **12.** (b) **13.** (d) **14.** (d) **15.** (c)

16. (c) **17.** (d) **18.** (a) **19.** (a) **20.** (b)

21. (d) **22.** (a) **23.** (c) **24.** (c) **25.** (b)

26. (a , c , d) **27.** (a , b , d) **28.** (b , c , d) **29.** (a , b , d) **30.** (b , d)

31. (c) **32.** (c) **33.** (b) **34.** (b) **35.** (a)

ANSWERS Exercise No. (2)

1. (c) **2.** (a) **3.** (c) **4.** (c) **5.** (c)

6. (a) **7.** (4) **8.** (7) **9.** (8) **10.** (8)

11. (a) → p , r , t
(b) → r , s , t
(c) → p
(d) → r , s , t

12. (a) → p
(b) → q
(c) → r
(d) → q